Biomechanics of Human Movement

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Introduction to Open Textbooks

This open access textbook was developed as an introductory resource to introduce basic concepts related to human biomechanics. It was edited and developed for students from Douglas College enrolled in Biomechanics (SPSC 1151). This book is best viewed online using the pressbooks format however, multiple formats (e.g., pdf, epub, mobi) are also made available.

A free textbook is great, but it can be even better with your help. please contact me if you find any errors or typos in the book.

PART I

CHAPTER 1: PREREQUISITE SKILLS FOR BIOMECHANICS

Chapter Objectives

After this chapter, you will be able to:

- List the pre-requisite skills for success in Biomechanics
- Define the field of Biomechanics
- Solve basic equations and algebra problems
- Understand the use of different units to quantify physical variables
- Convert values between commonly used units in Biomechanics
- Round your answers to two decimal places
- Graph simple sets of data in two-dimensions

In Biomechanics, we use principles of physics to quantify and understand human movement. In order to apply the concepts of physics, or more specifically, mechanics, students must have a mastery of certain skills such as trigonometry, algebra and graphing. In this chapter, we review some of these concepts.

1. 1.0 Introduction to Biomechanics

What is your first reaction when you hear the word “mechanics”? Do you imagine working through difficult equations or formulas that seem to have no real use in life outside the classroom? Many people come to the subject of biomechanics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that principles of mechanics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, think about how you walked to work or school this morning. You applied a force to the ground and the ground exerted a force back on your body, propelling you forwards. Aside from moving through the environment, professionals such as engineers, physicians, physical therapists and computer programmers apply biomechanics concepts in their daily work. For example, a physical therapist must understand how the muscles in the body experience forces as they move and bend. They must understand anatomy and the the effects of forces on the structures to understand the mechanism of injury and recovery.

Before we go any further, let’s define ‘**Biomechanics**‘. You can separate the word into two parts: ‘Bio’ which suggests that Biomechanics involves living or biological systems, and ‘Mechanics’ which suggests the analysis of forces and their effects. Biomechanics is the study of structure and function of biological systems by the means of mechanics (Hatze, 1974). The goal of biomechanics related to human movement is to improve physical performance (through improved technique, equipment or training) and injury prevention and rehabilitation.

Mechanics is a branch of physics concerned with the effects of forces acting on bodies. It can be divided into several branches including rigid-body mechanics, deformable body mechanics and fluid mechanics. This course will focus mostly on **rigid-body mechanics** since we make the assumption that the body is made of rigid segments linked together at the joints. In reality, these segments do deform under the action of forces but these deformation are considered negligible. You can further subdivide rigid-body mechanics into **statics** (the mechanics of bodies at rest or moving at a constant velocity) and **dynamics** (the mechanics of bodies under acceleration). Dynamics will be discussed in terms of **kinematics** and **kinetics.**

Kinematics is a branch of study focused on the description of motion (how high, how far, how fast!) and kinetics is a branch focused on the explanation of motion (the forces that cause of tend to cause changes in motion). This book with cover both the kinematics and the kinetics of angular and linear movements.

2. 1.1 Understanding Equations and Basic Math

Mathematics can sometimes feel like a different language. With enough practice, you can become fluent in the language of numbers. Let's review the concept of 'equation'. An equation says that two 'things' are equal with the use of an equal sign '='. For example:

$$\mathbf{x + 2 = 6}$$

That equation says: what is on the left (x+2) is equal to what is on the right (6) of the equal sign. This can be a powerful tool as we aim to understand human movement with data (numbers) collected. The equations may get a bit more complicated but the rules remain the same.

A formula is a 'rule' that use mathematical symbols. It usually consists of an equal sign and two or more variables. For example. the formula for force (as you will see later in the course) is:

$$\mathbf{F = ma}$$

This can be stated as: the force acting on an object is equal to the mass of the object multiplied by the acceleration of the object. It can be convenient with formulas to have basic algebra skills. In algebra, the goal is to get the letter or symbol (also called the unknown) on one side of the equation (usually the left) and the numbers on the other side.

The golden rule of algebra is: anything you do on one side of the equation, you must do on the other side. For example, if you want to add 10 from one side of the equation, you must add 10 from the other.

$$3x - 10 = 11$$
$$3x - 10\ (+\ 10) = 11\ (+\ 10)$$
$$3x = 21$$
$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$

In cases where the variable shows up twice in the equation, you should try to get all of the variables on one side of the equation and all of the numbers on the other:

x + 23 = 3x +45

1. Initial Equation / Problem	x + 23	=	3x + 45
2. Subtract x from each side	x – x + 23	=	3x – x + 45
Result	23	=	2x + 45
3. Subtract 45 from each side	23 – 45	=	2x + 45 – 45
Result	-22	=	2x
4. Divide both sides by 2	-22	=	2x
Result	x	=	-11

To become proficient in algebra, you should practice.

A note on the symbols used in this textbook

You'll notice that we used letters to symbolize a variable. For example, the value for 'force' will feature as 'F' in an equation. Although some symbols are universal ('m' for mass and 'a' for acceleration), some are not. Physics, engineering and biomechanics may use different symbols for the same thing (both p and L can be used for momentum). Sometimes, the same symbol can have different meanings in the same field. For example, a capital W can be used for both 'weight' and 'work'. You'll have to take great care in understanding the meaning of each variable based on the context they are presented in.

A symbol can have up to four parts: the main variable, a leading superscript, a following superscript and a following subscript:

$$^{x}p'_{1}$$

The main variable (p in the example above) represents the variable you are quantifying. The leading superscript (x) let's the reader know the frame of reference. X represents the horizontal axis and y the vertical axis. We'll discuss this in detail later in the book. The following superscript is important if you are keeping track of the variable over time. Time zero does not have a superscript, time at point 1 has a single prime, time at point 2 has a double prime and so on… The following subscript is important if you are quantifying the variable for more than one body. If you have two runners and you are reporting both their momentum, person 1 would have a moment p_1 and person 2, p_2.

3. 1.2 Physical Quantities and Units.

Mechanics is a quantitative science which means we will describe human movement and its causes using numbers. To provide information about a movement, we have to be able to specify how it is measured. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See Figure 1 below.)

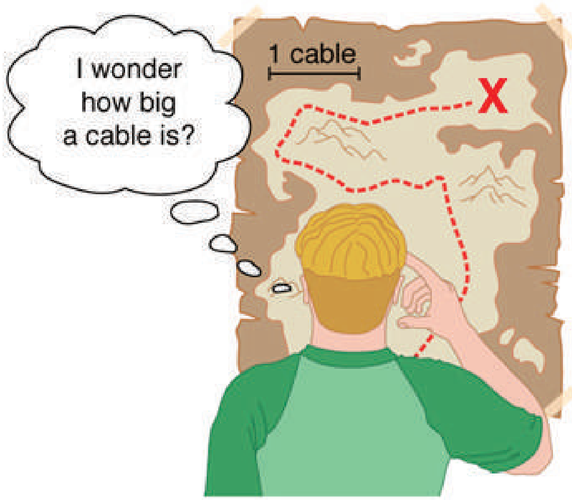


Figure 1. Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

The metric or SI system is administered in France by the Bureau International des Poids and Mesures or BIPM. You can read more about them at https://www.bipm.org/en/about-us/

Table 1 below shows the fundamental SI units that are used throughout this textbook.

Length	**Mass**	**Time**
meter (m)	kilogram (kg)	second (s)

Table 1. Fundamental SI Units.

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass and time. All other physical quantities, such as force and velocity, can be expressed as algebraic combinations of length, mass and time; these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the second (abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation).

The Meter

The SI unit for length is the meter (abbreviated m); its definition has also changed over time to become more accurate and precise. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

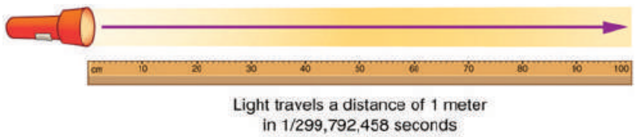


Figure 2. *The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.*

The Kilogram

The SI unit for mass is the kilogram (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris.

In Biomechanics, all pertinent physical quantities can be expressed in terms of these fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. The table below gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on.

Prefix	Symbol	Value	Example (some are approximate)			
kilo	**k**	**10^3**	**kilometer**	km	10^3 m	m
hecto	h	10^2	hectoliter	hL	10^2 L	
deka	da	10^1	dekagram	dag	10^1 g	of
—	—	**10^0** (=1)	**meter**			
deci	d	10^{-1}	deciliter	dL	10^{-1} L	ha

Prefix	Symbol	Value	Example (some are approximate)			
centi	**c**	**10^{-2}**	**centimeter**	cm	10^{-2} m	th
milli	**m**	**10^{-3}**	**millimeter**	mm	10^{-3} m	sh

Table 2. Select Metric Prefixes for Powers of 10 and their Symbols.

Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.

- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of three fundamental units.
- The three fundamental units we will use in

this text are the meter (for length), the kilogram (for mass) and the second (for time). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.

- The three fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.

Glossary

physical quantity
a characteristic or property of an object that can be measure or calculated from other measurements

units
a standard used for expressing and comparing measurements

SI units
the international system of units that scientist in most countries have agreed to use; includes units such as meters, liters, and grams

English units
system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units
units that can only be expressed relative to the procedure used to measure them

derived units
units that can be calculated using algebraic combinations of the fundamental units

second
the SI unit for time, abbreviated (s)

meter
the SI unit for length, abbreviated (m)

kilogram
the SI unit for mass, abbreviated (kg)

metric system
a system in which values can be calculated in factors of 10

4. 1.3 Converting Units

It is sometimes important to convert between different types of units. Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$\mathbf{80\ \cancel{m}} \times \frac{\mathbf{1\ km}}{\mathbf{1000\ \cancel{m}}} = \mathbf{0.080\ km}$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Here's another way to think about it:

Consider a simple example: how many cm are there in 4 meters? You may simply think there are 400cm in 4 meters. How did you make this determination? Well, if there are 100 cm in 1 m and there are 4 meters, then there are 4 × 100 = 400cm in 4 meters.

This is correct, of course, but it is informal. Let us formalize it in a way that can be applied more generally. We know that 1 m equals 100 cm:

1 m = 100 cm

In math, this expression is called an *equality*. The rules of algebra say that you can change (i.e., multiply or divide or add or subtract) the equality (as long as you don't divide by zero) and the new expression will still be an equality. For example, if we divide both sides by 2, we get

1/2 m = 100/2 cm

We see that one-half of a meter equals 100/2, or fifty cm—something we also know to be true, so the above equation is still an equality. Going back to the original equality, suppose we divide both sides of the equation by 1 meter (number *and* unit):

1/1m = 100cm/1m

The expression is still an equality, by the rules of algebra. The left fraction equals 1. It has the same quantity in the numerator and the denominator, so it must equal 1. The quantities in the numerator and denominator cancel, both the number *and* the unit. When everything cancels in a fraction, the fraction reduces to 1:

1 = 100cm/1m

We have an expression, 100 cm / 1m, that equals 1. This is a strange way to write 1, but it makes sense: 100 cm equal 1 m, so the quantities in the numerator and denominator are the same quantity, just expressed with different units. The expression 100 cm / 1m is called a **conversion factor**, and it is used to formally change the unit of a quantity into another unit. (The process of converting units in such a formal fashion is sometimes called *dimensional analysis* or the *factor label method*.)

To see how this happens, let us start with the original quantity:

4 m

Now let us multiply this quantity by 1. When you multiply anything by 1, you don't change the value of the quantity. Rather than multiplying by just 1, let us write 1 as 100cm/1m:

4 m X 100cm/1m

The 4 m term can be thought of as 400cm/1; that is, it can be thought of as a fraction with 1 in the denominator. We are essentially multiplying fractions. If the same thing appears in the numerator and denominator of a fraction, they cancel. In this case, what cancels is the unit *meter:*

400cm

That is all that we can cancel. Now, multiply and divide all the numbers to get the final answer:

Again, we get an answer of 400 cm, just as we did originally. But in this case, we used a more formal procedure that is applicable to a variety of problems.

Want more examples?

How many millimeters are in 14.66 m? To answer this, we need to construct a conversion factor between millimeters and meters and apply it correctly to the original quantity. We start with the definition of a millimeter, which is

1 mm = 1/1,000 m

The 1/1,000 is what the prefix *milli-* means. Most people are more comfortable working without fractions, so we will rewrite this equation by bringing the 1,000 into the numerator of the other side of the equation:

1,000 mm = 1 m

Now we construct a conversion factor by dividing one quantity into both sides. But now a question arises: which quantity do we divide by? It turns out that we have two choices, and the two choices will give us different conversion factors, both of which equal 1:

$$\frac{1000\,mm}{1000\,mm} = \frac{1\,m}{1000\,mm} \quad or \quad \frac{1000\,mm}{1\,m} = \frac{1\,m}{1\,m}$$

$$1 = \frac{1\,m}{1000\,mm} \quad or \quad \frac{1000\,mm}{1\,m} = 1$$

Which conversion factor do we use? The answer is based on *what unit you want to get rid of in your initial quantity*. The original unit of our quantity is meters, which we want to convert to millimeters. Because the original unit is assumed to be in the numerator, to get rid of it, we want the meter unit in the *denominator*; then they will cancel. Therefore, we will use the second conversion factor. Canceling units and performing the mathematics, we get

$$14.66\,\cancel{m} \times \frac{1000\,mm}{1\,\cancel{m}} = 14660\,mm$$

Note how m cancels, leaving mm, which is the unit of interest.

The ability to construct and apply proper conversion factors is a very powerful mathematical technique in biomechanics. You need to master this technique if you are going to be successful in this and future courses.

Applied Example: Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time

of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\textbf{average speed} = \frac{\textbf{distance}}{\textbf{time}}.$$

(2) Substitute the given values for distance and time.

$$\textbf{average speed} = \frac{\textbf{10.0 km}}{\textbf{20.0 min}} = \textbf{0.500}\frac{\textbf{km}}{\textbf{min}}$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,

$$\textbf{average speed} = \textbf{0.500}\frac{\textbf{km}}{\textbf{min}} \times \frac{\textbf{60 min}}{\textbf{1 h}} = \textbf{30.0}\frac{\textbf{km}}{\textbf{h}}.$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion.

If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\mathbf{km}}{\mathbf{min}} \times \frac{\mathbf{1\ hr}}{\mathbf{60\ min}} = \frac{\mathbf{1\ km \cdot hr}}{\mathbf{60\ min^2}},$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Round to two decimal places. The answer can be left as 30 km/h since it is a whole value.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\textbf{Average speed} = \mathbf{30.0\frac{km}{h}} \times \mathbf{\frac{1\ h}{3,600\ s}} \times \mathbf{\frac{1,000\ m}{1\ km}},$$

$$\textbf{Average speed} = \mathbf{8.33\frac{m}{s}}.$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

What if we have a derived unit that is the product of more than one unit, such as m^2? Suppose we want to convert square meters to square centimeters? The key is to remember that m^2 means m × m, which means we have *two* meter units in our derived unit. That means we have to include *two* conversion factors, one for each unit. For example, to convert 17.6 m^2 to square centimeters, we perform the conversion as follows:

$$17.6\, m^2 = 17.6\, (\cancel{m} \times \cancel{m}) \times \frac{100\, cm}{1\, \cancel{m}} \times \frac{100\, cm}{1\, \cancel{m}} = 176000\, cm \times cm = 1.76 \times 10^5\, cm^2$$

Practice Problem

How many cubic centimeters are in 0.883 m^3?

Solution

With an exponent of 3, we have three length units, so by extension we need to use three conversion factors between meters and centimeters. Thus, we have

$$0.833\, \cancel{m^3} \times \frac{100\, cm}{1\, \cancel{m}} \times \frac{100\, cm}{1\, \cancel{m}} \times \frac{100\, cm}{1\, \cancel{m}} = 883000\, cm^3 = 8.83 \times 10^5\, cm^3$$

You should demonstrate to yourself that the three meter units do indeed cancel.

Suppose the unit you want to convert is in the denominator of a derived unit; what then? Then, in the conversion factor, the unit you want to remove must be in the *numerator*. This will cancel with the original unit in the denominator and introduce a new unit in the denominator. The following example illustrates this situation.

Practice Problem

Convert 88.4 m/min to meters/second.

Solution

We want to change the unit in the denominator from minutes to seconds. Because there are 60 seconds in 1 minute (60 s = 1 min), we construct a conversion factor so that the unit we want to remove, minutes, is in the numerator: 1 min/60 s. Apply and perform the math:

$$\frac{88.4\ m}{\cancel{min}} \times \frac{1\ \cancel{min}}{60\ s} = 1.47\ m/s$$

Notice how the 88.4 automatically goes in the numerator. That's because any number can be thought of as being in the numerator of a fraction divided by 1.

Sometimes there will be a need to convert from one unit with one numerical prefix to another unit with a different numerical prefix. How do we handle those conversions? Well, you could memorize the conversion factors that interrelate all numerical prefixes. Or you can go the easier route: first convert the quantity to the base unit, the unit with no numerical prefix, using the definition of the original prefix. Then convert the quantity in the base unit to the desired unit using the definition of the second prefix. You can do the conversion in two separate steps or as one long algebraic step. For example, to convert 2.77 kg to milligrams:

$$2.77\ \cancel{kg} \times \frac{1000\ g}{1\ \cancel{kg}} = 2770\ g\ (convert\ to\ the\ base\ unit\ of\ grams)$$

$$2770\ \cancel{g} \times \frac{1000\ mg}{1\ \cancel{g}} = 2770000\ mg = 2.77 \times 10^6\ mg\ (convert\ to\ the\ desired\ unit)$$

Alternatively, it can be done in a single multistep process:

$$2.77\ \cancel{kg} \times \frac{1000\ \cancel{g}}{1\ \cancel{kg}} \times \frac{1000\ mg}{1\ \cancel{g}} = 2770000\ mg = 2.77 \times 10^6\ mg$$

You get the same answer either way.

How do I round my final answer?

For the purpose of this class, you can round your answer to two decimal places. For example, if your final answer of the force exerted during an Olympic deadlift is 783.4723 N, you can round to 783.47 N.

The third number after the decimal will decide how to proceed:

1. If the number you are rounding is followed by 5, 6, 7, 8, or 9, round the number up. Example: 38.345 will round to 38.35.
2. If the number you are rounding is followed by 0, 1, 2, 3, or 4, round the number down. Example: 38.342 will round to 38.34.
3. If the final answer is a whole number: 38.00000 you can write your answer as 38.

Key Takeaways

- Units can be converted to other units using the proper conversion factors.
- Conversion factors are constructed from equalities that relate two different units.
- Conversions can be a single step or multistep.
- Unit conversion is a powerful mathematical technique in biomechanics that must be mastered.
- In this course, you can round your answer to two decimal places.

5. 1.4 Accuracy and Precision of Measurements

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of a long jump. The jump was 7.2 m long. You measure the length of the jump three times and obtain the following measurements: 7.1 m., 7.3 m., and 7.2 m. These measurements are quite accurate because they are very close to the correct value of 7.2 m. In contrast, if you had obtained a measurement of 8 m, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the long jump measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 7.1 m. and the highest value was 7.3 m. Thus, the measured values deviated from each other by at most 0.2 m. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 7.1, 7.3, and 7.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the long jump example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a runner in a city. Think of the runners location as existing at the centre of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In Figure 1 you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the runner at the centre of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 2 the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

Figure 1. *A GPS system attempts to locate a runner at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the runner. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the runnert, indicating high accuracy. (credit: Dark Evil).*

Figure 2. *In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the runner, indicating low accuracy. (credit: Dark Evil).*

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the estimated time you will take to complete a 50 km trail race, you might say that it will take you 8 hours, plus or minus 30 minutes. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual time it may take you to complete the race might be as low as 7 and a half hours or as high as 8 and a half hours, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the long jump, we might say that the length of the jump is 7.2 m., plus or minus 0.1 m. The uncertainty in a measurement, **A**, is often denoted as **δA** ("delta **A**"), so the measurement result would be recorded as **A** ± **δA**. In our paper example, the length of the jump could be expressed as 7.2 m. ± 0.1.

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object/body being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 m. or the person using the ruler has bad eyesight.. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement **A** is expressed with uncertainty, **δA**, the **percent uncertainty** (%unc) is defined to be:

$$\% \text{ unc} = \frac{\delta \mathbf{A}}{\mathbf{A}} \times 100\%$$

Example 1: Calculating Percent Uncertainty: Angle of Take-off

You were told that to achieve maximum distance in your long jump, you should take-off at an angle of 45 degrees. You jump four times attempting to take-off at the optimal angle and measure the angle of take-off each time manually with a protractor. You obtain the following measurements:

- Jump 1 angle: **50 degrees**
- Jump 2 angle: **65 degrees**
- Jump 3 angle: **40 degrees**
- Jump 4 angle: **25 degrees**

You determine that the the average angle of take-off you manage to complete is 45 degrees ±20. What is the percent uncertainty of your take-off angle when using a protractor?

Strategy

First, observe that the expected value of the take-off angle, **A**, is 45 degrees. The uncertainty in this value, **δA**, is 20 degrees. We can use the following equation to determine the percent uncertainty of the weight:

$$\mathbf{\% \ unc} = \frac{\boldsymbol{\delta} \mathbf{A}}{\mathbf{A}} \times \mathbf{100\%}$$

Solution

Plug the known values into the equation:

$$\mathbf{\% \ unc} = \frac{\mathbf{20 \ lb}}{\mathbf{45 \ lb}} \times \mathbf{100\%} = \mathbf{44.4\%}$$

Discussion

We can conclude that the take-off angle is 45 degree ±44.4%. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Check Your Understanding 1

1: A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of ±0.05 s. Runners on the track coach's team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.

Glossary

accuracy
: the degree to which a measure value agrees with the correct value for that measurement

percent uncertainty
: the ratio of the uncertainty of a measurement to the measure value, express as a percentage

precision
: the degree to which repeated measurements agree with each other

Solutions

Check Your Understanding 1

1: No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

6. 1.5 Graphing Introduction

Biomechanics researchers collect a lot of data (numbers) to understand human movement. These numbers have to be interpreted and presented to the readers. For example, if researchers want to see if carbon insoles help you jump higher than regular insoles they would have to collect jump height from participants wearing both carbon and regular insoles. They would calculate average jump height in carbon and regular insole and be left with data to present to their audience. They may chose to build a graph.

Graphs are a simple and elegant way to express a lot of information. They allow you to visually display the relationship between two (or more) variables. A basic graph typically has two dimensions represented by a vertical line and a horizontal line that intersect at a point called the origin. The horizontal line (x-axis) represents the data from the independent variable (time, frame number, insole type, etc..) where as the vertical line (y-axis) represents the data from the dependent variable (displacement, velocity, acceleration, force, jump height…). The dependent variable is the variable that you are measuring and quantifying.

More often than not, you will put lower numbers at the left on the horizontal axis and at the bottom on the vertical axis. The result is that the graphed line, or the bars in a bar graph, go up—the most natural direction for most data. In reading graphs, however, you should consider the axes and the type of measure being plotted before you interpret the meaning of lines going up and down, just to make sure that "up" represents "more" or "better."

If your study involves multiple variables, you can represent it in your graph by plotting each level of the variable with a separate line. It is helpful to use different types of lines or different colors. For example, you might graph the reaction time of boys with a solid line and the reaction time of girls with a dashed line. Alternatively, you can use different symbols—say, an X or an O, or a triangle—for the different levels of the variable. You can graph data by hand but in this class, you'll be asked to use a graphing software called Excel.

Want help Graphing a line-graph in Excel?

Instructions will vary slightly depending on the graph. Begin by entering the data you collected onto an Excel Spreadsheet. You should have at least two columns of data. The first column contains the Independent variable (ex: time, trial number, frame number). The second column, just to the right of the first, should contain the Dependent variable (ex: distance, velocity…). Place the name of the variable at the top of the column and enter the data below. Now you are ready to graph:

- Highlight the area you wish to graph by using the mouse and mouse keys. You should highlight both the 'Independent Variable' and one 'Dependent Variable' (graph one dependent variable at a time. Click on 'Insert' and 'Chart'.
- On the menu at the top of the page, pick the type of graph you would like (**Straight Marked Scatter** for line graph).
- In the Chart Layout menu (at the top), type in chart title, x-axis title, y-axis title; make sure to include the units. Remove ALL gridlines.

The graph should have:

1. Labeled x and y axis with variable and unit.
2. A title
3. No gridlines

4. No wasted space (adjust the axes value accordingly)
5. Clear data lines

Resources

The PHET group in Colorado have a large, and growing, number of simulations to help you with your math basics.

You can check them all out at

https://phet.colorado.edu/en/simulations/category/math/mathconcepts

Play around with this PHET simulation. It will help you see what equations look like.

https://phet.colorado.edu/en/simulation/graphing-lines

For help with analyzing linear graphs, check out

https://phet.colorado.edu/en/simulation/graphing-slope-intercept

PART II

CHAPTER 2: ANATOMY BASICS

Chapter Objectives

After this chapter, you will be able to:

- Use planes and axis to describe motion in three dimensions
- Define and identify the different body movements
- Describe the functions of the skeletal system and define its two major subdivisions
- Discuss both functional and structural classifications for body joints
- Describe the characteristic features for fibrous, cartilaginous, and synovial joints and give examples of each
- Discuss the structure of specific body joints and the movements allowed by each
- Explain the criteria used to name skeletal muscles

This section provides you with an overview of the 'bio' in biomechanics. In order to properly communicate, biomechanists need a good understanding of human anatomy and related anatomical terms. You will quickly come to understand that structure dictates functions.

7. 2.0 The Bio in Biomechanics

A discussion of Biomechanics would not be possible without a brief presentation of anatomical terms. This section introduces the basics of muscles, joints and bones as these terms will be used for the rest of the course when discussing human movement.

Since the human body can move in different position, we will often discuss human movement relative to the anatomical position. The anatomical position refers to an upright body, facing the observer, feet flat and directed forward. The upper limbs are at the body's sides with the palms facing forward. Certain terms used to describe the body and segments include:

1. **Superior**: towards the head
   Inferior: away from the head
2. **Anterior**: towards the front of the body
   Posterior: towards the back of the body
3. **Medial** : towards the midline of the body
   Lateral : away from the midline of the body
4. **Proximal**: closest to the trunk
   Distal: away from the trunk
5. **Superficial**: towards the surface of the body
   Deep: away from the surface

8. 2.1. The Skeleton

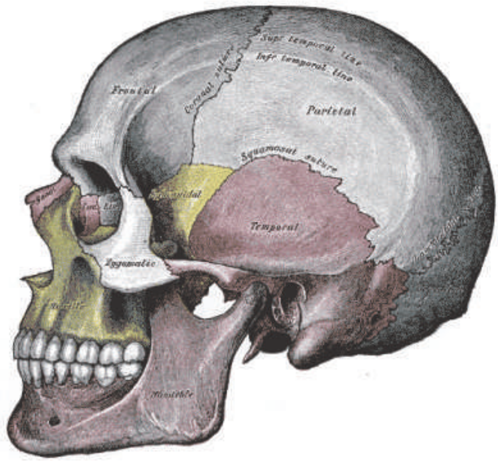


Figure 1. Lateral View of the Human Skull.

The skeletal system forms the rigid internal framework of the body. It consists of the bones, cartilages, and ligaments. Bones support the weight of the body, allow for body movements, and protect internal organs. Cartilage provides flexible strength and support for body structures such as the thoracic cage, the external ear, and the trachea and larynx. At joints of the body, cartilage can also unite adjacent bones or provide cushioning between them. Ligaments are the strong connective tissue bands that hold the bones at a moveable joint together and serve to prevent excessive movements of the joint that would result in injury. Providing movement of the skeleton are the muscles of the body, which are firmly attached to the skeleton via connective tissue structures called tendons. As muscles contract, they pull on the bones to produce movements of the body. Thus, without a skeleton, you would not be able to stand, run, or even feed yourself!

Each bone of the body serves a particular function, and therefore bones vary in size, shape, and strength based on these functions. For example, the bones of the lower back and lower limb are thick and strong to support your body weight. Similarly, the size of a bony landmark that serves as a muscle attachment site on an individual bone is related to the strength of this muscle. Muscles can apply very strong pulling forces to the bones of the skeleton. To resist these forces, bones have enlarged bony landmarks at sites where powerful muscles attach. This means that not only the size of a bone, but also its shape, is related to its function. For this reason, the identification of bony landmarks is important during your study of the skeletal system.

Bones are also dynamic organs that can modify their strength and thickness in response to changes in muscle strength or body weight. Thus, muscle attachment sites on bones will thicken if you begin a workout program that increases muscle strength. Similarly, the walls of weight-bearing bones will thicken if you gain body weight or begin pounding the pavement as part of a new running regimen. In contrast, a reduction in muscle strength or body weight will cause bones to become thinner. This may happen during a prolonged hospital stay, following limb immobilization in a cast, or going into the weightlessness of outer space. Even a change in diet, such as eating only soft food due to the loss of teeth, will result in a noticeable decrease in the size and thickness of the jaw bones.

The skeletal system includes all of the bones, cartilages, and ligaments of the body that support and give shape to the body and body structures. The **skeleton** consists of the bones of the body. For adults, there are 206 bones in the skeleton. Younger individuals have higher numbers of bones because some bones fuse together during childhood and adolescence to form an adult bone. The primary functions of the skeleton are to provide a rigid, internal structure that can support the weight of the body against the force of gravity, and to provide a structure upon which muscles can act to produce movements of the body. The lower portion of the skeleton is specialized for stability during walking or running. In contrast, the upper skeleton has greater mobility and ranges of motion, features that allow you to lift and carry objects or turn your head and trunk.

In addition to providing for support and movements of the body, the skeleton has protective and storage functions. It protects the internal organs, including the brain, spinal cord, heart, lungs, and pelvic organs. The bones of the skeleton serve as the primary storage site for important minerals such as calcium and phosphate. The bone marrow found within bones stores fat and houses the blood-cell producing tissue of the body.

The skeleton is subdivided into two major divisions—the axial and appendicular.

The Axial Skeleton

The **axial skeleton** forms the vertical, central axis of the body and includes all bones of the head, neck, chest, and back (Figure 1). It serves to protect the brain, spinal cord, heart, and lungs. It also serves as the attachment site for muscles that move the head, neck, and back, and for muscles that act across the shoulder and hip joints to move their corresponding limbs.

The axial skeleton of the adult consists of 80 bones, including the **skull**, the **vertebral column**, and the **thoracic cage**. The skull is formed by 22 bones. Also associated with the head are an additional seven bones, including the **hyoid bone** and the **ear ossicles** (three small bones found in each middle ear). The vertebral column consists of 24 bones, each called a **vertebra**, plus the **sacrum** and **coccyx**. The thoracic cage includes the 12 pairs of **ribs**, and the **sternum**, the flattened bone of the anterior chest.

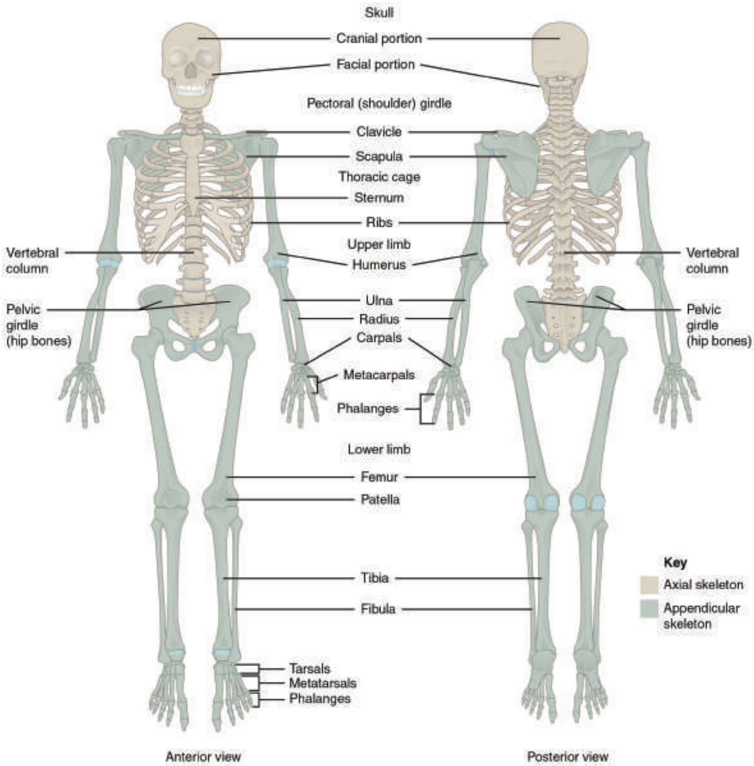


Figure 1. Axial and Appendicular Skeleton. The axial skeleton supports the head, neck, back, and chest and thus forms the vertical axis of the body. It consists of the skull, vertebral column (including the sacrum and coccyx), and the thoracic cage, formed by the ribs and sternum. The appendicular skeleton is made up of all bones of the upper and lower limbs.

The Appendicular Skeleton

The **appendicular skeleton** includes all bones of the upper and lower limbs, plus the bones that attach each limb to the axial skeleton. There are 126 bones in the appendicular skeleton of an adult.

Review

The skeletal system includes all of the bones, cartilages, and ligaments of the body. It serves to support the body, protect the brain and other internal organs, and provides a rigid structure upon which muscles can pull to generate body movements. It also stores fat and the tissue responsible for the production of blood cells. The skeleton is subdivided into two parts. The axial skeleton forms a vertical axis that includes the head, neck, back, and chest. It has 80 bones and consists of the skull, vertebral column, and thoracic cage. The adult vertebral column consists of 24 vertebrae plus the sacrum and coccyx. The thoracic cage is formed by 12 pairs of ribs and the sternum. The appendicular skeleton consists of 126 bones in the adult and includes all of the bones of the upper and lower limbs plus the bones that anchor each limb to the axial skeleton.

Glossary

appendicular skeleton
: all bones of the upper and lower limbs, plus the girdle bones that attach each limb to the axial skeleton

axial skeleton
: central, vertical axis of the body, including the skull,

vertebral column, and thoracic cage

coccyx
: small bone located at inferior end of the adult vertebral column that is formed by the fusion of four coccygeal vertebrae; also referred to as the “tailbone”

ear ossicles
: three small bones located in the middle ear cavity that serve to transmit sound vibrations to the inner ear

hyoid bone
: small, U-shaped bone located in upper neck that does not contact any other bone

ribs
: thin, curved bones of the chest wall

sacrum
: single bone located near the inferior end of the adult vertebral column that is formed by the fusion of five sacral vertebrae; forms the posterior portion of the pelvis

skeleton
: bones of the body

skull
: bony structure that forms the head, face, and jaws, and protects the brain; consists of 22 bones

sternum
: flattened bone located at the center of the anterior chest

thoracic cage
: consists of 12 pairs of ribs and sternum

vertebra
: individual bone in the neck and back regions of the vertebral column

vertebral column
: entire sequence of bones that extend from the skull to the tailbone

9. 2.2 Joints

Figure 1. Girl Kayaking. Without joints, body movements would be impossible. (credit: Graham Richardson/flickr.com)

The adult human body has 206 bones, and with the exception of the hyoid bone in the neck, each bone is connected to at least one other bone. Joints are the location where bones come together. Many joints allow for movement between the bones. At these joints, the articulating surfaces of the adjacent bones can move smoothly against each other. However, the bones of other joints may be joined to each other by connective tissue or cartilage. These joints are designed for stability and provide for little or no movement. Importantly, joint stability and movement are related to each other. This means that stable joints allow for little or no mobility between the adjacent bones.

Conversely, joints that provide the most movement between bones are the least stable. Understanding the relationship between joint structure and function will help to explain why particular types of joints are found in certain areas of the body.

The articulating surfaces of bones at stable types of joints, with little or no mobility, are strongly united to each other. For example, most of the joints of the skull are held together by fibrous connective tissue and do not allow for movement between the adjacent bones. This lack of mobility is important, because the skull bones serve to protect the brain. Similarly, other joints united by fibrous connective tissue allow for very little movement, which provides stability and weight-bearing support for the body. For example, the tibia and fibula of the leg are tightly united to give stability to the body when standing. At other joints, the bones are held together by cartilage, which permits limited movements between the bones. Thus, the joints of the vertebral column only allow for small movements between adjacent vertebrae, but when added together, these movements provide the flexibility that allows your body to twist, or bend to the front, back, or side. In contrast, at joints that allow for wide ranges of motion, the articulating surfaces of the bones are not directly united to each other. Instead, these surfaces are enclosed within a space filled with lubricating fluid, which allows the bones to move smoothly against each other. These joints provide greater mobility, but since the bones are free to move in relation to each other, the joint is less stable. Most of the joints between the bones of the appendicular skeleton are this freely moveable type of joint. These joints allow the muscles of the body to pull on a bone and thereby produce movement of that body region. Your ability to kick a soccer ball, pick up a fork, and dance the tango depend on mobility at these types of joints.

10. 2.2.1 Classification of Joints

A **joint**, also called an **articulation**, is any place where adjacent bones or bone and cartilage come together (articulate with each other) to form a connection. Joints are classified both structurally and functionally. Structural classifications of joints take into account whether the adjacent bones are strongly anchored to each other by fibrous connective tissue or cartilage, or whether the adjacent bones articulate with each other within a fluid-filled space called a **joint cavity**. Functional classifications describe the degree of movement available between the bones, ranging from immobile, to slightly mobile, to freely moveable joints. The amount of movement available at a particular joint of the body is related to the functional requirements for that joint. Thus immobile or slightly moveable joints serve to protect internal organs, give stability to the body, and allow for limited body movement. In contrast, freely moveable joints allow for much more extensive movements of the body and limbs.

Structural Classification of Joints

The structural classification of joints is based on whether the articulating surfaces of the adjacent bones are directly connected by fibrous connective tissue or cartilage, or whether the articulating surfaces contact each other within a fluid-filled joint cavity. These differences serve to divide the joints of the body into three structural classifications. A **fibrous joint** is where the adjacent bones are united by fibrous connective tissue. At a **cartilaginous joint**, the bones are joined by hyaline cartilage or fibrocartilage. At a **synovial joint**, the articulating surfaces of the bones are not directly connected, but instead come into contact with each other within a joint cavity that is filled with a lubricating fluid. Synovial joints allow for free movement between the bones and are the most common joints of the body.

Functional Classification of Joints

The functional classification of joints is determined by the amount of mobility found between the adjacent bones. Joints are thus functionally classified as a synarthrosis or immobile joint, an amphiarthrosis or slightly moveable joint, or as a diarthrosis, which is a freely moveable joint (arthroun = "to fasten by a joint"). Depending on their location, fibrous joints may be functionally classified as a synarthrosis (immobile joint) or an amphiarthrosis (slightly mobile joint). Cartilaginous joints are also functionally classified as either a synarthrosis or an amphiarthrosis joint. All synovial joints are functionally classified as a diarthrosis joint.

Synarthrosis

An immobile or nearly immobile joint is called a **synarthrosis**. The immobile nature of these joints provide for a strong union between the articulating bones. This is important at locations where the bones provide protection for internal organs. Examples include sutures, the fibrous joints between the bones of the skull that surround and protect the brain (Figure 1).

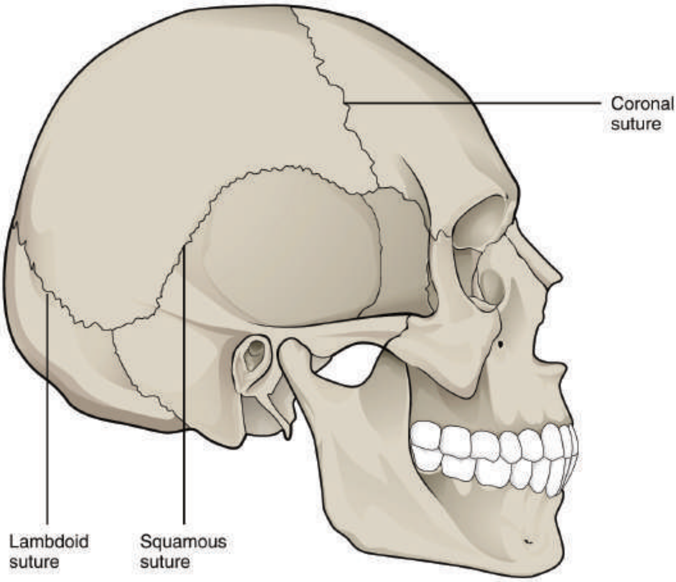


Figure 1. Suture Joints of Skull. The suture joints of the skull are an example of a synarthrosis, an immobile or essentially immobile joint.

Amphiarthrosis

An **amphiarthrosis** is a joint that has limited mobility. An example of this type of joint is the cartilaginous joint that unites the bodies of adjacent vertebrae. Filling the gap between the vertebrae is a thick pad of fibrocartilage called an intervertebral disc (Figure 2). Each intervertebral disc strongly unites the vertebrae but still allows for a limited amount of movement between them. However, the small movements available between adjacent vertebrae can sum together along the length of the vertebral column to provide for large ranges of body movements.

Another example of an amphiarthrosis is the pubic symphysis of the pelvis. This is a cartilaginous joint in which the pubic regions of the right and left hip bones are strongly anchored to each other by fibrocartilage. This joint normally has very little mobility. The strength of the pubic symphysis is important in conferring weight-bearing stability to the pelvis.

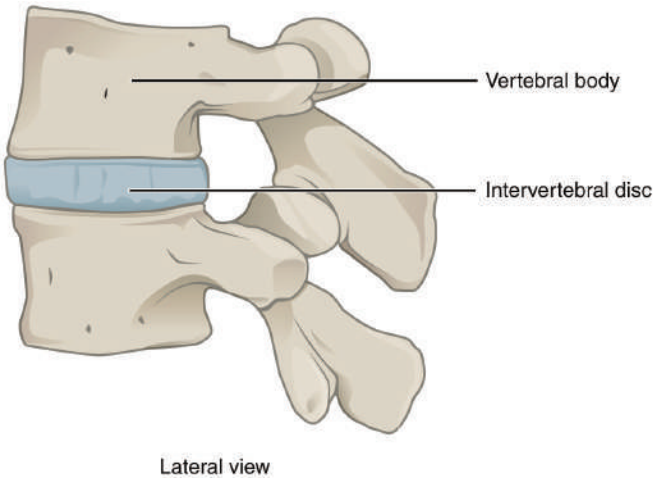


Figure 2. Intervertebral Disc. An intervertebral disc unites the bodies of adjacent vertebrae within the vertebral column. Each disc allows for limited movement between the vertebrae and thus functionally forms an amphiarthrosis type of joint. Intervertebral discs are made of fibrocartilage and thereby structurally form a symphysis type of cartilaginous joint.

Diarthrosis

A freely mobile joint is classified as a **diarthrosis**. These types of joints include all synovial joints of the body, which provide the majority of body movements. Most diarthrotic joints are found in the appendicular skeleton and thus give the limbs a wide range of motion. These joints are divided into three categories, based on the number of axes of motion provided by each. An axis in anatomy is described as the movements in reference to the three anatomical planes: transverse, frontal, and sagittal. Thus, diarthroses are classified as uniaxial (for movement in one plane- one degree of freedom), biaxial (for movement in two planes- two degrees of freedom), or multiaxial joints (for movement in all three anatomical planes – three degrees of freedom).

A **uniaxial joint** only allows for a motion in a single plane (around a single axis). The elbow joint, which only allows for bending or straightening, is an example of a uniaxial joint. A **biaxial joint** allows for motions within two planes. An example of a biaxial joint is a metacarpophalangeal joint (knuckle joint) of the hand. The joint allows for movement along one axis to produce bending or straightening of the finger, and movement along a second axis, which allows for spreading of the fingers away from each other and bringing them together. A joint that allows for the several directions of movement is called a **multiaxial joint** (polyaxial or triaxial joint). This type of diarthrotic joint allows for movement along three axes (Figure 3). The shoulder and hip joints are multiaxial joints. They allow the upper or lower limb to move in an anterior-posterior direction and a medial-lateral direction. In addition, the limb can also be rotated around its long axis. This third movement results in rotation of the limb so that its anterior surface is moved either toward or away from the midline of the body.

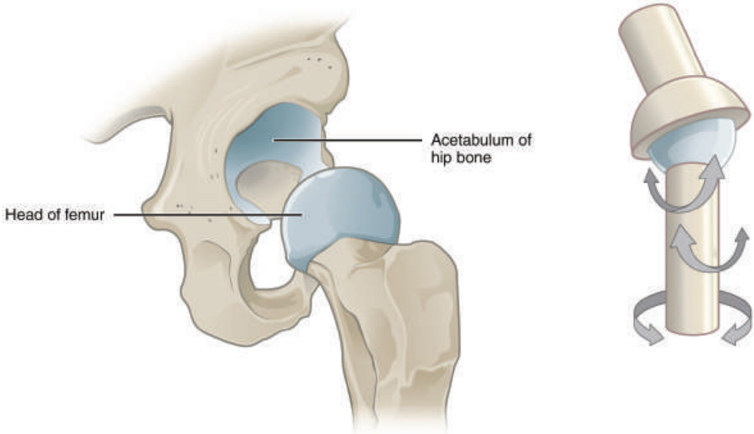


Figure 3. Multiaxial Joint. A multiaxial joint, such as the hip joint, allows for three types of movement: anterior-posterior, medial-lateral, and rotational.

Chapter Review

Structural classifications of the body joints are based on how the bones are held together and articulate with each other. At fibrous joints, the adjacent bones are directly united to each other by fibrous connective tissue. Similarly, at a cartilaginous joint, the adjacent bones are united by cartilage. In contrast, at a synovial joint, the articulating bone surfaces are not directly united to each other, but come together within a fluid-filled joint cavity.

The functional classification of body joints is based on the degree of movement found at each joint. A synarthrosis is a joint that is essentially immobile. This type of joint provides for a strong connection between the adjacent bones, which serves to protect internal structures such as the brain or heart. Examples include the fibrous joints of the skull sutures and the cartilaginous manubriosternal joint. A joint that allows for limited movement is an amphiarthrosis. An example is the pubic symphysis of the pelvis, the cartilaginous joint that strongly unites the right and left hip bones of the pelvis. The cartilaginous joints in which vertebrae are united by intervertebral discs provide for small movements between the adjacent vertebrae and are also an amphiarthrosis type of joint. Thus, based on their movement ability, both fibrous and cartilaginous joints are functionally classified as a synarthrosis or amphiarthrosis.

The most common type of joint is the diarthrosis, which is a freely moveable joint. All synovial joints are functionally classified as diarthroses. A uniaxial diarthrosis, such as the elbow, is a joint that only allows for movement within a single anatomical plane. Joints that allow for movements in two planes are biaxial joints, such as the metacarpophalangeal joints of the fingers. A multiaxial joint, such as the shoulder or hip joint, allows for three planes of motions.

Glossary

amphiarthrosis
: slightly mobile joint

articulation
: joint of the body

biaxial joint
: type of diarthrosis; a joint that allows for movements within two planes (two axes)

cartilaginous joint
: joint at which the bones are united by hyaline cartilage (synchondrosis) or fibrocartilage (symphysis)

diarthrosis
: freely mobile joint

fibrous joint
: joint where the articulating areas of the adjacent bones

are connected by fibrous connective tissue

joint
: site at which two or more bones or bone and cartilage come together (articulate)

joint cavity
: space enclosed by the articular capsule of a synovial joint that is filled with synovial fluid and contains the articulating surfaces of the adjacent bones

multiaxial joint
: type of diarthrosis; a joint that allows for movements within three planes (three axes)

synarthrosis
: immobile or nearly immobile joint

synovial joint
: joint at which the articulating surfaces of the bones are located within a joint cavity formed by an articular capsule

uniaxial joint
: type of diarthrosis; joint that allows for motion within only one plane (one axis)

Solutions

Answers for Review Questions

1. C
2. B
3. A
4. D

Answers for Critical Thinking Questions

1. Functional classification of joints is based on the degree of mobility exhibited by the joint. A

synarthrosis is an immobile or nearly immobile joint. An example is the manubriosternal joint or the joints between the skull bones surrounding the brain. An amphiarthrosis is a slightly moveable joint, such as the pubic symphysis or an intervertebral cartilaginous joint. A diarthrosis is a freely moveable joint. These are subdivided into three categories. A uniaxial diarthrosis allows movement within a single anatomical plane or axis of motion. The elbow joint is an example. A biaxial diarthrosis, such as the metacarpophalangeal joint, allows for movement along two planes or axes. The hip and shoulder joints are examples of a multiaxial diarthrosis. These allow movements along three planes or axes.

2. The functional needs of joints vary and thus joints differ in their degree of mobility. A synarthrosis, which is an immobile joint, serves to strongly connect bones thus protecting internal organs such as the heart or brain. A slightly moveable amphiarthrosis provides for small movements, which in the vertebral column can add together to yield a much larger overall movement. The freedom of movement provided by a diarthrosis can allow for large movements, such as is seen with most joints of the limbs.

11. 2.2.2 Synovial Joints

Synovial joints are the most common type of joint in the body (Figure 1). A key structural characteristic for a synovial joint that is not seen at fibrous or cartilaginous joints is the presence of a joint cavity. This fluid-filled space is the site at which the articulating surfaces of the bones contact each other. Also unlike fibrous or cartilaginous joints, the articulating bone surfaces at a synovial joint are not directly connected to each other with fibrous connective tissue or cartilage. This gives the bones of a synovial joint the ability to move smoothly against each other, allowing for increased joint mobility.

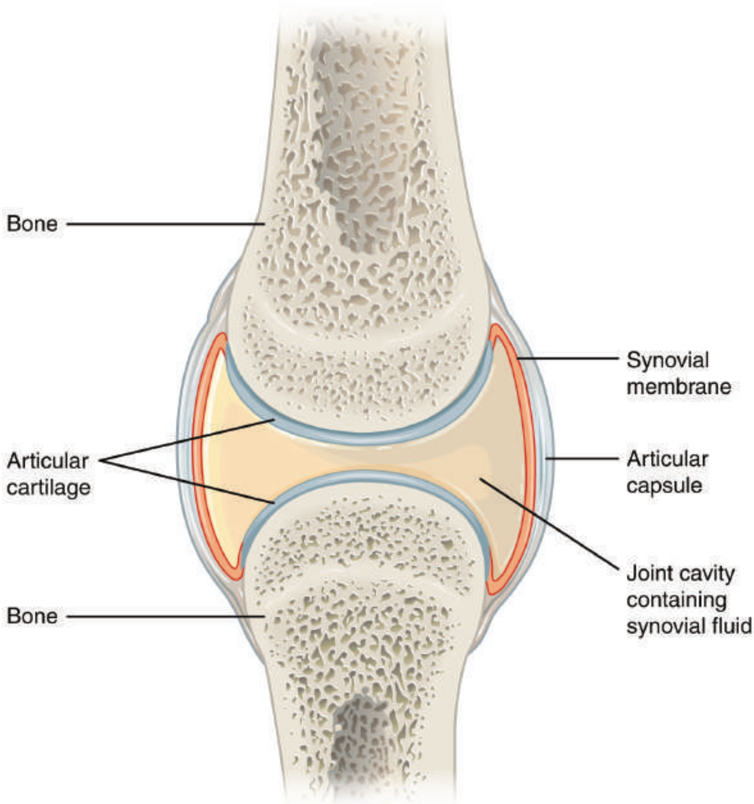


Figure 1. Synovial Joints. Synovial joints allow for smooth movements between the adjacent bones. The joint is surrounded by an articular capsule that defines a joint cavity filled with synovial fluid. The articulating surfaces of the bones are covered by a thin layer of articular cartilage. Ligaments support the joint by holding the bones together and resisting excess or abnormal joint motions.

Structural Features of Synovial Joints

Synovial joints are characterized by the presence of a joint cavity. The walls of this space are formed by the **articular**

capsule, a fibrous connective tissue structure that is attached to each bone just outside the area of the bone's articulating surface. The bones of the joint articulate with each other within the joint cavity.

Friction between the bones at a synovial joint is prevented by the presence of the **articular cartilage**, a thin layer of hyaline cartilage that covers the entire articulating surface of each bone. However, unlike at a cartilaginous joint, the articular cartilages of each bone are not continuous with each other. Instead, the articular cartilage acts like a Teflon® coating over the bone surface, allowing the articulating bones to move smoothly against each other without damaging the underlying bone tissue. Lining the inner surface of the articular capsule is a thin **synovial membrane**. The cells of this membrane secrete **synovial fluid** (synovia = "a thick fluid"), a thick, slimy fluid that provides lubrication to further reduce friction between the bones of the joint. This fluid also provides nourishment to the articular cartilage, which does not contain blood vessels. The ability of the bones to move smoothly against each other within the joint cavity, and the freedom of joint movement this provides, means that each synovial joint is functionally classified as a diarthrosis.

Outside of their articulating surfaces, the bones are connected together by ligaments, which are strong bands of fibrous connective tissue. These strengthen and support the joint by anchoring the bones together and preventing their separation. Ligaments allow for normal movements at a joint, but limit the range of these motions, thus preventing excessive or abnormal joint movements. Ligaments are classified based on their relationship to the fibrous articular capsule. An **extrinsic ligament** is located outside of the articular capsule, an **intrinsic ligament** is fused to or incorporated into the wall of the articular capsule, and an **intracapsular ligament** is located inside of the articular capsule.

At many synovial joints, additional support is provided by the muscles and their tendons that act across the joint. A **tendon** is the dense connective tissue structure that attaches a muscle to bone. As forces acting on a joint increase, the body will automatically increase the overall strength of contraction of the muscles crossing that joint, thus allowing the muscle and its tendon to serve as a "dynamic ligament" to resist forces and support the joint. This type of indirect support by muscles is very important at the shoulder joint, for example, where the ligaments are relatively weak.

Additional Structures Associated with Synovial Joints

A few synovial joints of the body have a fibrocartilage structure located between the articulating bones. This is called an **articular disc**, which is generally small and oval-shaped, or a **meniscus**, which is larger and C-shaped. These structures can serve several functions, depending on the specific joint. In some places, an articular disc may act to strongly unite the bones of the joint to each other. Examples of this include the articular discs found at the sternoclavicular joint or between the distal ends of the radius and ulna bones. At other synovial joints, the disc can provide shock absorption and cushioning between the bones, which is the function of each meniscus within the knee joint. Finally, an articular disc can serve to smooth the movements between the articulating bones, as seen at the temporomandibular joint. Some synovial joints also have a fat pad, which can serve as a cushion between the bones.

Additional structures located outside of a synovial joint serve to prevent friction between the bones of the joint and the overlying muscle tendons or skin. A **bursa** (plural = bursae) is a thin connective tissue sac filled with lubricating liquid. They are located in regions where skin, ligaments, muscles, or muscle tendons can rub against each other, usually near a body joint (Figure 2). Bursae reduce friction by separating the adjacent structures, preventing them from rubbing directly against each other. Bursae are classified by their location. A **subcutaneous bursa** is located between the skin and an underlying bone. It allows skin to move smoothly over the bone. Examples include the prepatellar bursa located over the kneecap and the olecranon bursa at the tip of the elbow. A **submuscular bursa** is found between a muscle and an underlying bone, or between adjacent muscles. These prevent rubbing of the muscle during movements. A large submuscular bursa, the trochanteric bursa, is found at the lateral hip, between the greater trochanter of the femur and the overlying gluteus maximus muscle. A **subtendinous bursa** is found between a tendon and a bone. Examples include the subacromial bursa that protects the tendon of shoulder muscle as it passes under the acromion of the scapula, and the suprapatellar bursa that separates the tendon of the large anterior thigh muscle from the distal femur just above the knee.

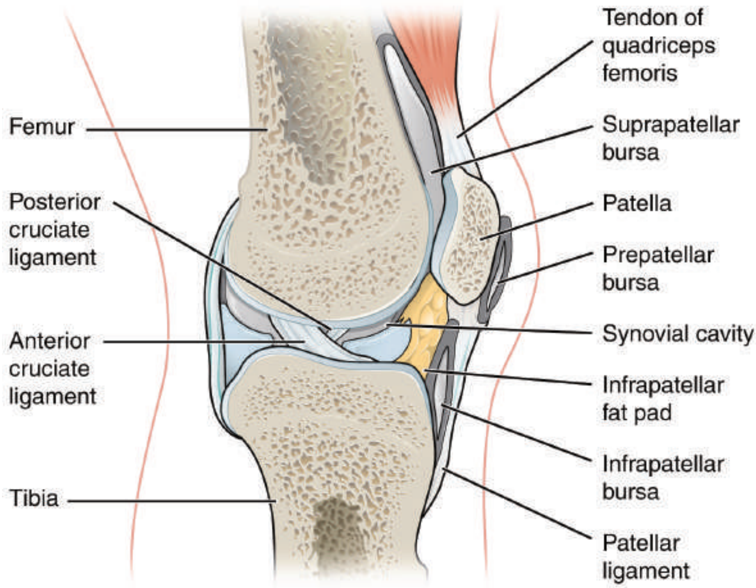


Figure 2. Bursae. Bursae are fluid-filled sacs that serve to prevent friction between skin, muscle, or tendon and an underlying bone. Three major bursae and a fat pad are part of the complex joint that unites the femur and tibia of the leg.

A **tendon sheath** is similar in structure to a bursa, but smaller. It is a connective tissue sac that surrounds a muscle tendon at places where the tendon crosses a joint. It contains a lubricating fluid that allows for smooth motions of the tendon during muscle contraction and joint movements.

Types of Synovial Joints

Synovial joints are subdivided based on the shapes of the articulating surfaces of the bones that form each joint. The six types of synovial joints are pivot, hinge, condyloid, saddle, plane, and ball-and socket-joints (Figure 3).

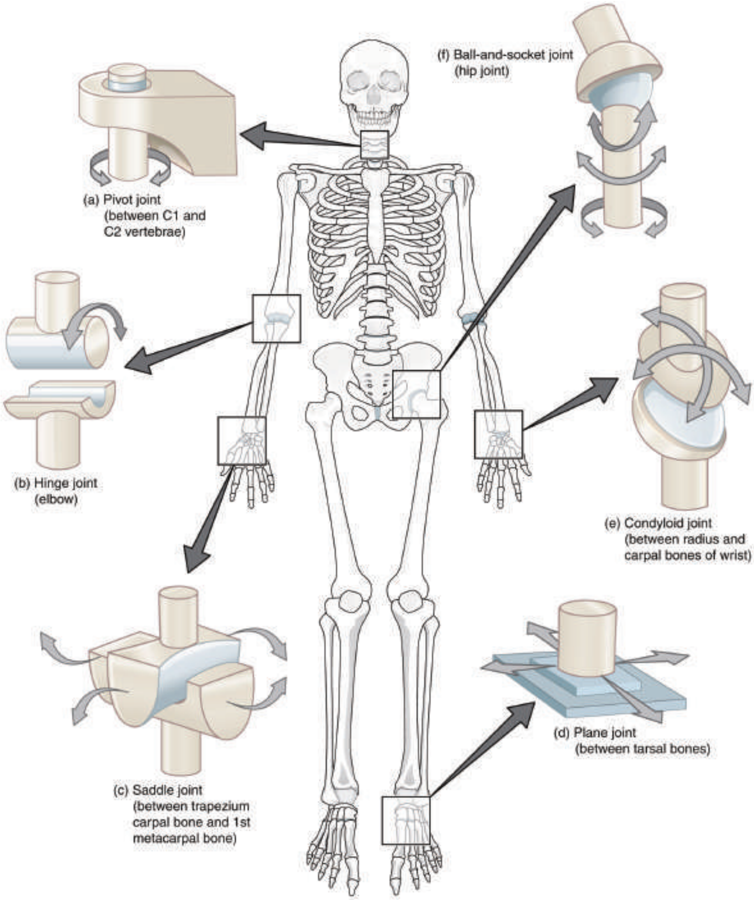


Figure 3. Types of Synovial Joints. The six types of synovial joints allow the body to move in a variety of ways. (a) Pivot joints allow for rotation around an axis, such as between the first and second cervical vertebrae, which allows for side-to-side rotation of the head. (b) The hinge joint of the elbow works like a door hinge. (c) The articulation between the trapezium carpal bone and the first metacarpal bone at the base of the thumb is a saddle joint. (d) Plane joints, such as those between the tarsal bones of the foot, allow for limited gliding movements between bones. (e) The radiocarpal joint of the wrist is a condyloid joint. (f) The hip and shoulder joints are the only ball-and-socket joints of the body.

Pivot Joint

At a **pivot joint**, a rounded portion of a bone is enclosed within a ring formed partially by the articulation with another bone and partially by a ligament (see Figure 3**a**). The bone rotates within this ring. Since the rotation is around a single axis, pivot joints are functionally classified as a uniaxial diarthrosis type of joint. An example of a pivot joint is the atlantoaxial joint, found between the C1 (atlas) and C2 (axis) vertebrae. Here, the upward projecting dens of the axis articulates with the inner aspect of the atlas, where it is held in place by a ligament. Rotation at this joint allows you to turn your head from side to side. A second pivot joint is found at the **proximal radioulnar joint**. Here, the head of the radius is largely encircled by a ligament that holds it in place as it articulates with the radial notch of the ulna. Rotation of the radius allows for forearm movements.

Hinge Joint

In a **hinge joint**, the convex end of one bone articulates with the concave end of the adjoining bone (see Figure 3**b**). This type of joint allows only for bending and straightening motions along a single axis, and thus hinge joints are functionally classified as uniaxial joints. A good example is the elbow joint, with the articulation between the trochlea of the humerus and the trochlear notch of the ulna. Other hinge joints of the body include the knee, ankle, and interphalangeal joints between the phalanx bones of the fingers and toes.

Condyloid Joint

At a **condyloid joint** (ellipsoid joint), the shallow depression at the end of one bone articulates with a rounded structure from an adjacent bone or bones (see Figure 3**e**). The knuckle (metacarpophalangeal) joints of the hand between the distal end of a metacarpal bone and the proximal phalanx bone are condyloid joints. Another example is the radiocarpal joint of the wrist, between the shallow depression at the distal end of the radius bone and the rounded scaphoid, lunate, and triquetrum carpal bones. In this case, the articulation area has a more oval (elliptical) shape. Functionally, condyloid joints are biaxial joints that allow for two planes of movement. One movement involves the bending and straightening of the fingers or the anterior-posterior movements of the hand. The second movement is a side-to-side movement, which allows you to spread your fingers apart and bring them together, or to move your hand in a medial-going or lateral-going direction.

Saddle Joint

At a **saddle joint**, both of the articulating surfaces for the bones have a saddle shape, which is concave in one direction and convex in the other (see Figure 3**c**). This allows the two bones to fit together like a rider sitting on a saddle. Saddle joints are functionally classified as biaxial joints. The primary example is the first carpometacarpal joint, between the trapezium (a carpal bone) and the first metacarpal bone at the base of the thumb. This joint provides the thumb the ability to move away from the palm of the hand along two planes. Thus, the thumb can move within the same plane as the palm of the hand, or it can jut out anteriorly, perpendicular to the palm. This movement of the first carpometacarpal joint is what gives humans their distinctive “opposable” thumbs. The sternoclavicular joint is also classified as a saddle joint.

Plane Joint

At a **plane joint** (gliding joint), the articulating surfaces of the bones are flat or slightly curved and of approximately the same size, which allows the bones to slide against each other (see Figure 3**d**). The motion at this type of joint is usually small and tightly constrained by surrounding ligaments. Based only on their shape, plane joints can allow multiple movements, including rotation. Thus plane joints can be functionally classified as a multiaxial joint. However, not all of these movements are available to every plane joint due to limitations placed on it by ligaments or neighboring bones. Thus, depending upon the specific joint of the body, a plane joint may exhibit only a single type of movement or several movements. Plane joints are found between the carpal bones (intercarpal joints) of the wrist or tarsal bones (intertarsal joints) of the foot, between the clavicle and acromion of the scapula (acromioclavicular joint), and between the superior and inferior articular processes of adjacent vertebrae (zygapophysial joints).

Ball-and-Socket Joint

The joint with the greatest range of motion is the **ball-and-socket joint**. At these joints, the rounded head of one bone (the ball) fits into the concave articulation (the socket) of the adjacent bone (see Figure 3**f**). The hip joint and the glenohumeral (shoulder) joint are the only ball-and-socket joints of the body. At the hip joint, the head of the femur articulates with the acetabulum of the hip bone, and at the shoulder joint, the head of the humerus articulates with the glenoid cavity of the scapula.

Ball-and-socket joints are classified functionally as multiaxial joints. The femur and the humerus are able to move in both anterior-posterior and medial-lateral directions and they can also rotate around their long axis. The shallow socket formed by the glenoid cavity allows the shoulder joint an extensive range of motion. In contrast, the deep socket of the acetabulum and the strong supporting ligaments of the hip joint serve to constrain movements of the femur, reflecting the need for stability and weight-bearing ability at the hip.

Watch this video to see an animation of synovial joints in action.

Chapter Review

Synovial joints are the most common type of joints in the body. They are characterized by the presence of a joint cavity, inside of which the bones of the joint articulate with each other. The articulating surfaces of the bones at a synovial joint are not directly connected to each other by connective tissue or cartilage, which allows the bones to move freely against each other. The walls of the joint cavity are formed by the articular capsule. Friction between the bones is reduced by a thin layer of articular cartilage covering the surfaces of the bones, and by a lubricating synovial fluid, which is secreted by the synovial membrane.

Synovial joints are strengthened by the presence of ligaments, which hold the bones together and resist excessive or abnormal movements of the joint. Ligaments are classified as extrinsic ligaments if they are located outside of the articular capsule, intrinsic ligaments if they are fused to the wall of the articular capsule, or intracapsular ligaments if they are located inside the articular capsule. Some synovial joints also have an articular disc (meniscus), which can provide padding between the bones, smooth their movements, or strongly join the bones together to strengthen the joint. Muscles and their tendons acting across a joint can also increase their contractile strength when needed, thus providing indirect support for the joint.

Bursae contain a lubricating fluid that serves to reduce friction between structures. Subcutaneous bursae prevent friction between the skin and an underlying bone, submuscular bursae protect muscles from rubbing against a bone or another muscle, and a subtendinous bursa prevents friction between bone and a muscle tendon. Tendon sheaths contain a lubricating fluid and surround tendons to allow for smooth movement of the tendon as it crosses a joint.

Based on the shape of the articulating bone surfaces and the types of movement allowed, synovial joints are classified into six types. At a pivot joint, one bone is held within a ring by a ligament and its articulation with a second bone. Pivot joints only allow for rotation around a single axis. These are found at the articulation between the C1 (atlas) and the dens of the C2 (axis) vertebrae, which provides the side-to-side rotation of the head, or at the proximal radioulnar joint between the head of the radius and the radial notch of the ulna, which allows for rotation of the radius during forearm movements. Hinge joints, such as at the elbow, knee, ankle, or interphalangeal joints between phalanx bones of the fingers and toes, allow only for bending and straightening of the joint. Pivot and hinge joints are functionally classified as uniaxial joints.

Condyloid joints are found where the shallow depression of one bone receives a rounded bony area formed by one or two bones. Condyloid joints are found at the base of the fingers (metacarpophalangeal joints) and at the wrist (radiocarpal joint). At a saddle joint, the articulating bones fit together like a rider and a saddle. An example is the first carpometacarpal joint located at the base of the thumb. Both condyloid and saddle joints are functionally classified as biaxial joints.

Plane joints are formed between the small, flattened surfaces of adjacent bones. These joints allow the bones to slide or rotate against each other, but the range of motion is usually slight and tightly limited by ligaments or surrounding bones. This type of joint is found between the articular processes of adjacent vertebrae, at the acromioclavicular joint, or at the intercarpal joints of the hand and intertarsal joints of the foot. Ball-and-socket joints, in which the rounded head of a bone fits into a large depression or socket, are found at the shoulder and hip joints. Both plane and ball-and-sockets joints are classified functionally as multiaxial joints. However, ball-and-socket joints allow for large movements, while the motions between bones at a plane joint are small.

Glossary

articular capsule
: connective tissue structure that encloses the joint cavity of a synovial joint

articular cartilage
: thin layer of hyaline cartilage that covers the articulating surfaces of bones at a synovial joint

articular disc
: meniscus; a fibrocartilage structure found between the bones of some synovial joints; provides padding or smooths movements between the bones; strongly unites the bones together

ball-and-socket joint
: synovial joint formed between the spherical end of one bone (the ball) that fits into the depression of a second bone (the socket); found at the hip and shoulder joints; functionally classified as a multiaxial joint

bursa
: connective tissue sac containing lubricating fluid that prevents friction between adjacent structures, such as skin and bone, tendons and bone, or between muscles

condyloid joint
: synovial joint in which the shallow depression at the end of one bone receives a rounded end from a second bone or a rounded structure formed by two bones; found at the metacarpophalangeal joints of the fingers or the radiocarpal joint of the wrist; functionally classified as a biaxial joint

extrinsic ligament
: ligament located outside of the articular capsule of a synovial joint

hinge joint
: synovial joint at which the convex surface of one bone articulates with the concave surface of a second bone; includes the elbow, knee, ankle, and interphalangeal joints; functionally classified as a uniaxial joint

intracapsular ligament
: ligament that is located within the articular capsule of a synovial joint

intrinsic ligament
: ligament that is fused to or incorporated into the wall of the articular capsule of a synovial joint

meniscus
: articular disc

pivot joint
: synovial joint at which the rounded portion of a bone rotates within a ring formed by a ligament and an

articulating bone; functionally classified as uniaxial joint

plane joint
: synovial joint formed between the flattened articulating surfaces of adjacent bones; functionally classified as a multiaxial joint

proximal radioulnar joint
: articulation between head of radius and radial notch of ulna; uniaxial pivot joint that allows for rotation of radius during pronation/supination of forearm

saddle joint
: synovial joint in which the articulating ends of both bones are convex and concave in shape, such as at the first carpometacarpal joint at the base of the thumb; functionally classified as a biaxial joint

subcutaneous bursa
: bursa that prevents friction between skin and an underlying bone

submuscular bursa
: bursa that prevents friction between bone and a muscle or between adjacent muscles

subtendinous bursa
: bursa that prevents friction between bone and a muscle tendon

synovial fluid
: thick, lubricating fluid that fills the interior of a synovial joint

synovial membrane
: thin layer that lines the inner surface of the joint cavity at a synovial joint; produces the synovial fluid

tendon
: dense connective tissue structure that anchors a muscle to bone

tendon sheath
: connective tissue that surrounds a tendon at places where the tendon crosses a joint; contains a lubricating fluid to

prevent friction and allow smooth movements of the tendon.

12. 2.2.3 Types of Body Movements

Synovial joints allow the body a tremendous range of movements. Each movement at a synovial joint results from the contraction or relaxation of the muscles that are attached to the bones on either side of the articulation. The type of movement that can be produced at a synovial joint is determined by its structural type. While the ball-and-socket joint gives the greatest range of movement at an individual joint, in other regions of the body, several joints may work together to produce a particular movement. Overall, each type of synovial joint is necessary to provide the body with its great flexibility and mobility. There are many types of movement that can occur at synovial joints (Table 1).

Human movements are complex. In order to describe movements we typically break down the movement and describe what is occurring at every joint. At each joint, we can break down the movement into three planes. Planes describe the direction of the movement. The sagittal plane lies vertically and divides the body into right and left parts. Forward and backward movements fall into this plane (flexion, extension). The frontal plane also lies vertically but divides the body into anterior and posterior parts. Lateral movements that involves the limbs moving away and towards the body fall under this plane (adduction, abduction). The transverse plane lies horizontally and divides the body into superior and inferior. Rotations and twisting motions fall under this plane (internal rotation, external rotation).

An axis is a straight line around which a limb rotates. Movement at a joint takes place in a plane about an axis. There are three axes of rotation that correspond to each of the three planes:

1. Sagittal plane: medio-lateral axis
2. Frontal plane: anteroposterior axis
3. Transverse plane: longitudinal axis

There is a tendency when describing a movement to refer it to the particular plane that it is dominated by. For example, running is often considered to be a movement in the sagittal plane. In reality, all movements involves movements in more than one dimension.

Movement types are generally paired, with one being the opposite of the other. Body movements are always described in relation to the anatomical position of the body: upright stance, with upper limbs to the side of body and palms facing forward. Refer to Figure 1 as you go through this section.

Watch this video to learn about anatomical motions. What motions involve increasing or decreasing the angle of the foot at the ankle?

Watch this video to learn about anatomical motions. What motions involve increasing or decreasing the angle of the foot at the ankle?

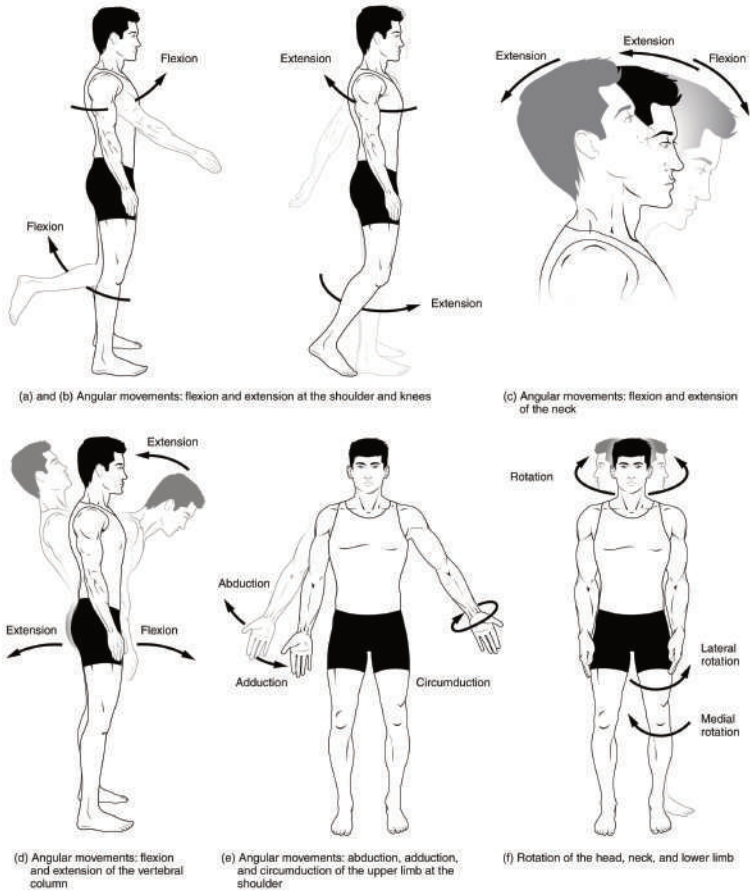


Figure 1. Movements of the Body, Part 1. Synovial joints give the body many ways in which to move. (a)–(b) Flexion and extension motions are in the sagittal (anterior–posterior) plane of motion. These movements take place at the shoulder, hip, elbow, knee, wrist, metacarpophalangeal, metatarsophalangeal, and interphalangeal joints. (c)–(d) Anterior bending of the head or vertebral column is flexion, while any posterior-going movement is extension. (e) Abduction and adduction are motions of the limbs, hand, fingers, or toes in the coronal (medial–lateral) plane of movement. Moving the limb or hand laterally away from the body, or spreading the fingers or toes, is abduction. Adduction brings the limb or hand toward or across the midline of the body, or brings the fingers or toes together. Circumduction is the movement of the limb, hand, or fingers in a circular pattern, using the sequential combination of flexion, adduction, extension, and abduction motions. Adduction/abduction and circumduction take place at the shoulder, hip, wrist,

metacarpophalangeal, and metatarsophalangeal joints. (f) Turning of the head side to side or twisting of the body is rotation. Medial and lateral rotation of the upper limb at the shoulder or lower limb at the hip involves turning the anterior surface of the limb toward the midline of the body (medial or internal rotation) or away from the midline (lateral or external rotation).

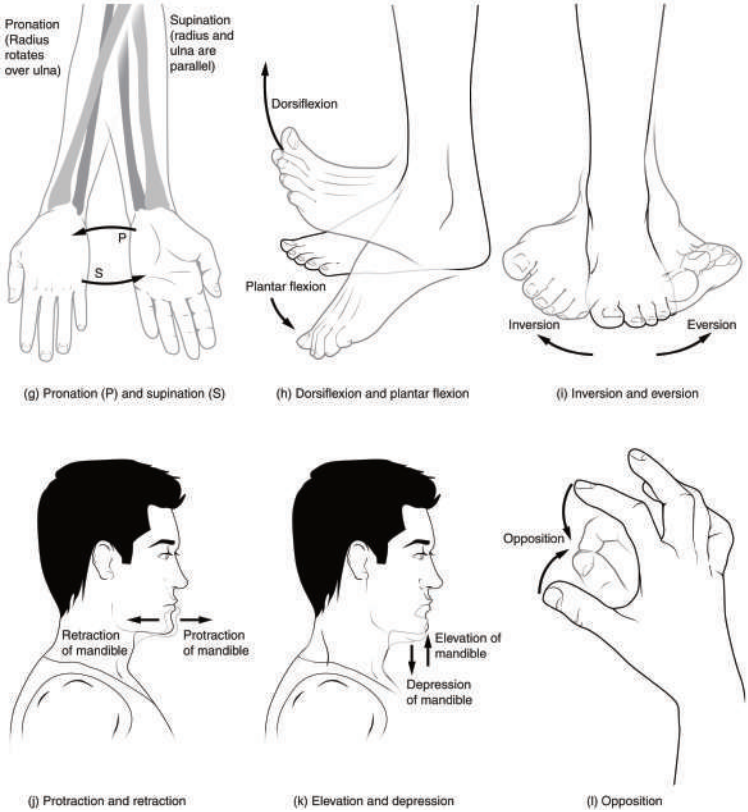


Figure 2. Movements of the Body, Part 2. (g) Supination of the forearm turns the hand to the palm forward position in which the radius and ulna are parallel, while forearm pronation turns the hand to the palm backward position in which the radius crosses over the ulna to form an "X." (h) Dorsiflexion of the foot at the ankle joint moves the top of the foot toward the leg, while plantar flexion lifts the heel and points the toes. (i) Eversion of the foot moves the bottom (sole) of the foot away from the midline of the body, while foot inversion faces the sole toward the midline. (j) Protraction of the mandible pushes the chin forward, and retraction pulls the chin back. (k) Depression of the mandible opens the mouth, while elevation closes it. (l) Opposition of the thumb brings the tip of the thumb into contact with the tip of the fingers of the same hand and reposition brings the thumb back next to the index finger.

Flexion and Extension

Flexion and **extension** are movements that take place within the sagittal plane and involve anterior or posterior movements of the body or limbs. For the vertebral column, flexion (anterior flexion) is an anterior (forward) bending of the neck or body, while extension involves a posterior-directed motion, such as straightening from a flexed position or bending backward. **Lateral flexion** is the bending of the neck or body toward the right or left side. These movements of the vertebral column involve both the symphysis joint formed by each intervertebral disc, as well as the plane type of synovial joint formed between the inferior articular processes of one vertebra and the superior articular processes of the next lower vertebra.

In the limbs, flexion decreases the angle between the bones (bending of the joint), while extension increases the angle and straightens the joint. For the upper limb, all anterior-going motions are flexion and all posterior-going motions are extension. These include anterior-posterior movements of the arm at the shoulder, the forearm at the elbow, the hand at the wrist, and the fingers at the metacarpophalangeal and interphalangeal joints. For the thumb, extension moves the thumb away from the palm of the hand, within the same plane as the palm, while flexion brings the thumb back against the index finger or into the palm. These motions take place at the first carpometacarpal joint. In the lower limb, bringing the thigh forward and upward is flexion at the hip joint, while any posterior-going motion of the thigh is extension. Note that extension of the thigh beyond the anatomical (standing) position is greatly limited by the ligaments that support the hip joint. Knee flexion is the bending of the knee to bring the foot toward the posterior thigh, and extension is the straightening of the knee. Flexion and extension movements are seen at the hinge, condyloid, saddle, and ball-and-socket joints of the limbs (see Figure 1**a-d**).

Hyperextension is the abnormal or excessive extension of a joint beyond its normal range of motion, thus resulting in injury. Similarly, **hyperflexion** is excessive flexion at a joint. Hyperextension injuries are common at hinge joints such as the knee or elbow. In cases of “whiplash” in which the head is suddenly moved backward and then forward, a patient may experience both hyperextension and hyperflexion of the cervical region.

Abduction and Adduction

Abduction and **adduction** motions occur within the coronal plane and involve medial-lateral motions of the limbs, fingers, toes, or thumb. Abduction moves the limb laterally away from the midline of the body, while adduction is the opposing movement that brings the limb toward the body or across the midline. For example, abduction is raising the arm at the shoulder joint, moving it laterally away from the body, while adduction brings the arm down to the side of the body. Similarly, abduction and adduction at the wrist moves the hand away from or toward the midline of the body. Spreading the fingers or toes apart is also abduction, while bringing the fingers or toes together is adduction. For the thumb, abduction is the anterior movement that brings the thumb to a 90° perpendicular position, pointing straight out from the palm. Adduction moves the thumb back to the anatomical position, next to the index finger. Abduction and adduction movements are seen at condyloid, saddle, and ball-and-socket joints (see Figure 1**e**).

Circumduction

Circumduction is the movement of a body region in a circular manner, in which one end of the body region being moved stays relatively stationary while the other end describes a circle. It involves the sequential combination of flexion, adduction, extension, and abduction at a joint. This type of motion is found at biaxial condyloid and saddle joints, and at multiaxial ball-and-sockets joints (see Figure 1**e**).

Rotation

Rotation can occur within the vertebral column, at a pivot joint, or at a ball-and-socket joint. Rotation of the neck or body is the twisting movement produced by the summation of the small rotational movements available between adjacent vertebrae. At a pivot joint, one bone rotates in relation to another bone. This is a uniaxial joint, and thus rotation is the only motion allowed at a pivot joint. For example, at the atlantoaxial joint, the first cervical (C1) vertebra (atlas) rotates around the dens, the upward projection from the second cervical (C2) vertebra (axis). This allows the head to rotate from side to side as when shaking the head "no." The proximal radioulnar joint is a pivot joint formed by the head of the radius and its articulation with the ulna. This joint allows for the radius to rotate along its length during pronation and supination movements of the forearm.

Rotation can also occur at the ball-and-socket joints of the shoulder and hip. Here, the humerus and femur rotate around their long axis, which moves the anterior surface of the arm or thigh either toward or away from the midline of the body. Movement that brings the anterior surface of the limb toward the midline of the body is called **medial (internal) rotation**. Conversely, rotation of the limb so that the anterior surface moves away from the midline is **lateral (external) rotation** (see Figure 1**f**). Be sure to distinguish medial and lateral rotation, which can only occur at the multiaxial shoulder and hip joints, from circumduction, which can occur at either biaxial or multiaxial joints.

Supination and Pronation

Supination and pronation are movements of the forearm. In the anatomical position, the upper limb is held next to the body with the palm facing forward. This is the **supinated position** of the forearm. In this position, the radius and ulna are parallel to each other. When the palm of the hand faces backward, the forearm is in the **pronated position**, and the radius and ulna form an X-shape.

Supination and pronation are the movements of the forearm that go between these two positions. **Pronation** is the motion that moves the forearm from the supinated (anatomical) position to the pronated (palm backward) position. This motion is produced by rotation of the radius at the proximal radioulnar joint, accompanied by movement of the radius at the distal radioulnar joint. The proximal radioulnar joint is a pivot joint that allows for rotation of the head of the radius. Because of the slight curvature of the shaft of the radius, this rotation causes the distal end of the radius to cross over the distal ulna at the distal radioulnar joint. This crossing over brings the radius and ulna into an X-shape position. **Supination** is the opposite motion, in which rotation of the radius returns the bones to their parallel positions and moves the palm to the anterior facing (supinated) position. It helps to remember that supination is the motion you use when scooping up soup with a spoon (see Figure 2**g**).

Dorsiflexion and Plantar Flexion

Dorsiflexion and **plantar flexion** are movements at the ankle joint, which is a hinge joint. Lifting the front of the foot, so that the top of the foot moves toward the anterior leg is dorsiflexion, while lifting the heel of the foot from the ground or pointing the toes downward is plantar flexion. These are the only movements available at the ankle joint (see Figure 2**h**).

Inversion and Eversion

Inversion and eversion are complex movements that involve the multiple plane joints among the tarsal bones of the posterior foot (intertarsal joints) and thus are not motions that take place at the ankle joint. **Inversion** is the turning of the foot to angle the bottom of the foot toward the midline, while **eversion** turns the bottom of the foot away from the midline. The foot has a greater range of inversion than eversion motion. These are important motions that help to stabilize the foot when walking or running on an uneven surface and aid in the quick side-to-side changes in direction used during active sports such as basketball, racquetball, or soccer (see Figure 2**i**).

Protraction and Retraction

Protraction and **retraction** are anterior-posterior movements of the scapula or mandible. Protraction of the scapula occurs when the shoulder is moved forward, as when pushing against something or throwing a ball. Retraction is the opposite motion, with the scapula being pulled posteriorly and medially, toward the vertebral column. For the mandible, protraction occurs when the lower jaw is pushed forward, to stick out the chin, while retraction pulls the lower jaw backward. (See Figure 2**j**.)

Depression and Elevation

Depression and **elevation** are downward and upward movements of the scapula or mandible. The upward movement of the scapula and shoulder is elevation, while a downward movement is depression. These movements are used to shrug your shoulders. Similarly, elevation of the mandible is the upward movement of the lower jaw used to close the mouth or bite on something, and depression is the downward movement that produces opening of the mouth (see Figure 2**k**).

Excursion

Excursion is the side to side movement of the mandible. **Lateral excursion** moves the mandible away from the midline, toward either the right or left side. **Medial excursion** returns the mandible to its resting position at the midline.

Superior Rotation and Inferior

Rotation

Superior and inferior rotation are movements of the scapula and are defined by the direction of movement of the glenoid cavity. These motions involve rotation of the scapula around a point inferior to the scapular spine and are produced by combinations of muscles acting on the scapula. During **superior rotation**, the glenoid cavity moves upward as the medial end of the scapular spine moves downward. This is a very important motion that contributes to upper limb abduction. Without superior rotation of the scapula, the greater tubercle of the humerus would hit the acromion of the scapula, thus preventing any abduction of the arm above shoulder height. Superior rotation of the scapula is thus required for full abduction of the upper limb. Superior rotation is also used without arm abduction when carrying a heavy load with your hand or on your shoulder. You can feel this rotation when you pick up a load, such as a heavy book bag and carry it on only one shoulder. To increase its weight-bearing support for the bag, the shoulder lifts as the scapula superiorly rotates. **Inferior rotation** occurs during limb adduction and involves the downward motion of the glenoid cavity with upward movement of the medial end of the scapular spine.

Opposition and Reposition

Opposition is the thumb movement that brings the tip of the thumb in contact with the tip of a finger. This movement is produced at the first carpometacarpal joint, which is a saddle joint formed between the trapezium carpal bone and the first metacarpal bone. Thumb opposition is produced by a combination of flexion and abduction of the thumb at this joint. Returning the thumb to its anatomical position next to the index finger is called **reposition** (see Figure 2**l**).

Movements of the Joints (Table 1)

Type of Joint	**Movement**	**Example**
Pivot	Uniaxial joint; allows rotational movement	Atlantoaxial joint (C1–C2 vertebrae articulation); proximal radioulnar joint
Hinge	Uniaxial joint; allows flexion/extension movements	Knee; elbow; ankle; interphalangeal joints of fingers and toes
Condyloid	Biaxial joint; allows flexion/extension, abduction/adduction, and circumduction movements	Metacarpophalangeal (knuckle) joints of fingers; radiocarpal joint of wrist; metatarsophalangeal joints for toes
Saddle	Biaxial joint; allows flexion/extension, abduction/adduction, and circumduction movements	First carpometacarpal joint of the thumb; sternoclavicular joint
Plane	Multiaxial joint; allows inversion and eversion of foot, or flexion, extension, and lateral flexion of the vertebral column	Intertarsal joints of foot; superior-inferior articular process articulations between vertebrae
Ball-and-socket	Multiaxial joint; allows flexion/extension, abduction/adduction, circumduction, and medial/lateral rotation movements	Shoulder and hip joints

Chapter Review

The variety of movements provided by the different types of synovial joints allows for a large range of body motions and gives you tremendous mobility. These movements allow you to flex or extend your body or limbs, medially rotate and adduct your arms and flex your elbows to hold a heavy object against your chest, raise your arms above your head, rotate or shake your head, and bend to touch the toes (with or without bending your knees).

Each of the different structural types of synovial joints also allow for specific motions. The atlantoaxial pivot joint provides side-to-side rotation of the head, while the proximal radioulnar articulation allows for rotation of the radius during pronation and supination of the forearm. Hinge joints, such as at the knee and elbow, allow only for flexion and extension. Similarly, the hinge joint of the ankle only allows for dorsiflexion and plantar flexion of the foot.

Condyloid and saddle joints are biaxial. These allow for flexion and extension, and abduction and adduction. The sequential combination of flexion, adduction, extension, and abduction produces circumduction. Multiaxial plane joints provide for only small motions, but these can add together over several adjacent joints to produce body movement, such as inversion and eversion of the foot. Similarly, plane joints allow for flexion, extension, and lateral flexion movements of the vertebral column. The multiaxial ball and socket joints allow for flexion-extension, abduction-adduction, and circumduction. In addition, these also allow for medial (internal) and lateral (external) rotation. Ball-and-socket joints have the greatest range of motion of all synovial joints.

Interactive Link Questions

Watch this video to learn about anatomical motions. What motions involve increasing or decreasing the angle of the foot at the ankle?

Dorsiflexion of the foot at the ankle decreases the angle of the ankle joint, while plantar flexion increases the angle of the ankle joint.

Glossary

abduction
: movement in the coronal plane that moves a limb laterally away from the body; spreading of the fingers

adduction
: movement in the coronal plane that moves a limb medially toward or across the midline of the body; bringing fingers together

circumduction
: circular motion of the arm, thigh, hand, thumb, or finger that is produced by the sequential combination of flexion, abduction, extension, and adduction

depression
: downward (inferior) motion of the scapula or mandible

dorsiflexion
: movement at the ankle that brings the top of the foot toward the anterior leg

elevation
: upward (superior) motion of the scapula or mandible

eversion
: foot movement involving the intertarsal joints of the foot in which the bottom of the foot is turned laterally, away from the midline

extension
: movement in the sagittal plane that increases the angle of a joint (straightens the joint); motion involving posterior bending of the vertebral column or returning to the upright position from a flexed position

flexion
: movement in the sagittal plane that decreases the angle of a joint (bends the joint); motion involving anterior bending of the vertebral column

hyperextension
: excessive extension of joint, beyond the normal range of movement

hyperflexion
: excessive flexion of joint, beyond the normal range of movement

inferior rotation
: movement of the scapula during upper limb adduction in which the glenoid cavity of the scapula moves in a downward direction as the medial end of the scapular spine moves in an upward direction

inversion
: foot movement involving the intertarsal joints of the foot in which the bottom of the foot is turned toward the midline

lateral excursion
: side-to-side movement of the mandible away from the midline, toward either the right or left side

lateral flexion
: bending of the neck or body toward the right or left side

lateral (external) rotation
: movement of the arm at the shoulder joint or the thigh at the hip joint that moves the anterior surface of the limb away from the midline of the body

medial excursion
: side-to-side movement that returns the mandible to the midline

medial (internal) rotation
: movement of the arm at the shoulder joint or the thigh at the hip joint that brings the anterior surface of the limb toward the midline of the body

opposition

thumb movement that brings the tip of the thumb in contact with the tip of a finger

plantar flexion

foot movement at the ankle in which the heel is lifted off of the ground

pronated position

forearm position in which the palm faces backward

pronation

forearm motion that moves the palm of the hand from the palm forward to the palm backward position

protraction

anterior motion of the scapula or mandible

reposition

movement of the thumb from opposition back to the anatomical position (next to index finger)

retraction

posterior motion of the scapula or mandible

rotation

movement of a bone around a central axis (atlantoaxial joint) or around its long axis (proximal radioulnar joint; shoulder or hip joint); twisting of the vertebral column resulting from the summation of small motions between adjacent vertebrae

superior rotation

movement of the scapula during upper limb abduction in which the glenoid cavity of the scapula moves in an upward direction as the medial end of the scapular spine moves in a downward direction

supinated position

forearm position in which the palm faces anteriorly (anatomical position)

supination

forearm motion that moves the palm of the hand from the palm backward to the palm forward position

Solutions

Answers for Review Questions

1. A
2. C
3. D
4. A
5. C

Answers for Critical Thinking Questions

1. Ball-and-socket joints are multiaxial joints that allow for flexion and extension, abduction and adduction, circumduction, and medial and lateral rotation.
2. To cross your arms, you need to use both your shoulder and elbow joints. At the shoulder, the arm would need to flex and medially rotate. At the elbow, the forearm would need to be flexed.

13. 2.2.4 Anatomy of Selected Synovial Joints

Each synovial joint of the body is specialized to perform certain movements. The movements that are allowed are determined by the structural classification for each joint. For example, a multiaxial ball-and-socket joint has much more mobility than a uniaxial hinge joint. However, the ligaments and muscles that support a joint may place restrictions on the total range of motion available. Thus, the ball-and-socket joint of the shoulder has little in the way of ligament support, which gives the shoulder a very large range of motion. In contrast, movements at the hip joint are restricted by strong ligaments, which reduce its range of motion but confer stability during standing and weight bearing.

This section will examine the anatomy of selected synovial joints of the body. Anatomical names for most joints are derived from the names of the bones that articulate at that joint, although some joints, such as the elbow, hip, and knee joints are exceptions to this general naming scheme.

Shoulder Joint

The shoulder joint is called the **glenohumeral joint**. This is a ball-and-socket joint formed by the articulation between the head of the humerus and the glenoid cavity of the scapula (Figure 3). This joint has the largest range of motion of any joint in the body. However, this freedom of movement is due to the lack of structural support and thus the enhanced mobility is offset by a loss of stability.

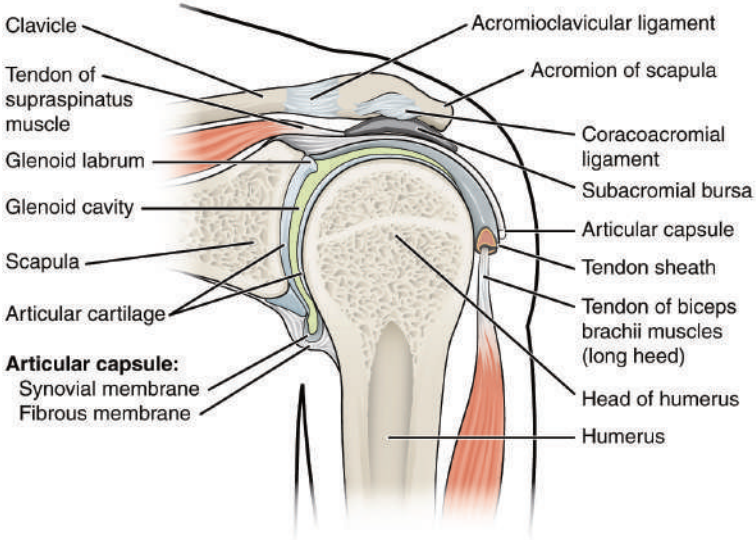


Figure 3. Glenohumeral Joint. The glenohumeral (shoulder) joint is a ball-and-socket joint that provides the widest range of motions. It has a loose articular capsule and is supported by ligaments and the rotator cuff muscles.

The large range of motions at the shoulder joint is provided by the articulation of the large, rounded humeral head with the small and shallow glenoid cavity, which is only about one third of the size of the humeral head. The socket formed by the glenoid cavity is deepened slightly by a small lip of fibrocartilage called the **glenoid labrum**, which extends around the outer margin of the cavity. The articular capsule that surrounds the glenohumeral joint is relatively thin and loose to allow for large motions of the upper limb. Some structural support for the joint is provided by thickenings of the articular capsule wall that form weak intrinsic ligaments. These include the **coracohumeral ligament**, running from the coracoid process of the scapula to the anterior humerus, and three ligaments, each called a **glenohumeral ligament**, located on the anterior side of the articular capsule. These ligaments help to strengthen the superior and anterior capsule walls.

However, the primary support for the shoulder joint is provided by muscles crossing the joint, particularly the four rotator cuff muscles. These muscles (supraspinatus, infraspinatus, teres minor, and subscapularis) arise from the scapula and attach to the greater or lesser tubercles of the humerus. As these muscles cross the shoulder joint, their tendons encircle the head of the humerus and become fused to the anterior, superior, and posterior walls of the articular capsule. The thickening of the capsule formed by the fusion of these four muscle tendons is called the **rotator cuff**. Two bursae, the **subacromial bursa** and the **subscapular bursa**, help to prevent friction between the rotator cuff muscle tendons and the scapula as these tendons cross the glenohumeral joint. In addition to their individual actions of moving the upper limb, the rotator cuff muscles also serve to hold the head of the humerus in position within the glenoid cavity. By constantly adjusting their strength of contraction to resist forces acting on the shoulder, these muscles serve as "dynamic ligaments" and thus provide the primary structural support for the glenohumeral joint.

Injuries to the shoulder joint are common. Repetitive use of the upper limb, particularly in abduction such as during throwing, swimming, or racquet sports, may lead to acute or chronic inflammation of the bursa or muscle tendons, a tear of the glenoid labrum, or degeneration or tears of the rotator cuff. Because the humeral head is strongly supported by muscles and ligaments around its anterior, superior, and posterior aspects, most dislocations of the humerus occur in an inferior direction. This can occur when force is applied to the humerus when the upper limb is fully abducted, as when diving to catch a baseball and landing on your hand or elbow. Inflammatory responses to any shoulder injury can lead to the formation of scar tissue between the articular capsule and surrounding structures, thus reducing shoulder mobility, a condition called adhesive capsulitis ("frozen shoulder").

Watch this video for a tutorial on the anatomy of the shoulder joint. What movements are available at the shoulder joint?

Watch this video for a tutorial on the anatomy of the shoulder joint. What movements are available at the shoulder joint?

Watch this video to learn more about the anatomy of the shoulder joint, including bones, joints, muscles, nerves, and blood vessels.

Watch this video to learn more about the anatomy of the shoulder joint, including bones, joints, muscles, nerves, and blood vessels. What is the shape of the glenoid labrum in cross-section, and what is the importance of this shape?

Elbow Joint

The **elbow joint** is a uniaxial hinge joint formed by the **humeroulnar joint**, the articulation between the trochlea of the humerus and the trochlear notch of the ulna. Also associated with the elbow are the **humeroradial joint** and the proximal radioulnar joint. All three of these joints are enclosed within a single articular capsule (Figure 4).

The articular capsule of the elbow is thin on its anterior and posterior aspects, but is thickened along its outside margins by strong intrinsic ligaments. These ligaments prevent side-to-side movements and hyperextension. On the medial side is the triangular **ulnar collateral ligament**. This arises from the medial epicondyle of the humerus and attaches to the medial side of the proximal ulna. The strongest part of this ligament is the anterior portion, which resists hyperextension of the elbow. The ulnar collateral ligament may be injured by frequent, forceful extensions of the forearm, as is seen in baseball pitchers. Reconstructive surgical repair of this ligament is referred to as Tommy John surgery, named for the former major league pitcher who was the first person to have this treatment.

The lateral side of the elbow is supported by the **radial collateral ligament**. This arises from the lateral epicondyle of the humerus and then blends into the lateral side of the annular ligament. The **annular ligament** encircles the head of the radius. This ligament supports the head of the radius as it articulates with the radial notch of the ulna at the proximal radioulnar joint. This is a pivot joint that allows for rotation of the radius during supination and pronation of the forearm.

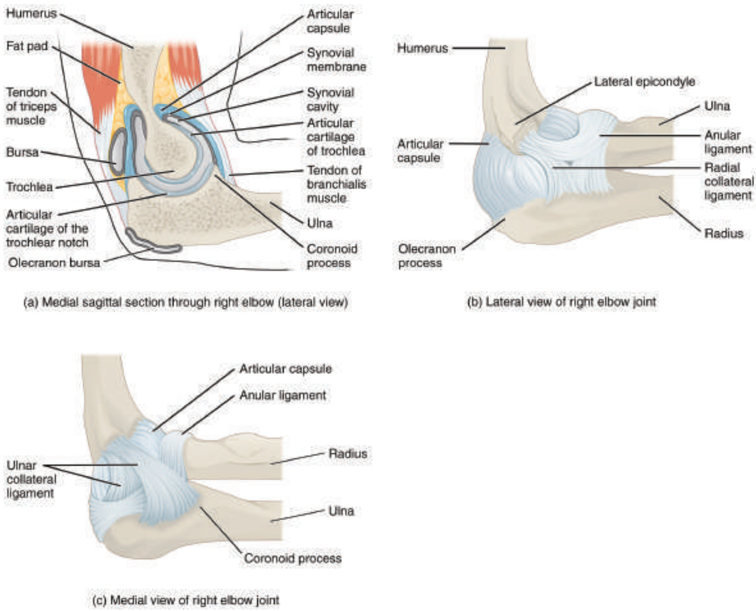


Figure 4. Elbow Joint. (a) The elbow is a hinge joint that allows only for flexion and extension of the forearm. (b) It is supported by the ulnar and radial collateral ligaments. (c) The annular ligament supports the head of the radius at the proximal radioulnar joint, the pivot joint that allows for rotation of the radius.

Watch this animation to learn more about the anatomy of the elbow joint. Which structures provide the main stability for the elbow?

Watch this animation to learn more about the anatomy of the elbow joint. Which structures provide the main stability for the elbow?

Watch this video to learn more about the anatomy of the elbow joint, including bones, joints, muscles, nerves, and blood vessels. What are the functions of the articular cartilage?

Watch this video to learn more about the anatomy of the elbow joint, including bones, joints, muscles, nerves, and blood vessels. What are the functions of the articular cartilage?

Hip Joint

The hip joint is a multiaxial ball-and-socket joint between the head of the femur and the acetabulum of the hip bone (Figure 5). The hip carries the weight of the body and thus requires strength and stability during standing and walking. For these reasons, its range of motion is more limited than at the shoulder joint.

The acetabulum is the socket portion of the hip joint. This space is deep and has a large articulation area for the femoral head, thus giving stability and weight bearing ability to the joint. The acetabulum is further deepened by the **acetabular labrum**, a fibrocartilage lip attached to the outer margin of the acetabulum. The surrounding articular capsule is strong, with several thickened areas forming intrinsic ligaments. These ligaments arise from the hip bone, at the margins of the acetabulum, and attach to the femur at the base of the neck. The ligaments are the **iliofemoral ligament**, **pubofemoral ligament**, and **ischiofemoral ligament**, all of which spiral around the head and neck of the femur. The ligaments are tightened by extension at the hip, thus pulling the head of the femur tightly into the acetabulum when in the upright, standing position. Very little additional extension of the thigh is permitted beyond this vertical position. These ligaments thus stabilize the hip joint and allow you to maintain an upright standing position with only minimal muscle contraction. Inside of the articular capsule, the **ligament of the head of the femur** (ligamentum teres) spans between the acetabulum and femoral head. This intracapsular ligament is normally slack and does not provide any significant joint support, but it does provide a pathway for an important artery that supplies the head of the femur.

The hip is prone to osteoarthritis, and thus was the first joint for which a replacement prosthesis was developed. A common injury in elderly individuals, particularly those with weakened bones due to osteoporosis, is a “broken hip,” which is actually a fracture of the femoral neck. This may result from a fall, or it may cause the fall. This can happen as one lower limb is taking a step and all of the body weight is placed on the other limb, causing the femoral neck to break and producing a fall. Any accompanying disruption of the blood supply to the femoral neck or head can lead to necrosis of these areas, resulting in bone and cartilage death. Femoral fractures usually require surgical treatment, after which the patient will need mobility assistance for a prolonged period, either from family members or in a long-term care facility. Consequentially, the associated health care costs of “broken hips” are substantial. In addition, hip fractures are associated with increased rates of morbidity (incidences of disease) and mortality (death). Surgery for a hip fracture followed by prolonged bed rest may lead to life-threatening complications, including pneumonia, infection of pressure ulcers (bedsores), and thrombophlebitis (deep vein thrombosis; blood clot formation) that can result in a pulmonary embolism (blood clot within the lung).

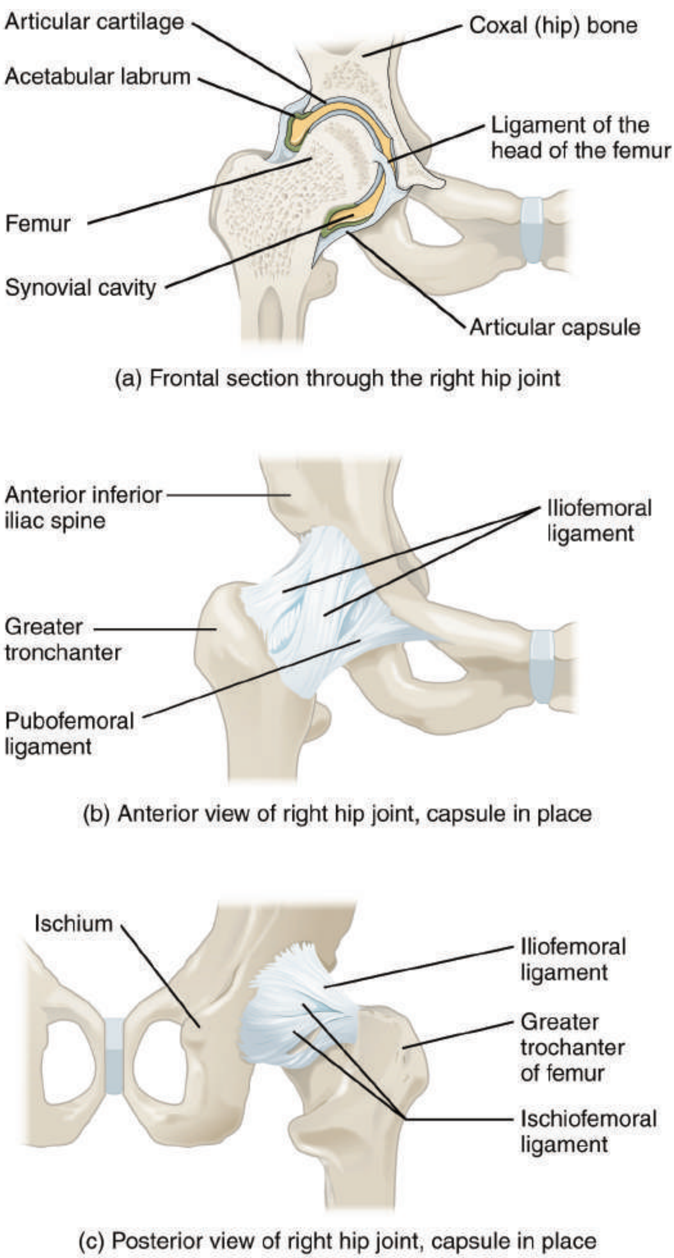


Figure 5. Hip Joint. (a) The ball-and-socket joint of the hip is a multiaxial joint that provides both stability and a wide range

of motion. (b–c) When standing, the supporting ligaments are tight, pulling the head of the femur into the acetabulum.

Watch this video for a tutorial on the anatomy of the hip joint. What is a possible consequence following a fracture of the femoral neck within the capsule of the hip joint?

Watch this video for a tutorial on the anatomy of the hip joint. What is a possible consequence following a fracture of the femoral neck within the capsule of the hip joint?

Watch this video to learn more about the anatomy of the hip joint, including bones, joints, muscles, nerves, and blood vessels. Where is the articular cartilage thickest within the hip joint?

Watch this video to learn more about the anatomy of the hip joint, including bones, joints, muscles, nerves, and blood vessels. Where is the articular cartilage thickest within the hip joint?

Knee Joint

The knee joint is the largest joint of the body (Figure 6). It actually consists of three articulations. The **femoropatellar joint** is found between the patella and the distal femur. The **medial tibiofemoral joint** and **lateral tibiofemoral joint** are located between the medial and lateral condyles of the femur and the medial and lateral condyles of the tibia. All of these articulations are enclosed within a single articular capsule. The knee functions as a hinge joint, allowing flexion and extension of the leg. This action is generated by both rolling and gliding motions of the femur on the tibia. In addition, some rotation of the leg is available when the knee is flexed, but not when extended. The knee is well constructed for weight bearing in its extended position, but is vulnerable to injuries associated with hyperextension, twisting, or blows to the medial or lateral side of the joint, particularly while weight bearing.

At the femoropatellar joint, the patella slides vertically within a groove on the distal femur. The patella is a sesamoid bone incorporated into the tendon of the quadriceps femoris muscle, the large muscle of the anterior thigh. The patella serves to protect the quadriceps tendon from friction against the distal femur. Continuing from the patella to the anterior tibia just below the knee is the **patellar ligament**. Acting via the patella and patellar ligament, the quadriceps femoris is a powerful muscle that acts to extend the leg at the knee. It also serves as a “dynamic ligament” to provide very important support and stabilization for the knee joint.

The medial and lateral tibiofemoral joints are the articulations between the rounded condyles of the femur and the relatively flat condyles of the tibia. During flexion and extension motions, the condyles of the femur both roll and glide over the surfaces of the tibia. The rolling action produces flexion or extension, while the gliding action serves to maintain the femoral condyles centered over the tibial condyles, thus ensuring maximal bony, weight-bearing support for the femur in all knee positions. As the knee comes into full extension, the femur undergoes a slight medial rotation in relation to tibia. The rotation results because the lateral condyle of the femur is slightly smaller than the medial condyle. Thus, the lateral condyle finishes its rolling motion first, followed by the medial condyle. The resulting small medial rotation of the femur serves to “lock” the knee into its fully extended and most stable position. Flexion of the knee is initiated by a slight lateral rotation of the femur on the tibia, which “unlocks” the knee.

This lateral rotation motion is produced by the popliteus muscle of the posterior leg.

Located between the articulating surfaces of the femur and tibia are two articular discs, the **medial meniscus** and **lateral meniscus** (see Figure 6**b**). Each is a C-shaped fibrocartilage structure that is thin along its inside margin and thick along the outer margin. They are attached to their tibial condyles, but do not attach to the femur. While both menisci are free to move during knee motions, the medial meniscus shows less movement because it is anchored at its outer margin to the articular capsule and tibial collateral ligament. The menisci provide padding between the bones and help to fill the gap between the round femoral condyles and flattened tibial condyles. Some areas of each meniscus lack an arterial blood supply and thus these areas heal poorly if damaged.

The knee joint has multiple ligaments that provide support, particularly in the extended position (see Figure 6**c**). Outside of the articular capsule, located at the sides of the knee, are two extrinsic ligaments. The **fibular collateral ligament** (lateral collateral ligament) is on the lateral side and spans from the lateral epicondyle of the femur to the head of the fibula. The **tibial collateral ligament** (medial collateral ligament) of the medial knee runs from the medial epicondyle of the femur to the medial tibia. As it crosses the knee, the tibial collateral ligament is firmly attached on its deep side to the articular capsule and to the medial meniscus, an important factor when considering knee injuries. In the fully extended knee position, both collateral ligaments are taut (tight), thus serving to stabilize and support the extended knee and preventing side-to-side or rotational motions between the femur and tibia.

The articular capsule of the posterior knee is thickened by intrinsic ligaments that help to resist knee hyperextension. Inside the knee are two intracapsular ligaments, the **anterior cruciate ligament** and **posterior cruciate ligament**. These ligaments are anchored inferiorly to the tibia at the intercondylar eminence, the roughened area between the tibial condyles. The cruciate ligaments are named for whether they are attached anteriorly or posteriorly to this tibial region. Each ligament runs diagonally upward to attach to the inner aspect of a femoral condyle. The cruciate ligaments are named for the X-shape formed as they pass each other (cruciate means "cross"). The posterior cruciate ligament is the stronger ligament. It serves to support the knee when it is flexed and weight bearing, as when walking downhill. In this position, the posterior cruciate ligament prevents the femur from sliding anteriorly off the top of the tibia. The anterior cruciate ligament becomes tight when the knee is extended, and thus resists hyperextension.

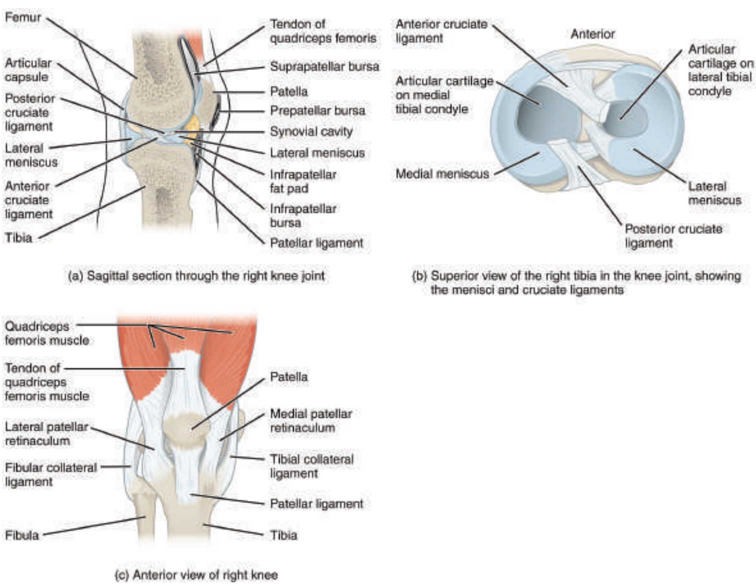


Figure 6. Knee Joint. (a) The knee joint is the largest joint of the body. (b)–(c) It is supported by the tibial and fibular collateral ligaments located on the sides of the knee outside of the articular capsule, and the anterior and posterior cruciate ligaments found inside the capsule. The medial and lateral menisci provide padding and support between the femoral condyles and tibial condyles.

Watch this video to learn more about the flexion and extension of the knee, as the femur both rolls and glides on the tibia to maintain stable contact between the bones in all knee positions.

Watch this video to learn more about the flexion and extension of the knee, as the femur both rolls and glides on the tibia to maintain stable contact between the bones in all knee positions. The patella glides along a groove on the anterior side of the distal femur. The collateral ligaments on the sides of the knee become tight in the fully extended position to help stabilize the knee. The posterior cruciate ligament supports the knee when flexed and the anterior cruciate ligament becomes tight when the knee comes into full extension to resist hyperextension. What are the ligaments that support the knee joint?

Watch this video to learn more about the anatomy of the knee joint, including bones, joints, muscles, nerves, and blood vessels.

Watch this video to learn more about the anatomy of the knee joint, including bones, joints, muscles, nerves, and blood vessels. Which ligament of the knee keeps the tibia from sliding too far forward in relation to the femur and which ligament keeps the tibia from sliding too far backward?
Disorders of the…

Joints
Injuries to the knee are common. Since this joint is primarily supported by muscles and ligaments, injuries to any of these structures will result in pain or knee instability. Injury to the posterior cruciate ligament occurs when the knee is flexed and the tibia is driven posteriorly, such as falling and landing on the tibial tuberosity or hitting the tibia on the dashboard when not wearing a seatbelt during an automobile accident. More commonly, injuries occur when forces are applied to the extended knee, particularly when the foot is planted and unable to move. Anterior cruciate ligament injuries can result with a forceful blow to the anterior knee, producing hyperextension, or when a runner makes a quick change of

direction that produces both twisting and hyperextension of the knee.

A worse combination of injuries can occur with a hit to the lateral side of the extended knee (Figure 7). A moderate blow to the lateral knee will cause the medial side of the joint to open, resulting in stretching or damage to the tibial collateral ligament. Because the medial meniscus is attached to the tibial collateral ligament, a stronger blow can tear the ligament and also damage the medial meniscus. This is one reason that the medial meniscus is 20 times more likely to be injured than the lateral meniscus. A powerful blow to the lateral knee produces a "terrible triad" injury, in which there is a sequential injury to the tibial collateral ligament, medial meniscus, and anterior cruciate ligament.

Arthroscopic surgery has greatly improved the surgical treatment of knee injuries and reduced subsequent recovery times. This procedure involves a small incision and the insertion into the joint of an arthroscope, a pencil-thin instrument that allows for visualization of the joint interior. Small surgical instruments are also inserted via additional incisions. These tools allow a surgeon to remove or repair a torn meniscus or to reconstruct a ruptured cruciate ligament. The current method for anterior cruciate ligament replacement involves using a portion of the patellar ligament. Holes are drilled into the cruciate ligament attachment points on the tibia and femur, and the patellar ligament graft, with small areas of attached bone still intact at each end, is inserted into these holes. The bone-to-bone sites at each end of the graft heal rapidly and strongly, thus enabling a rapid recovery.

Knee Injury

A strong blow to the lateral side of the extended knee will cause three injuries, in sequence: tearing of the tibial collateral ligament, damage to the medial meniscus, and rupture of the anterior cruciate ligament.

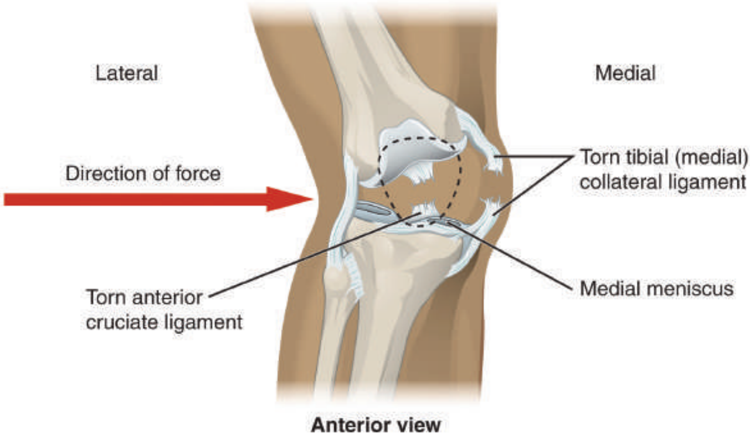


Figure 7. Knee Injury. A strong blow to the lateral side of the extended knee will cause three injuries, in sequence: tearing of the tibial collateral ligament, damage to the medial meniscus, and rupture of the anterior cruciate ligament.

Watch this video to learn more about different knee injuries and diagnostic testing of the knee. What are the most common causes of anterior cruciate ligament injury?

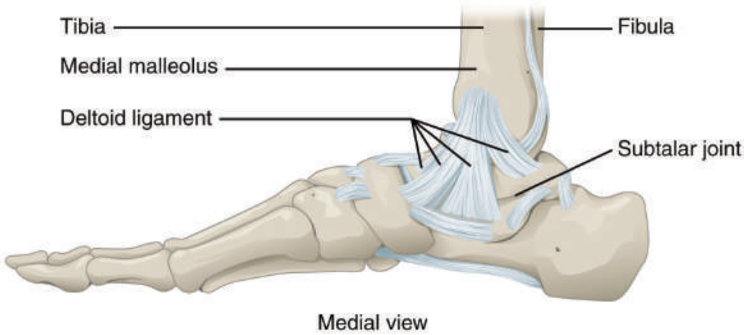
Watch this video to learn more about different knee injuries and diagnostic testing of the knee. What are the most common causes of anterior cruciate ligament injury?

Ankle and Foot Joints

The ankle is formed by the **talocrural joint** (Figure 8). It consists of the articulations between the talus bone of the foot and the distal ends of the tibia and fibula of the leg (crural = "leg"). The superior aspect of the talus bone is square-shaped and has three areas of articulation. The top of the talus articulates with the inferior tibia. This is the portion of the ankle joint that carries the body weight between the leg and foot. The sides of the talus are firmly held in position by the articulations with the medial malleolus of the tibia and the lateral malleolus of the fibula, which prevent any side-to-side motion of the talus. The ankle is thus a uniaxial hinge joint that allows only for dorsiflexion and plantar flexion of the foot.

Additional joints between the tarsal bones of the posterior foot allow for the movements of foot inversion and eversion. Most important for these movements is the **subtalar joint**, located between the talus and calcaneus bones. The joints between the talus and navicular bones and the calcaneus and cuboid bones are also important contributors to these movements. All of the joints between tarsal bones are plane joints. Together, the small motions that take place at these joints all contribute to the production of inversion and eversion foot motions.

Like the hinge joints of the elbow and knee, the talocrural joint of the ankle is supported by several strong ligaments located on the sides of the joint. These ligaments extend from the medial malleolus of the tibia or lateral malleolus of the fibula and anchor to the talus and calcaneus bones. Since they are located on the sides of the ankle joint, they allow for dorsiflexion and plantar flexion of the foot. They also prevent abnormal side-to-side and twisting movements of the talus and calcaneus bones during eversion and inversion of the foot. On the medial side is the broad **deltoid ligament**. The deltoid ligament supports the ankle joint and also resists excessive eversion of the foot. The lateral side of the ankle has several smaller ligaments. These include the **anterior talofibular ligament** and the **posterior talofibular ligament**, both of which span between the talus bone and the lateral malleolus of the fibula, and the **calcaneofibular ligament**, located between the calcaneus bone and fibula. These ligaments support the ankle and also resist excess inversion of the foot.



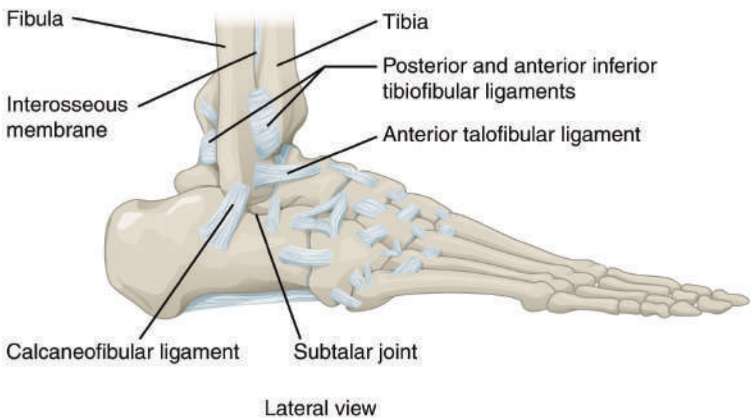


Figure 8. Ankle Joint. The talocrural (ankle) joint is a uniaxial hinge joint that only allows for dorsiflexion or plantar flexion of the foot. Movements at the subtalar joint, between the talus and calcaneus bones, combined with motions at other intertarsal joints, enables eversion/inversion movements of the foot. Ligaments that unite the medial or lateral malleolus with the talus and calcaneus bones serve to support the talocrural joint and to resist excess eversion or inversion of the foot.

Watch this video for a tutorial on the anatomy of the ankle joint. What are the three ligaments found on the lateral side of the ankle joint?

Watch this video for a tutorial on the anatomy of the ankle joint. What are the three ligaments found on the lateral side of the ankle joint?

Watch this video to learn more about the anatomy of the ankle joint, including bones, joints, muscles, nerves, and blood vessels. Which type of joint used in woodworking does the ankle joint resemble?

Watch this video to learn more about the anatomy of the ankle joint, including bones, joints, muscles, nerves, and blood vessels. Which type of joint used in woodworking does the ankle joint resemble?
Disorders of the…

Joints

The ankle is the most frequently injured joint in the body, with the most common injury being an inversion ankle sprain. A sprain is the stretching or tearing of the supporting ligaments. Excess inversion causes the talus bone to tilt laterally, thus damaging the ligaments on the lateral side of the ankle. The anterior talofibular ligament is most commonly injured, followed by the calcaneofibular ligament. In severe inversion injuries, the forceful lateral movement of the talus not only ruptures the lateral ankle ligaments, but also fractures the distal fibula.

Less common are eversion sprains of the ankle, which involve stretching of the deltoid ligament on the medial side of the ankle. Forcible eversion of the foot, for example, with an awkward landing from a jump or when a football player has a foot planted and is hit on the lateral ankle, can result in a Pott's fracture and dislocation of the ankle joint. In this injury, the very strong deltoid ligament does not tear, but instead shears off the medial malleolus of the tibia. This frees the talus, which moves laterally and fractures the distal fibula. In extreme cases, the posterior margin of the tibia may also be sheared off.

Above the ankle, the distal ends of the tibia and fibula are united by a strong syndesmosis formed by the interosseous membrane and ligaments at the distal tibiofibular joint. These connections prevent separation between the distal ends of the tibia and fibula and maintain the talus locked into position between the medial malleolus and lateral malleolus. Injuries that produce a lateral twisting of the leg on top of the planted foot can result in stretching or tearing of the tibiofibular ligaments, producing a syndesmotic ankle sprain or “high ankle sprain.”

Most ankle sprains can be treated using the RICE technique: Rest, Ice, Compression, and Elevation. Reducing joint mobility using a brace or cast may be required for a period of time. More severe injuries involving ligament tears or bone fractures may require surgery.

Watch this video to learn more about the ligaments of the ankle joint, ankle sprains, and treatment.

Watch this video to learn more about the ligaments of the ankle joint, ankle sprains, and treatment. During an inversion ankle sprain injury, all three ligaments that resist excessive inversion of the foot may be injured. What is the sequence in which these three ligaments are injured?

Chapter Review

Although synovial joints share many common features, each joint of the body is specialized for certain movements and activities. The joints of the upper limb provide for large ranges of motion, which give the upper limb great mobility, thus enabling actions such as the throwing of a ball or typing on a keyboard. The joints of the lower limb are more robust, giving them greater strength and the stability needed to support the body weight during running, jumping, or kicking activities.

The glenohumeral (shoulder) joint is a multiaxial ball-and-socket joint that provides flexion/extension, abduction/adduction, circumduction, and medial/lateral rotation of the humerus. The head of the humerus articulates with the glenoid cavity of the scapula. The glenoid labrum extends around the margin of the glenoid cavity. Intrinsic ligaments, including the coracohumeral ligament and glenohumeral ligaments, provide some support for the shoulder joint. However, the primary support comes from muscles crossing the joint whose tendons form the rotator cuff. These muscle tendons are protected from friction against the scapula by the subacromial bursa and subscapular bursa.

The elbow is a uniaxial hinge joint that allows for flexion/extension of the forearm. It includes the humeroulnar joint and the humeroradial joint. The medial elbow is supported by the ulnar collateral ligament and the radial collateral ligament supports the lateral side. These ligaments prevent side-to-side movements and resist hyperextension of the elbow. The proximal radioulnar joint is a pivot joint that allows for rotation of the radius during pronation/supination of the forearm. The annular ligament surrounds the head of the radius to hold it in place at this joint.

The hip joint is a ball-and-socket joint whose motions are more restricted than at the shoulder to provide greater stability during weight bearing. The hip joint is the articulation between the head of the femur and the acetabulum of the hip bone. The acetabulum is deepened by the acetabular labrum. The iliofemoral, pubofemoral, and ischiofemoral ligaments strongly support the hip joint in the upright, standing position. The ligament of the head of the femur provides little support but carries an important artery that supplies the femur.

The knee includes three articulations. The femoropatellar joint is between the patella and distal femur. The patella, a sesamoid bone incorporated into the tendon of the quadriceps femoris muscle of the anterior thigh, serves to protect this tendon from rubbing against the distal femur during knee movements. The medial and lateral tibiofemoral joints, between the condyles of the femur and condyles of the tibia, are modified hinge joints that allow for knee extension and flexion. During these movements, the condyles of the femur both roll and glide over the surface of the tibia. As the knee comes into full extension, a slight medial rotation of the femur serves to "lock" the knee into its most stable, weight-bearing position. The reverse motion, a small lateral rotation of the femur, is required to initiate knee flexion. When the knee is flexed, some rotation of the leg is available.

Two extrinsic ligaments, the tibial collateral ligament on the medial side and the fibular collateral ligament on the lateral side, serve to resist hyperextension or rotation of the extended knee joint. Two intracapsular ligaments, the anterior cruciate ligament and posterior cruciate ligament, span between the tibia and the inner aspects of the femoral condyles. The anterior cruciate ligament resists hyperextension of the knee, while the posterior cruciate ligament prevents anterior sliding of the femur, thus supporting the knee when it is flexed and weight bearing. The medial and lateral menisci, located between the femoral and tibial condyles, are articular discs that provide padding and improve the fit between the bones.

The talocrural joint forms the ankle. It consists of the articulation between the talus bone and the medial malleolus of the tibia, the distal end of the tibia, and the lateral malleolus of the fibula. This is a uniaxial hinge joint that allows only dorsiflexion and plantar flexion of the foot. Gliding motions at the subtalar and intertarsal joints of the foot allow for inversion/eversion of the foot. The ankle joint is supported on the medial side by the deltoid ligament, which prevents side-to-side motions of the talus at the talocrural joint and resists excessive eversion of the foot. The lateral ankle is supported by the anterior and posterior talofibular ligaments and the calcaneofibular ligament. These support the ankle joint and also resist excess inversion of the foot. An inversion ankle sprain, a common injury, will result in injury to one or more of these lateral ankle ligaments.

Interactive Link Questions

The first motion is rotation (hinging) of the mandible, but this only produces about 20 mm (0.78 in) of mouth opening.

Watch this video for a tutorial on the anatomy of the shoulder joint. What movements are available at the shoulder joint?

The shoulder joint is a ball-and-socket joint that allows for flexion-extension, abduction-adduction, medial rotation, lateral rotation, and circumduction of the humerus.

Watch this video to learn about the anatomy of the shoulder joint, including bones, joints, muscles, nerves, and blood vessels. What is the shape of the glenoid labrum in cross-section, and what is the importance of this shape?

The glenoid labrum is wedge-shaped in cross-section. This is important because it creates an elevated rim around the glenoid cavity, which creates a deeper socket for the head of the humerus to fit into.

Watch this animation to learn more about the anatomy of the elbow joint. What structures provide the main stability for the elbow?

The structures that stabilize the elbow include the coronoid process, the radial (lateral) collateral ligament, and the anterior portion of the ulnar (medial) collateral ligament.

Watch this video to learn more about the anatomy of the elbow joint, including bones, joints, muscles, nerves, and blood vessels. What are the functions of the articular cartilage?

The articular cartilage functions to absorb shock and to provide an extremely smooth surface that makes movement between bones easy, without damaging the bones.

Watch this video for a tutorial on the anatomy of the hip joint. What is a possible consequence following a fracture of the femoral neck within the capsule of the hip joint?

An intracapsular fracture of the neck of the femur can result in disruption of the arterial blood supply to the head of the femur, which may lead to avascular necrosis of the femoral head.

Watch this video to learn more about the anatomy of the hip joint, including bones, joints, muscles, nerves, and blood vessels. Where is the articular cartilage thickest within the hip joint?

The articular cartilage is thickest in the upper and back part of the acetabulum, the socket portion of the hip joint. These regions receive most of the force from the head of the femur during walking and running.

Watch this video to learn more about the flexion and extension of the knee, as the femur both rolls and glides on the tibia to maintain stable contact between the bones in all knee positions. The patella glides along a groove on the anterior side of the distal femur. The collateral ligaments on the sides of the knee become tight in the fully extended position to help stabilize the knee. The posterior cruciate ligament supports the knee when flexed and the anterior cruciate ligament becomes tight when the knee comes into full extension to resist hyperextension. What are the ligaments that support the knee joint?

There are five ligaments associated with the knee joint. The tibial collateral ligament is located on the medial side of the knee and the fibular collateral ligament is located on the lateral side. The anterior and posterior cruciate ligaments are located inside the knee joint.

Watch this video to learn more about the anatomy of the knee joint, including bones, joints, muscles, nerves, and blood vessels. Which ligament of the knee keeps the tibia from sliding too far forward in relation to the femur and which ligament keeps the tibia from sliding too far backward?

The anterior cruciate ligament prevents the tibia from sliding too far forward in relation to the femur and the posterior cruciate ligament keeps the tibia from sliding too far backward.

Watch this video to learn more about different knee injuries and diagnostic testing of the knee. What are the most causes of anterior cruciate ligament injury?

The anterior cruciate ligament (ACL) is most commonly injured when traumatic force is applied to the knee during a twisting motion or when side standing or landing from a jump.

Watch this video for a tutorial on the anatomy of the ankle joint. What are the three ligaments found on the lateral side of the ankle joint?

The ligaments of the lateral ankle are the anterior and posterior talofibular ligaments and the calcaneofibular ligament. These ligaments support the ankle joint and resist excess inversion of the foot.

Watch this video to learn more about the anatomy of the ankle joint, including bones, joints, muscles, nerves, and blood vessels. The ankle joint resembles what type of joint used in woodworking?

Because of the square shape of the ankle joint, it has been compared to a mortise-and-tendon type of joint.

Watch this video to learn about the ligaments of the ankle joint, ankle sprains, and treatment. During an inversion ankle sprain injury, all three ligaments that resist excessive inversion of the foot may be injured. What is the sequence in which these three ligaments are injured?

An inversion ankle sprain may injure all three ligaments located on the lateral side of the ankle. The sequence of injury would be the anterior talofibular ligament first, followed by the calcaneofibular ligament second, and finally, the posterior talofibular ligament third.

Review Questions

1. The primary support for the glenohumeral joint is provided by the ________.

A. coracohumeral ligament
B. glenoid labrum
C. rotator cuff muscles
D. subacromial bursa

2. The proximal radioulnar joint ________.

A. is supported by the annular ligament
B. contains an articular disc that strongly unites

the bones

C. is supported by the ulnar collateral ligament

D. is a hinge joint that allows for flexion/extension of the forearm

3. Which statement is true concerning the knee joint?

A. The lateral meniscus is an intrinsic ligament located on the lateral side of the knee joint.

B. Hyperextension is resisted by the posterior cruciate ligament.

C. The anterior cruciate ligament supports the knee when it is flexed and weight bearing.

D. The medial meniscus is attached to the tibial collateral ligament.

4. The ankle joint ________.

A. is also called the subtalar joint

B. allows for gliding movements that produce inversion/eversion of the foot

C. is a uniaxial hinge joint

D. is supported by the tibial collateral ligament on the lateral side

Critical Thinking Questions

1. Discuss the structures that contribute to support of the shoulder joint.

2. Describe the sequence of injuries that may occur if the extended, weight-bearing knee receives a very strong blow to the lateral side of the knee.

Glossary

acetabular labrum
: lip of fibrocartilage that surrounds outer margin of the acetabulum on the hip bone

annular ligament
: intrinsic ligament of the elbow articular capsule that surrounds and supports the head of the radius at the proximal radioulnar joint

anterior cruciate ligament
: intracapsular ligament of the knee; extends from anterior, superior surface of the tibia to the inner aspect of the lateral condyle of the femur; resists hyperextension of knee

anterior talofibular ligament
: intrinsic ligament located on the lateral side of the ankle joint, between talus bone and lateral malleolus of fibula; supports talus at the talocrural joint and resists excess inversion of the foot

calcaneofibular ligament
: intrinsic ligament located on the lateral side of the ankle joint, between the calcaneus bone and lateral malleolus of the fibula; supports the talus bone at the ankle joint and resists excess inversion of the foot

coracohumeral ligament
: intrinsic ligament of the shoulder joint; runs from the coracoid process of the scapula to the anterior humerus

deltoid ligament
broad intrinsic ligament located on the medial side of the ankle joint; supports the talus at the talocrural joint and resists excess eversion of the foot

elbow joint
humeroulnar joint

femoropatellar joint
portion of the knee joint consisting of the articulation between the distal femur and the patella

fibular collateral ligament
extrinsic ligament of the knee joint that spans from the lateral epicondyle of the femur to the head of the fibula; resists hyperextension and rotation of the extended knee

glenohumeral joint
shoulder joint; articulation between the glenoid cavity of the scapula and head of the humerus; multiaxial ball-and-socket joint that allows for flexion/extension, abduction/adduction, circumduction, and medial/lateral rotation of the humerus

glenohumeral ligament
one of the three intrinsic ligaments of the shoulder joint that strengthen the anterior articular capsule

glenoid labrum
lip of fibrocartilage located around the outside margin of the glenoid cavity of the scapula

humeroradial joint
articulation between the capitulum of the humerus and head of the radius

humeroulnar joint
articulation between the trochlea of humerus and the trochlear notch of the ulna; uniaxial hinge joint that allows for flexion/extension of the forearm

iliofemoral ligament
intrinsic ligament spanning from the ilium of the hip bone to the femur, on the superior-anterior aspect of the hip

joint

ischiofemoral ligament

intrinsic ligament spanning from the ischium of the hip bone to the femur, on the posterior aspect of the hip joint

lateral meniscus

C-shaped fibrocartilage articular disc located at the knee, between the lateral condyle of the femur and the lateral condyle of the tibia

lateral tibiofemoral joint

portion of the knee consisting of the articulation between the lateral condyle of the tibia and the lateral condyle of the femur; allows for flexion/extension at the knee

ligament of the head of the femur

intracapsular ligament that runs from the acetabulum of the hip bone to the head of the femur

medial meniscus

C-shaped fibrocartilage articular disc located at the knee, between the medial condyle of the femur and medial condyle of the tibia

medial tibiofemoral joint

portion of the knee consisting of the articulation between the medial condyle of the tibia and the medial condyle of the femur; allows for flexion/extension at the knee

patellar ligament

ligament spanning from the patella to the anterior tibia; serves as the final attachment for the quadriceps femoris muscle

posterior cruciate ligament

intracapsular ligament of the knee; extends from the posterior, superior surface of the tibia to the inner aspect of the medial condyle of the femur; prevents anterior displacement of the femur when the knee is flexed and weight bearing

posterior talofibular ligament

intrinsic ligament located on the lateral side of the ankle joint, between the talus bone and lateral malleolus of the fibula; supports the talus at the talocrural joint and resists excess inversion of the foot

pubofemoral ligament
intrinsic ligament spanning from the pubis of the hip bone to the femur, on the anterior-inferior aspect of the hip joint

radial collateral ligament
intrinsic ligament on the lateral side of the elbow joint; runs from the lateral epicondyle of humerus to merge with the annular ligament

rotator cuff
strong connective tissue structure formed by the fusion of four rotator cuff muscle tendons to the articular capsule of the shoulder joint; surrounds and supports superior, anterior, lateral, and posterior sides of the humeral head

subacromial bursa
bursa that protects the supraspinatus muscle tendon and superior end of the humerus from rubbing against the acromion of the scapula

subscapular bursa
bursa that prevents rubbing of the subscapularis muscle tendon against the scapula

subtalar joint
articulation between the talus and calcaneus bones of the foot; allows motions that contribute to inversion/eversion of the foot

talocrural joint
ankle joint; articulation between the talus bone of the foot and medial malleolus of the tibia, distal tibia, and lateral malleolus of the fibula; a uniaxial hinge joint that allows only for dorsiflexion and plantar flexion of the foot

tibial collateral ligament
extrinsic ligament of knee joint that spans from the medial epicondyle of the femur to the medial tibia; resists

hyperextension and rotation of extended knee

ulnar collateral ligament

intrinsic ligament on the medial side of the elbow joint; spans from the medial epicondyle of the humerus to the medial ulna

Solutions

Answers for Review Questions

1. C
2. A
3. D
4. C

Answers for Critical Thinking Questions

1. The shoulder joint allows for a large range of motion. The primary support for the shoulder joint is provided by the four rotator cuff muscles. These muscles serve as “dynamic ligaments” and thus can modulate their strengths of contraction as needed to hold the head of the humerus in position at the glenoid fossa. Additional but weaker support comes from the coracohumeral ligament, an intrinsic ligament that supports the superior aspect of the shoulder joint, and the glenohumeral ligaments, which are intrinsic ligaments that support the anterior side of the joint.
2. A strong blow to the lateral side of the extended knee will cause the medial side of the knee joint to open, resulting in a sequence of three injuries.

First will be damage to the tibial collateral ligament. Since the medial meniscus is attached to the tibial collateral ligament, the meniscus is also injured. The third structure injured would be the anterior cruciate ligament.

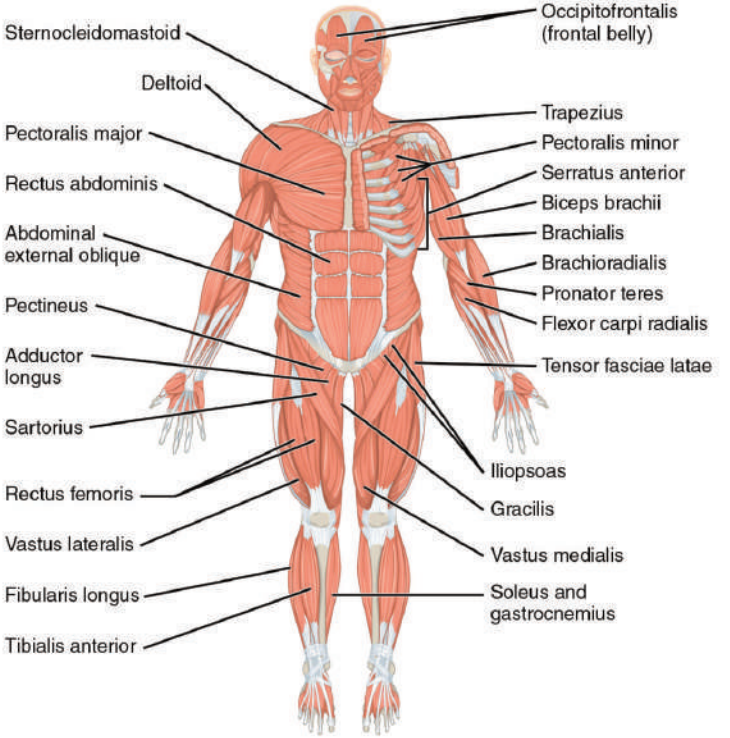
14. 2.3 Muscular Anatomy

Figure 1. A Body in Motion. The muscular system allows us to move, flex and contort our bodies. Practicing yoga, as pictured here, is a good example of the voluntary use of the muscular system. (credit: Dmitry Yanchylenko)

Think about the things that you do each day—talking, walking, sitting, standing, and running—all of these activities require movement of particular skeletal muscles. Skeletal muscles are even used during sleep. The diaphragm is a sheet of skeletal muscle that has to contract and relax for you to breathe day and night. If you recall from your study of the skeletal system and joints, body movement occurs around the joints in the body. The focus of this chapter is on skeletal muscle organization. The system to name skeletal muscles will be explained; in some cases, the muscle is named by its shape, and in other cases it is named by its location or attachments to the skeleton. If you understand the meaning of the name of the muscle, often it will help you remember its location and/or what it does.

15. 2.3.1 Naming Skeletal Muscles

The Greeks and Romans conducted the first studies done on the human body in Western culture. The educated class of subsequent societies studied Latin and Greek, and therefore the early pioneers of anatomy continued to apply Latin and Greek terminology or roots when they named the skeletal muscles. The large number of muscles in the body and unfamiliar words can make learning the names of the muscles in the body seem daunting, but understanding the etymology can help. Etymology is the study of how the root of a particular word entered a language and how the use of the word evolved over time. Taking the time to learn the root of the words is crucial to understanding the vocabulary of anatomy and physiology. When you understand the names of muscles it will help you remember where the muscles are located and what they do (Figure 1, Figure 2, and Table 2). Pronunciation of words and terms will take a bit of time to master, but after you have some basic information; the correct names and pronunciations will become easier.



Major muscles of the body. Right side: superficial; left side: deep (anterior view)

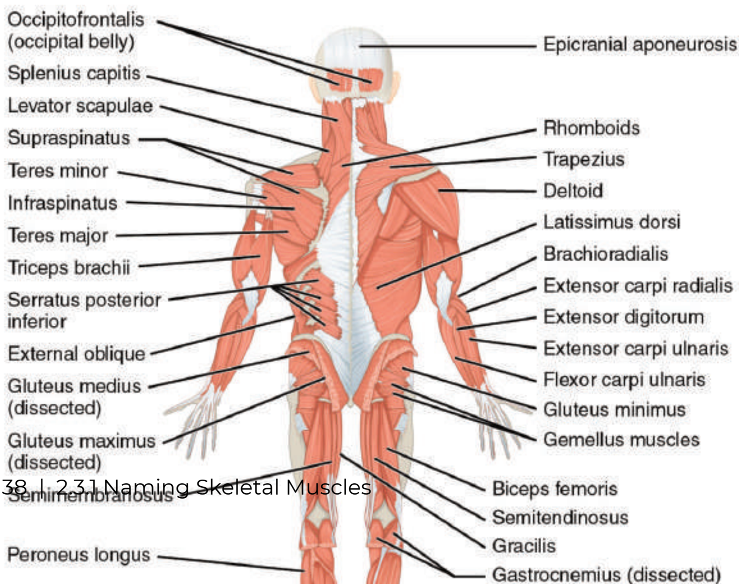


Figure 1. Overview of the Muscular System. On the anterior and posterior views of the muscular system above, superficial muscles (those at the surface) are shown on the right side of the body while deep muscles (those underneath the superficial muscles) are shown on the left half of the body. For the legs, superficial muscles are shown in the anterior view while the posterior view shows both superficial and deep muscles.

Example	Word	Latin Root 1	Latin Root 2	Meaning	Translation
abductor digiti minimi	abductor	ab = away from	duct = to move	a muscle that moves away from	A muscle that moves the little finger or toe away
	digiti	digitus = digit		refers to a finger or toe	
	minimi	minimus = mini, tiny		little	
adductor digiti minimi	adductor	ad = to, toward	duct = to move	a muscle that moves towards	A muscle that moves the little finger or toe toward
	digiti	digitus = digit		refers to a finger or toe	
	minimi	minimus = mini, tiny		little	

Figure 2. Understanding a Muscle Name from the Latin

Mnemonic Device for Latin Roots (Table 2)

Example	Latin or Greek Translation	Mnemonic Device
ad	to; toward	ADvance toward your goal
ab	away from	n/a
sub	under	SUBmarines move under water.
ductor	something that moves	A conDUCTOR makes a train move.
anti	against	If you are antisocial, you are against engaging in social activities.
epi	on top of	n/a
apo	to the side of	n/a
longissimus	longest	“Longissimus” is longer than the word “long.”
longus	long	long
brevis	short	brief
maximus	large	max
medius	medium	“Medius” and “medium” both begin with “med.”
minimus	tiny; little	mini
rectus	straight	To RECTify a situation is to straighten it out.
multi	many	If something is MULTIcolored, it has many colors.
uni	one	A UNIcorn has one horn.
bi/di	two	If a ring is DIcast, it is made of two metals.
tri	three	TRIple the amount of money is three times as much.
quad	four	QUADruplets are four children born at one birth.
externus	outside	EXternal
internus	inside	INternal

Anatomists name the skeletal muscles according to a number of criteria, each of which describes the muscle in some way. These include naming the muscle after its shape, its size compared to other muscles in the area, its location in the body or the location of its attachments to the skeleton, how many origins it has, or its action.

The skeletal muscle's anatomical location or its relationship to a particular bone often determines its name. For example, the frontalis muscle is located on top of the frontal bone of the skull. Similarly, the shapes of some muscles are very distinctive and the names, such as orbicularis, reflect the shape. For the buttocks, the size of the muscles influences the names: gluteus **maximus** (largest), gluteus **medius** (medium), and the gluteus **minimus** (smallest). Names were given to indicate length—**brevis** (short), **longus** (long)—and to identify position relative to the midline: **lateralis** (to the outside away from the midline), and **medialis** (toward the midline). The direction of the muscle fibers and fascicles are used to describe muscles relative to the midline, such as the **rectus** (straight) abdominis, or the **oblique** (at an angle) muscles of the abdomen.

Some muscle names indicate the number of muscles in a group. One example of this is the quadriceps, a group of four muscles located on the anterior (front) thigh. Other muscle names can provide information as to how many origins a particular muscle has, such as the biceps brachii. The prefix **bi** indicates that the muscle has two origins and **tri** indicates three origins.

The location of a muscle's attachment can also appear in its name. When the name of a muscle is based on the attachments, the origin is always named first. For instance, the sternocleidomastoid muscle of the neck has a dual origin on the sternum (sterno) and clavicle (cleido), and it inserts on the mastoid process of the temporal bone. The last feature by which to name a muscle is its action. When muscles are named for the movement they produce, one can find action words in their name. Some examples are **flexor** (decreases the angle at the joint), **extensor** (increases the angle at the joint), **abductor** (moves the bone away from the midline), or **adductor** (moves the bone toward the midline).

Chapter Review

Muscle names are based on many characteristics. The location of a muscle in the body is important. Some muscles are named based on their size and location, such as the gluteal muscles of the buttocks. Other muscle names can indicate the location in the body or bones with which the muscle is associated, such as the tibialis anterior. The shapes of some muscles are distinctive; for example, the direction of the muscle fibers is used to describe muscles of the body midline. The origin and/or insertion can also be features used to name a muscle; examples are the biceps brachii, triceps brachii, and the pectoralis major.

Glossary

abductor
: moves the bone away from the midline

adductor
: moves the bone toward the midline

bi
: two

brevis
: short

extensor
: muscle that increases the angle at the joint

flexor
: muscle that decreases the angle at the joint

lateralis
: to the outside

longus
: long

maximus
: largest

medialis
: to the inside

medius
: medium

minimus
: smallest

oblique
: at an angle

rectus
: straight

tri
: three

Solutions

Answers for Review Questions

1. A
2. C
3. D
4. C

Answers for Critical Thinking Questions

1. In anatomy and physiology, many word roots

are Latin or Greek. Portions, or roots, of the word give us clues about the function, shape, action, or location of a muscle.

16. 2.4 Human Dimensions and Joint Angles

It is sometimes important to know the person you are assessing's height or limb length. The following are brief description of the measurements:

Height (Length)

Height is a common body measurement typically measured in meters **(m)** or centimeters (**cm**). These are length measurements, so the SI unit would be meters. Height is typically measured with the participant standing straight, near a wall, with both feet flat on the ground.

Segment Length

Segments are measured in centimeters (**cm**) or millimeters (**mm**). Each segment (limb) is measured by identifying bony protuberances on each end of the segment. For example, the length of the tibia is measured from the medial condyle at the knee to the medial malleolus at the ankle. The fibula would be measured from the head of the fibula at the knee to the lateral malleolus at the ankle.

Joint Angles

Biomechanists quantify different types of joint angles:

1. Relative – This represents the angle between two segments. It can also be called a joint angle. For example, the angle between the shank (lower leg) and foot is called the ankle angle. There are two sub-types:
    1. Included: The absolute angle between two segments
    2. Anatomical: The angle between two segments relative to the anatomical position.
2. Absolute – An absolute angle, also called a segment angle, is the angle of a segment relative to the perfect horizontal. It is calculated by drawing a horizontal line at the distal end of the segment and measuring the angle from the right horizontal to the segment in a counterclockwise direction.

Range of Motion

Range of motion is a common body measurement, especially while diagnosing injury or disease, tracking progress during physical therapy, or working to improve flexibility or form. It is usually measured with a goniometer. Range of motion is calculate in each plane of movement respectively (sagittal, frontal or transverse). Range of motion can be defined as an angle measured in *degrees* (°)through which a joint moves away from a reference position as seen in this video demonstration of how to use a goniometer for range of motion measurement. It can also be measured as the difference between two extremes of motion (relative abduction vs adduction angle for example).

For example, to calculate the total range of motion at the knee in the sagittal plane you measure the angle between the thigh and the lower leg (knee angle) at full extension. You then measure the same knee angle at full flexion. The difference between full extension and full flexion, represents the range of motion at the knee.

Range of motion is an important predictor of injury prevention and performance in many sport. For example, the range of motion at the Hallux (the big toe) should be 75-85 degrees for extension and 35-45 degrees for flexion. A reduction in range of motion can lead to pain (toe, knee and hip) and difficulty with certain activities such as squatting and running. An ideal range of motion was established for each joint and can be found in Physical Therapy manuals.

Reinforcement Activity

What range of motion do you have at the shoulder? Is the range of motion on your left side the same as your right? How could you explain a difference between the range of motion of the right and left side?

PART III

CHAPTER 3: LINEAR KINEMATICS IN ONE-DIMENSION

Chapter Objectives

After this chapter, you will be able to:

- Define the term 'kinematics'
- Be able to describe movement in one dimension with time, displacement, velocity and acceleration
- Differentiate between vector and scalar variables
- Identify coordinates within a coordinate system
- Graphically analyze movement in one-dimension
- Predict the trajectory of projectiles

In this Chapter we will begin our discussion of kinematics. Remember that kinematics is concerned with the description of motion. The outcome of many sporting events are kinematic measures therefore an understanding of those measures and the way to appropriately report them is mandatory in a biomechanics course. For example, a 100-m winner is determined by the **time** is takes to complete the distance. A high jumper wins by jumping a longer **displacement** than his/

her opponent. Time and displacement are both kinematic measures. We will also discuss **speed**, **velocity** and **acceleration** as variables used to describe movement. At the end of this chapter you will be able to manipulate these variables to predict the movement of a projectile.

17. 3.0 Introduction

Figure 1. The results of many track and field events are the outcome of kinematic variables. Runners will try to cover the distance in the shortest time possible whereas jumpers will attempt to cover a greater displacement than their opponent.

Movement is everywhere we look. When we approach the topic of human movement, we can consider the movement of the body itself (walking, jumping etc..) or of object's that move as a consequence of human motion (tennis ball, hockey stick etc..). Questions about motion are interesting in and of themselves: *How long will it take for me to walk to work this morning? Where will a football land if it is thrown at a certain angle*? But an understanding of motion is also key to understanding other concepts in biomechanics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of biomechanics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word “kinematics” comes from a Greek term meaning motion and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion).

Movement can be separated into two main types: Linear and Angular. Linear motion refers to motion of a body along a straight or curved line. Angular motion refers to the movement of a body about a fixed axis. Let’s begin with the analysis of linear movement.

In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In following chapters, we apply concepts developed here to study motion along curved paths (two-dimensional motion); for example, that of a bicycle rounding a curve.

18. 3.1 Displacement

Figure 1. These cyclists can be described by their position relative to each other or to the start line. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Boris Stefanik, Unsplash).

Position

In order to describe the motion of an object or body, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a high jump would be described in terms of the position of the jumper with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. In biomechanics, we typically agree on a reference frame relative to an origin. If we are describing motion based off a picture or video, we use the bottom left-hand corner as our origin and describe movement relative to that point.Please see the figures 2 and 3 below.

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word "displacement" implies that an object has moved, or has been displaced.

DISPLACEMENT

Displacement is the *change in position* of an object:

$$\Delta\overrightarrow{\mathbf{p}} = \overrightarrow{\mathbf{p_f}} - \overrightarrow{\mathbf{p_i}}$$

where Δp is displacement, p_f is the final position (f), and p_i is the initial position (i).

In this text the upper case Greek letter **Δ** (delta) always means "change in" whatever quantity follows it; thus, $\Delta\overrightarrow{\mathbf{p}}$ means *change in position*. Always solve for displacement by subtracting initial position ($\overrightarrow{\mathbf{p_i}}$) from final position ($\overrightarrow{\mathbf{p_f}}$).

Note that the standard unit for displacement is the meter (m), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

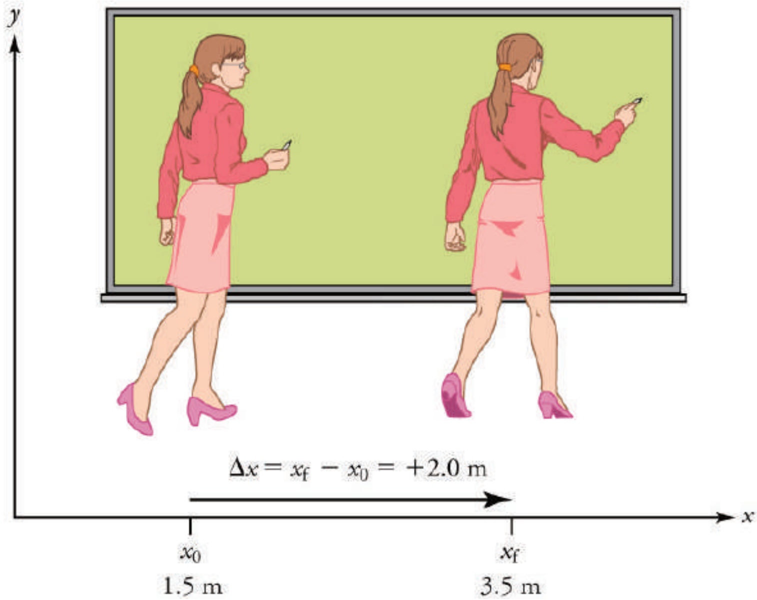


Figure 2. A professor paces left and right while lecturing. Her position relative to Earth is given by p $_x$. The +2.0 m displacement of the professor relative to Earth is represented by an arrow pointing to the right. ***Note: this figure uses the symbols x instead of p_x to denote position. Both conventions can be used.***

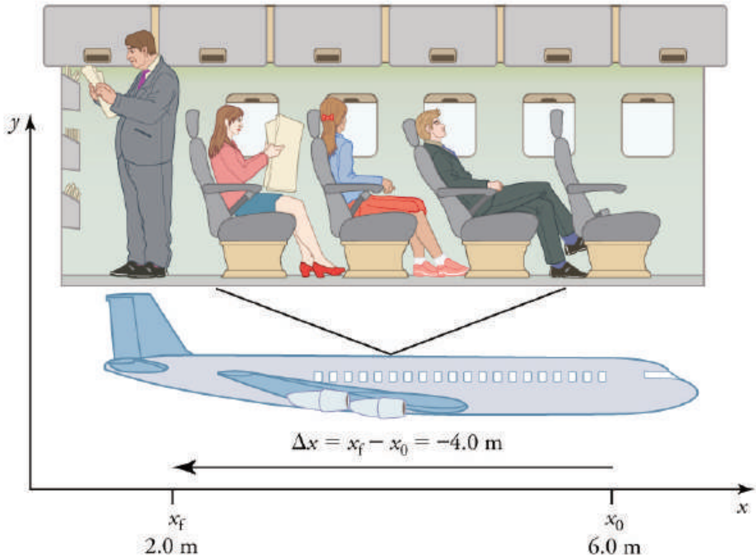


Figure 3. *A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by* **p_x**. *The* ***-4.0-m*** *displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor shown above (he moves twice as far).* ***Note: this figure uses the symbols x instead of p_x to denote position. Both conventions can be used.***

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is **p_{xi} = 1.5 m** and her final position is **p_{xf} = 3.5 m**. Thus her horizontal displacement is

$$\Delta\overrightarrow{\mathbf{p_x}} = \overrightarrow{\mathbf{p_{xf}}} - \overrightarrow{\mathbf{p_{xi}}} = \mathbf{3.5\ m} - \mathbf{1.5\ m} = \mathbf{+2.0\ m}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $\mathbf{p_{xi} = 6.0\ m}$ and his final position is $\mathbf{p_{xf} = 2.0\ m}$, so his displacement is

$$\Delta\overrightarrow{\mathbf{p_x}} = \overrightarrow{\mathbf{p_{xf}}} - \overrightarrow{\mathbf{p_{xi}}} = \mathbf{2.0\ m - 6.0\ m = -4.0\ m}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Note that movement in the vertical direction (up and down) would by specified with a 'y' instead of an 'x'. For example, if a jump moves from 0 m to 0.35 m, her displacement would be calculated as follows:

$$\Delta\overrightarrow{\mathbf{p_y}} = \overrightarrow{\mathbf{p_{yf}}} - \overrightarrow{\mathbf{p_{yi}}} = \mathbf{0.35\ m - 0\ m = 0.35\ m\ or\ 35\ cm}$$

More details on the symbols 'x' for horizontal movements and 'y' for vertical movements will be provided in the next section when we introduce coordinate systems.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

MISCONCEPTION ALERT: DISTANCE TRAVELED VS. MAGNITUDE OF DISPLACEMENT

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding 1

1: A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement $\Delta\overrightarrow{\mathbf{p}}$ is defined to be

$$\Delta\overrightarrow{\mathbf{p}} = \overrightarrow{\mathbf{p_f}} - \overrightarrow{\mathbf{p_i}}$$

where $\overrightarrow{\mathbf{p_i}}$ is the initial position and $\overrightarrow{\mathbf{p_f}}$ is the final position. In this text, the Greek letter **Δ** (delta) always means "change in" what ever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.

If you are describing movement along the horizontal axis (left to right or right to left) you can express the equation relative to the 'x' axis:

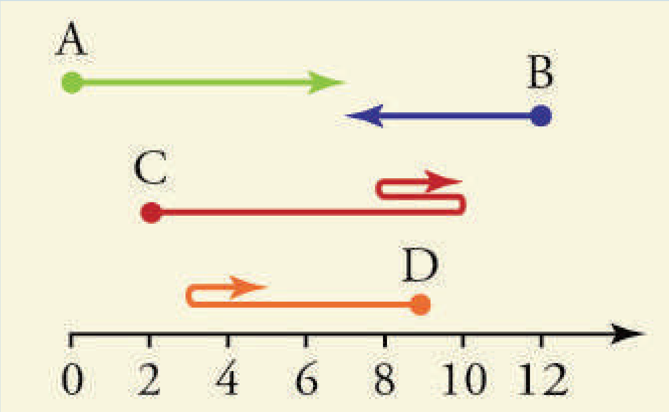
$$\Delta\overrightarrow{\mathbf{p}} = \overrightarrow{\mathbf{p_{xf}}} - \overrightarrow{\mathbf{p_{xi}}}$$

If you are describing movement along the vertical axis (up and down, down and up) you can express the equation relative to the 'y' axis:

$$\Delta\overrightarrow{\mathbf{p}} = \overrightarrow{\mathbf{p_{yf}}} - \overrightarrow{\mathbf{p_{yi}}}$$

- When you start a problem, assign which direction will be positive (typically 'right' or 'up').
- Distance is the length of the path travelled
- Displacement is the difference between the final position and the initial position.

Problems & Exercises



Four objects travelling along a one dimensional path, with the distance axis labelled.

1: Find the following for path A: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2: Find the following for path B: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

3: Find the following for path C: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

4: Find the following for path D: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Glossary

kinematics
: the study of motion without considering its causes

position
: the location of an object at a particular time

displacement
: the change in position of an object

distance
: the magnitude of displacement between two positions

distance traveled
: the total length of the path traveled between two positions

Solutions

Check Your Understanding 1

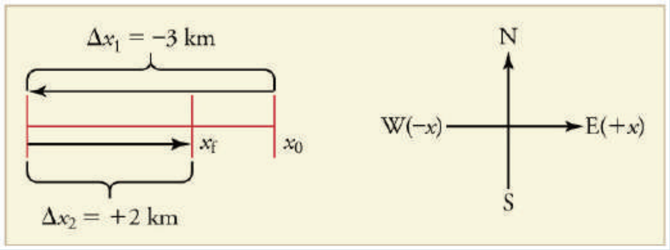


Figure 5.

1: (a) The rider's displacement is $\Delta \overrightarrow{\mathbf{p}} = \overrightarrow{\mathbf{p}_{\mathbf{xf}}} - \overrightarrow{\mathbf{p}_{\mathbf{xi}}}$. The displacement is negative because we take east to be positive and west to be negative. Or you could just say “1 km to the West”. Note that the drawing clearly showed that West was chosen to be negative. (b) The distance traveled is 3 km + 2 km = 5 km. (c) The magnitude of the displacement is 1 km.

Problems & Exercises

1: (a) 7 m (b) 7 m (c) + 7 m

2: (a) 5 m (b) 5 m (c) – 5 m

3: This is badly drawn so the answers are debatable. Assuming it went from a position of 2 m to 10 then back to 8 and then back again to 10 m that gives a) distance of 12 m b) magnitude of the displacement as 8 m and c) a displacement of +8 m or 8 metres ot the right.

4: (a) 8 m (b) 4 m (c) – 4 m

19. 3.2 Vectors, Scalars, and Coordinate Systems

Figure 1. The motion of this skateboarder can be described in terms of the distance he traveled (a scalar quantity) or his displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. (credit: Julien Lanoy, Unsplash).

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector. When writing a vector quantity, a horizontal arrow is used over the top of the variable. For example, $\overrightarrow{\mathbf{p}}$ indicates that position is a vector variable, having both a magnitude and direction associated with it.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20 °C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20 °C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet shown above, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

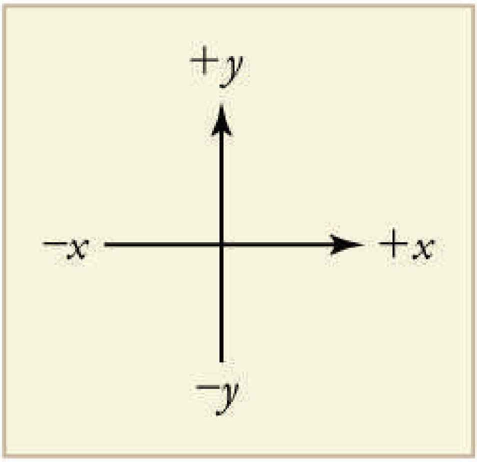


Figure 2. *It is usually convenient to consider motion upward or to the right as positive* ***(+)*** *and motion downward or to the left as negative* ***(−)****.*

This are called the Cartesian Coordinates in honour of Rene Descartes who first proposed them in the 17th Century.

Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

Conceptual Questions

1: A student writes, "*A diver heads towards the water at a speed of* -10 m/s." What is wrong with the student's statement? What has the student actually described? Explain.

2: What is the speed of the diver in the previous question?

3: Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

4: A weather forecast states that the temperature is predicted to be -5 °C the following day. Is this temperature a vector or a scalar quantity? Explain.

Glossary

scalar
: a quantity that is described by magnitude, but not direction

vector
: a quantity that is described by both magnitude and direction

Solutions

Check Your Understanding: Conceptual Questions

1: Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

2. The speed is 10 m/s.

3. A vector

4. Scalar. Temperature doesn't have a direction. The – means "less than".

20. 3.3 Time, Velocity, and Speed

Figure 1. The motion of these racing longboarders can be described by their speeds and their velocities. (credit: Alternate Skate, Unsplash).

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed earlier, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. In biomechanics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The standard unit for time is the second, abbreviated 's'.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time Δt** is the difference between the ending time and beginning time,

$$\Delta t = t_f - t_i$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_i is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_i is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_i=0$, then $\Delta t = t_f \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $t_i = 0$
- the symbol t is used for elapsed time unless otherwise specified ($\Delta t = t_i = t$)

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as kilometers per hour.

AVERAGE VELOCITY

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\overrightarrow{\mathbf{v}}_{average} = \frac{\Delta \overrightarrow{\mathbf{p}}}{\Delta \mathbf{t}} = \frac{\overrightarrow{\mathbf{p_f}} - \overrightarrow{\mathbf{p_i}}}{\mathbf{t_f} - \mathbf{t_i}}$$

where $\overrightarrow{\mathbf{v}}$ is velocity, $\Delta \overrightarrow{\mathbf{p}}$ is the change in position (or displacement), and $\overrightarrow{\mathbf{p_f}}$ and $\overrightarrow{\mathbf{p_i}}$ are the final and beginning positions at times t_f and t_i, respectively. If the starting time t_i is taken to be zero, then the average velocity is simply

$$\overrightarrow{\mathbf{v}}_{average} = \frac{\Delta \overrightarrow{\mathbf{p}}}{\Delta \mathbf{t}} = \frac{\overrightarrow{\mathbf{p_f}} - \overrightarrow{\mathbf{p_i}}}{\mathbf{t_f}}$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The standard unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\vec{\mathbf{v}}_{average} = \frac{\mathbf{displacement}}{\mathbf{time}} = \frac{-4\ \mathbf{m}}{5\ \mathbf{s}} = -0.8\ \mathbf{m/s}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

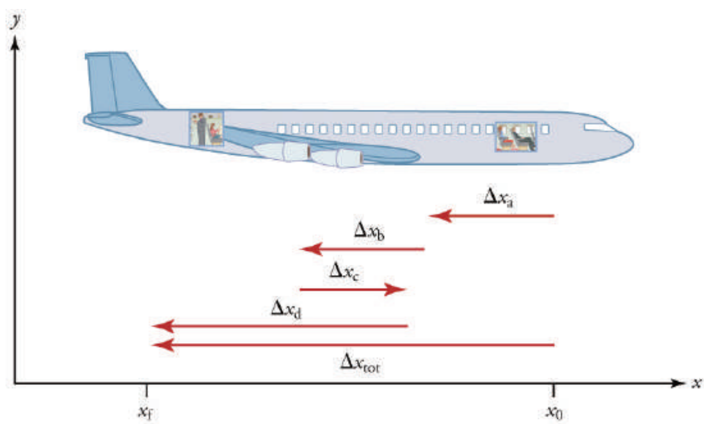


Figure 2. *A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.* ***Note: this figure uses the symbols x instead of p_x to denote position. Both conventions can be used.***

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity or the velocity at a specific instant.* A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity, $\vec{\mathbf{v}}$,** is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, $\vec{\mathbf{v}}$, at a precise instant ***t*** can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In biomechanics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction.

Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you run to a store and return home in half an hour, and the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

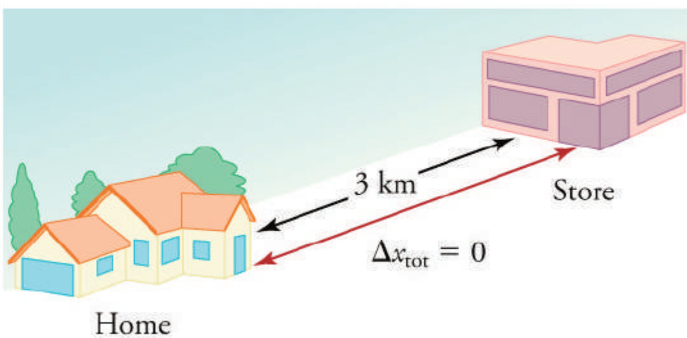
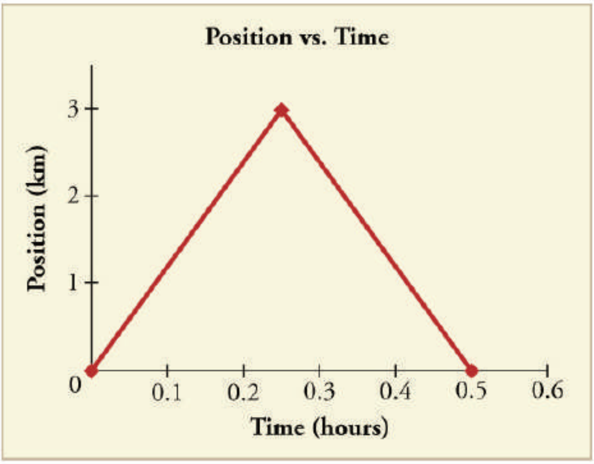


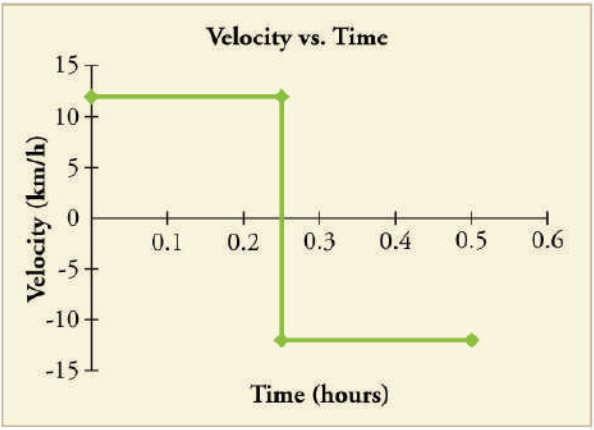
Figure 3. *During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.* ***Note: this figure uses the symbols x instead of p_x to denote position. Both conventions can be used.***

Graphing

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful.

For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in Figure 4 below. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)


Position vs. Time
Position (km)
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
Time (hours)


Velocity vs. Time
Velocity (km/h)
15
10
5
0
-5
-10
-15
0.1
0.2
0.3
0.4
0.5
0.6
Time (hours)

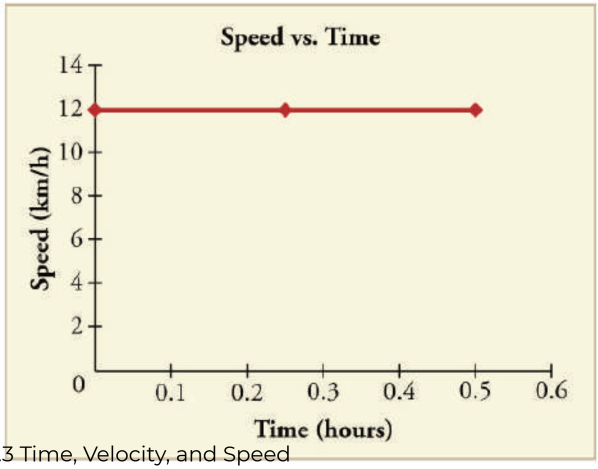

Speed vs. Time
Speed (km/h)
14
12
10
8
6
4
2
0
0.1
0.2
0.3
0.4
0.5
0.6
Time (hours)

__Figure 4.__ Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION — GETTING A SENSE OF SPEED

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 km per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h

Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is $\Delta t = t_f - t_i$

where t_f is the final time and t_i is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just ***t*.**

- Average velocity is defined as displacement divided by the travel time. In symbols, average velocity is

$$v_{average} = \Delta p/\Delta t = (p_f - p_i)/(t_f - t_i)$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity ***v*** is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

Glossary

average speed
: distance traveled divided by time during which motion occurs

average velocity
: displacement divided by time over which displacement occurs

instantaneous velocity
: velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed
: magnitude of the instantaneous velocity

time
: change, or the interval over which change occurs

model
: simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time
: the difference between the ending time and beginning time

21. 3.4 Acceleration

In everyday conversation, to accelerate means to speed up. Perhaps the most common demonstration of acceleration happens when you press on the gas pedal in your car. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

AVERAGE ACCELERATION

Average acceleration is *the rate at which velocity changes,*

$$\overrightarrow{\mathbf{a}}_{average} = \frac{\Delta\overrightarrow{\mathbf{v}}}{\Delta\mathbf{t}} = \frac{\overrightarrow{\mathbf{v_f}} - \overrightarrow{\mathbf{v_i}}}{\mathbf{t_f} - \mathbf{t_i}}$$

where

$< spanid =" import - auto - id2579029 "> \overrightarrow{\mathbf{a}}$

is average acceleration,

$< spanid =" import - auto - id2579029 "> \overrightarrow{\mathbf{v}}$

is velocity, and ***t*** is time.

Because acceleration is velocity in m/s divided by time in s, the standard units for acceleration are **m/s^2**, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a cyclist turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

ACCELERATION AS A VECTOR

Acceleration is a vector in the same direction as the *change* in velocity, $< spanid =" import - auto - id2579029 "> \Delta \vec{\mathbf{v}}$. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

MISCONCEPTION ALERT: DECELERATION VS. NEGATIVE ACCELERATION

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider the car shown below here:

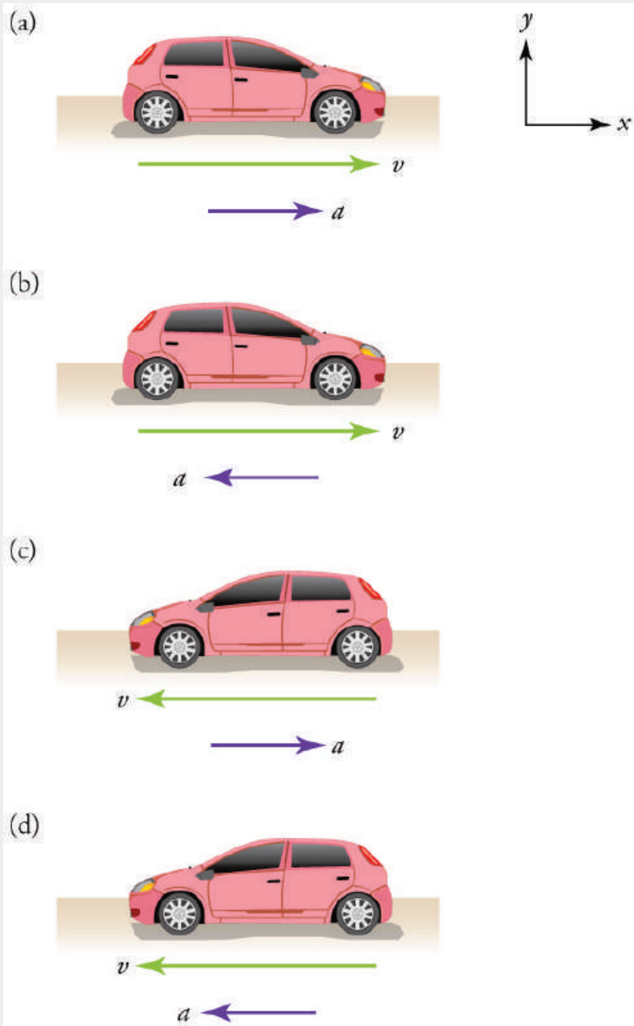


Figure 3. *(a) This car is speeding up as it moves toward the*

right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (not decelerating).

Example 1: Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

Figure 4. *(credit: Jon Sullivan, PD Photo.org).*

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

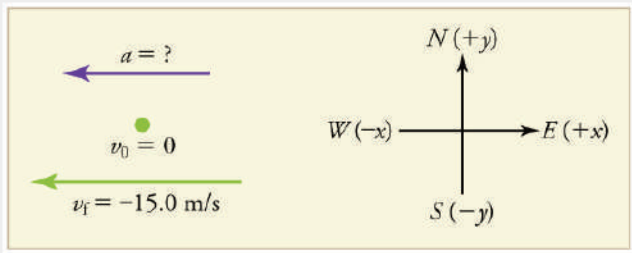


Figure 5.

We can solve this problem by identifying $< spanid =" import - auto - id2579029 "> \Delta \overrightarrow{\mathbf{v}}$ and **Δ*t*** from the given information and then calculating the average acceleration directly from the equation:

$$\overrightarrow{\mathbf{a}}_{average} = \frac{\Delta \overrightarrow{\mathbf{v}}}{\Delta \mathbf{t}} = \frac{\overrightarrow{\mathbf{v_f}} - \overrightarrow{\mathbf{v_i}}}{\mathbf{t_f} - \mathbf{t_i}}$$

Solution

1. Identify the knowns $\overrightarrow{\mathbf{v_i}} = 0 \; \mathbf{m/s}$, $\overrightarrow{\mathbf{v_f}} = -15 \; \mathbf{m/s}$, (the negative sign indicates direction toward the west), **Δt = 1.80 s**.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity:

\Delta\overrightarrow{\mathbf{v}} = \overrightarrow{\mathbf{v_f}} = -15.0 \textbf{m/s}
.

3. Plug in the known values ($\Delta\overrightarrow{\mathbf{v}}$ and $\boldsymbol{\Delta t}$) and solve for the unknown $\overrightarrow{\mathbf{a}}$.

$$\overrightarrow{\mathbf{a}}_{average} = \frac{\Delta\overrightarrow{\mathbf{v}}}{\Delta\mathbf{t}} = \frac{\overrightarrow{\mathbf{v_f}} - \overrightarrow{\mathbf{v_i}}}{\mathbf{t_f} - \mathbf{t_i}} = \frac{(-15\ \mathbf{m/s} - 0\ \mathbf{m/s})}{1.80\ \mathbf{s}} = -8.33\ \mathbf{m/s^2}$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration, $\overrightarrow{\mathbf{a}}$, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in the earlier section on speed and velocity —that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion.

Figure 6 below hows graphs of instantaneous acceleration versus time for two very different motions. In Figure 6 a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In Figure 6 b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of +3.0 m/s^2 and -2.0 m/s^2, respectively.

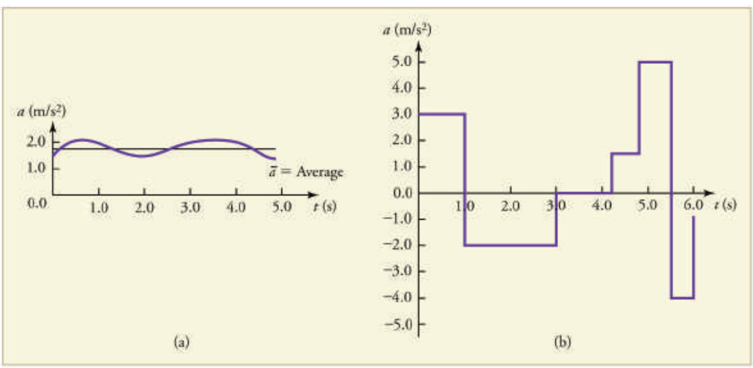


Figure 6. *Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.*

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. The crucial distinction if the acceleration is in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

PHET EXPLORATIONS: MOVING MAN SIMULATION

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

https://phet.colorado.edu/en/simulation/moving-man

PhET Interactive Simulation

Figure 13. *Moving Man.*

Section Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** is
$$\vec{\mathbf{a}}_{average} = \frac{\Delta \vec{\mathbf{v}}}{\Delta \mathbf{t}} = \frac{\vec{\mathbf{v}_\mathbf{f}} - \vec{\mathbf{v}_\mathbf{i}}}{\mathbf{t}_\mathbf{f} - \mathbf{t}_\mathbf{i}}$$
- The standard unit for acceleration is m/s^2.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration, $\vec{\mathbf{a}}$, is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

Glossary

acceleration
: the rate of change in velocity; the change in velocity over time

average acceleration
: the change in velocity divided by the time over which it changes

instantaneous acceleration
: acceleration at a specific point in time

deceleration
: acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

22. 3.5 Graphical Analysis of One-Dimensional Motion

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of displacement, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in Figure 1 a straight-line graph has the general form

$$y = mx + b$$

Here m is the **slope,** defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the **y-intercept**, which is the point at which the line crosses the vertical axis.

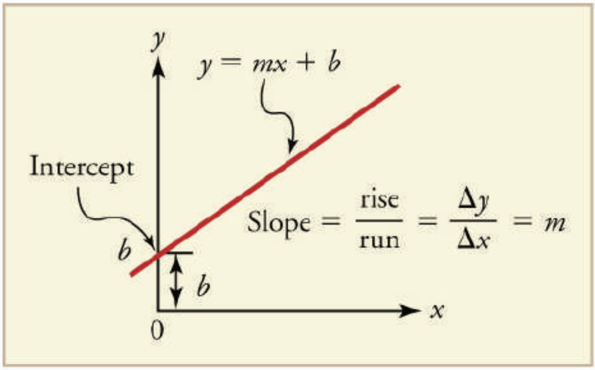


Figure 1. *A straight-line graph. The equation for a straight line is **y** = **mx** + **b.***

Graph of Displacement vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as displacement, depend upon. A graph of displacement versus time would, thus, have $\mathbf{p}_x$ *(displayed as x on graph below)* on the vertical axis and ***t*** on the horizontal axis. Figure 2 shown below is just such a straight-line graph. It shows a graph of displacement versus time for a jet-powered car on a very flat dry lake bed in Nevada**.**

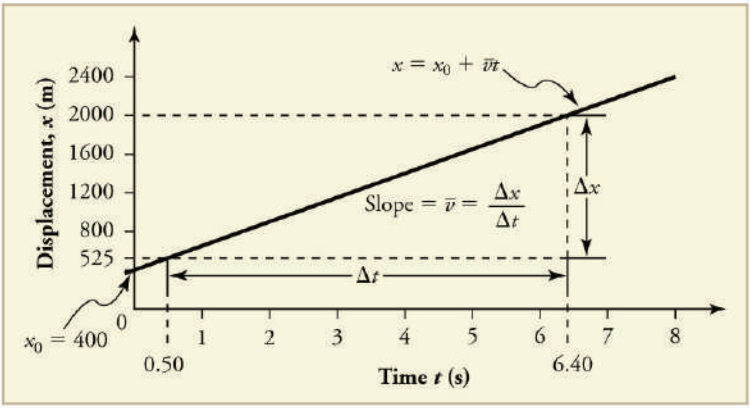


Figure 2. *Graph of displacement versus time for a jet-powered car on the Bonneville Salt Flats.* ***Note: this figure uses the symbols x instead of p_X to denote position. Both conventions can be used.***

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity or $\bar{v}$ and the intercept is displacement at time zero—that is, $\mathbf{p_{X0}}$. Substituting these symbols into ***y = mx + b*** gives

$$p_X = \text{(average velocity)} + p_{Xo}$$

Thus a graph of displacement versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation.

THE SLOPE OF X VS. T

The slope of the graph of displacement $\mathbf{p}_{\boldsymbol{x}}$ vs. time ***t*** is velocity ***v***.

slope = rise / run = $\Delta p_x / \Delta t = v_{x\ \text{average}}$ or v_x bar (by definition)

Notice that this equation is the same as that derived algebraically from other motion equations the earlier section.

From the figure we can see that the car has a displacement of 25 m at 0.50 s and 2000 m at 6.40 s. Its displacement at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example 1: Determining Average Velocity from a Graph of Displacement versus Time: Jet Car

Find the average velocity of the car whose position is graphed above in Figure 2.

Strategy

The slope of a graph of $\mathbf{p}_{\mathbf{x}}$ vs. $\boldsymbol{t}$ is average velocity, since slope equals rise over run. In this case, rise = change in displacement and run = change in time, so that

$$\text{slope} = \text{rise} / \text{run} = \Delta p_x / \Delta t = v_{x \text{ average}}$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the $\mathbf{p}_{\mathbf{x}}$ and $\boldsymbol{t}$ values of the chosen points into the equation. Remember in calculating change (**Δ**) we always use final value minus initial value.

$$\text{slope} = \text{rise} / \text{run} = \Delta p_x / \Delta t = v_{x \text{ average}} = (2000 \text{ m/s} - 525 \text{ m/s}) / (6.4 \text{ s} - 0.50 \text{ s})$$

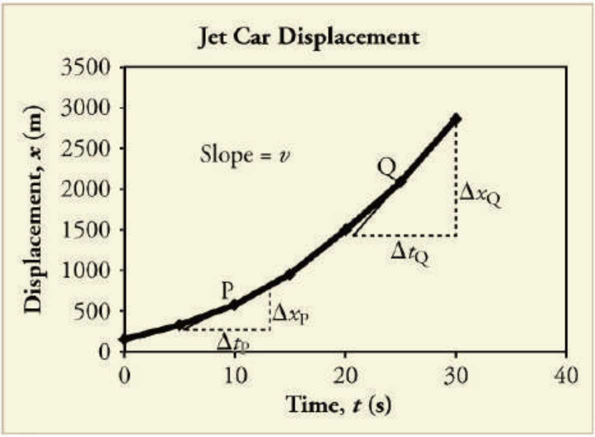
$$\text{yielding the average velocity}_x = 250 \text{ m/s}$$

Discussion

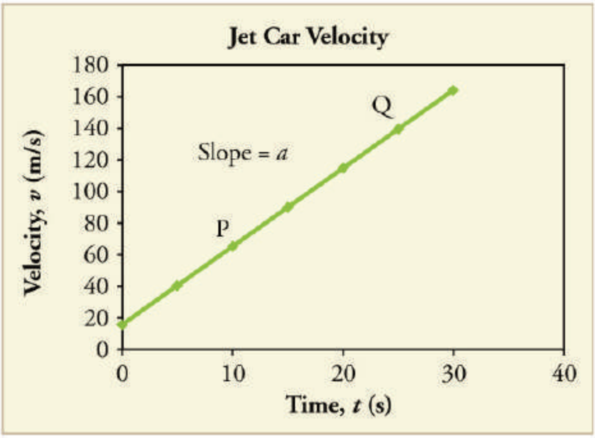
This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when α is constant but α≠0

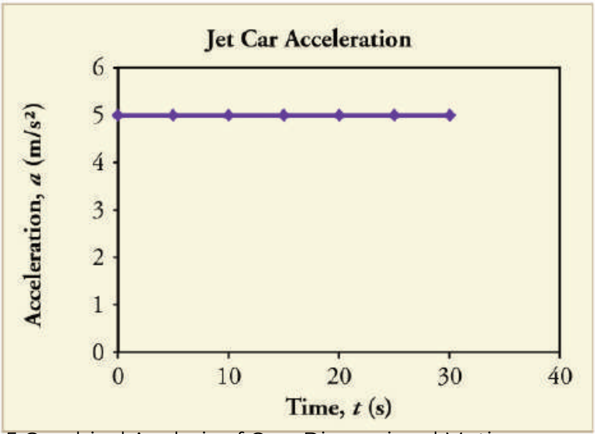
The graphs in Figure 3 below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.


Jet Car Displacement
Slope = v
P
Q
Δx_P
Δt_P
Δx_Q
Δt_Q
Displacement, x (m)
Time, t (s)
0
500
1000
1500
2000
2500
3000
3500
0
10
20
30
40

(a)


Jet Car Velocity
Slope = a
P
Q
Velocity, v (m/s)
Time, t (s)
0
20
40
60
80
100
120
140
160
180
0
10
20
30
40

(b)


Jet Car Acceleration
Acceleration, a (m/s²)
Time, t (s)
0
1
2
3
4
5
6
0
10
20
30
40

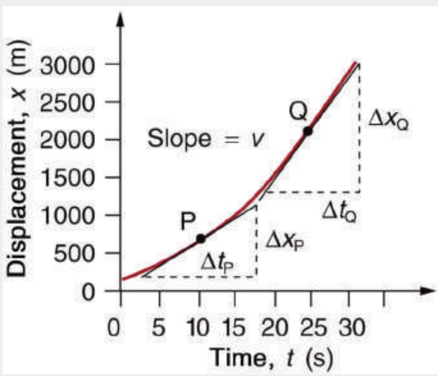
(c)

Figure 3. *Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an* $\boldsymbol{p_x}$ *vs.* $\boldsymbol{t}$ *graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the* $\boldsymbol{v_x}$ *vs.* $\boldsymbol{t}$ *graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of* $\boldsymbol{5.0\ m/s^2}$ *over the time interval plotted.*

The graph of displacement versus time in Figure 3 (a) shown above is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a displacement-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown above for two points in Figure 3 (a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in Figure 3 (b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure 3(c).

Example 2: Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the $\mathbf{p}_X$ vs. $\boldsymbol{t}$ graph in the graph below.



t (s)	x (m)
0	200
5	338
10	600
15	988
20	1500
25	2138
30	2900

Figure 5. *The slope of an* $\mathbf{p}_X$ *vs.* $\boldsymbol{t}$ *graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.*

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in the figure shown above where Q is the point at ***t*=25 s**.

Solution

1. Find the tangent line to the curve at ***t* = 25 s**.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, ***v***.

$$\mathbf{v_{X\ Q} = \Delta p_{x\ Q} / \Delta t_{Q}} = (3120\text{ m} - 1300\text{ m}) / (32\text{ s} - 19\text{ s})$$

$$\text{Thus } \mathbf{v_{XQ}} = (1820\text{ m}) / (13\text{ s}) = 140\text{ m/s}$$

Discussion

This is the value given in this figure's table for $\boldsymbol{v_X}$ at ***t* = 25 s**. The value of 140 m/s for $\mathbf{v_{X\ Q}}$ is plotted in Figure 5(b) . The entire graph of $\boldsymbol{v_X}$ vs. ***t*** can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a ***v*** vs. ***t*** graph, rise = change in velocity **Δ*v*** and run = change in time **Δ*t***.

THE SLOPE OF V VS. T

The slope of a graph of velocity v vs. time t is acceleration a.

$$\textbf{slope = average acceleration} = \Delta v \;/\; \Delta t$$

Since the velocity versus time graph in Figure 3 (b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in Figure 3(c)

Additional general information can be obtained from Figure 6 and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis p_x is t. Substituting these symbols yields

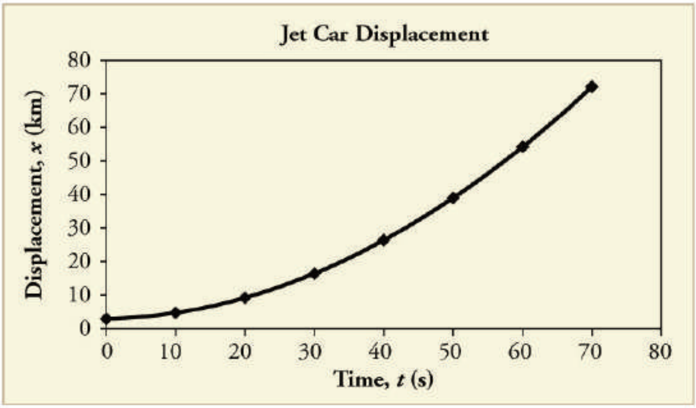
$$\mathbf{v = a\,t + v_o} \quad \text{or often written} \quad \mathbf{v = v_o + a\,t}$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in earlier sections.

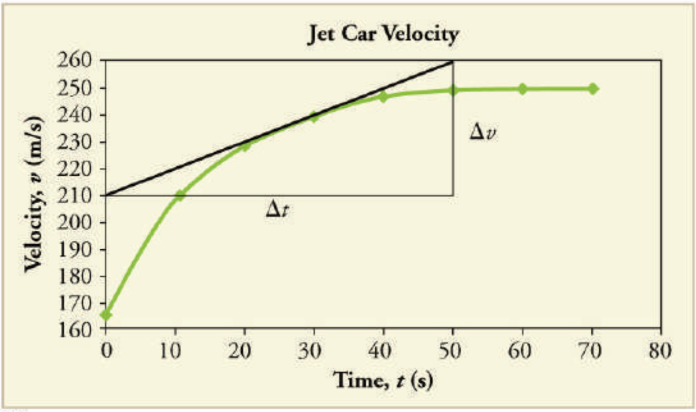
It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

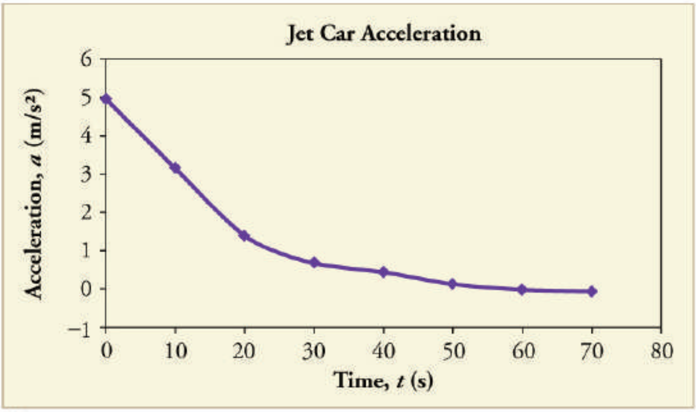
Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in Figure 6 below. Time again starts at zero, and the initial displacement and velocity are 2900 m and 165 m/s, respectively. (These were the final displacement and velocity of the car in the motion graphed in Figure 3. Acceleration gradually decreases from 5.0 m/s^2 to zero when the car hits 250 m/s. The slope of the ***x*** vs. ***t*** graph increases until ***t* = 55 s**, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.


Jet Car Displacement
Displacement, x (km)
80
70
60
50
40
30
20
10
0
0 10 20 30 40 50 60 70 80
Time, t (s)

(a)


Jet Car Velocity
Velocity, v (m/s)
260
250
240
230
220
210
200
190
180
170
160
Δv
Δt
0 10 20 30 40 50 60 70 80
Time, t (s)

(b)


Jet Car Acceleration
Acceleration, a (m/s²)
6
5
4
3
2
1
0
−1
0 10 20 30 40 50 60 70 80
Time, t (s)

(c)

Figure 6. *Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in Figure 3 ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.*

Example 3: Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the ***v*** vs. ***t*** graph in Figure 6(b).

Strategy

The slope of the curve at ***t* = 25 s** is equal to the slope of the line tangent at that point, as illustrated in Figure 6(b)

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, ***a***.

$$\text{slope} = \Delta v / \Delta t = (260\text{ m/s} - 210\text{ m/s}) / (51 - 1.0\text{ s}) = 1.0\text{ m/s}^2$$

Discussion: Note that this value for **a** is consistent with the value plotted in Figure 6 (c) at t **= 25 s**.

A graph of displacement versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Section Summary

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement $\mathbf{p}_x$ vs. time t is velocity v_x.
- The slope of a graph of velocity v_x vs. time t graph is acceleration a_x.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Conceptual Questions

1: (a) Explain how you can use the graph of position versus time in Figure 8 below o describe the change in velocity over time. Identify (b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

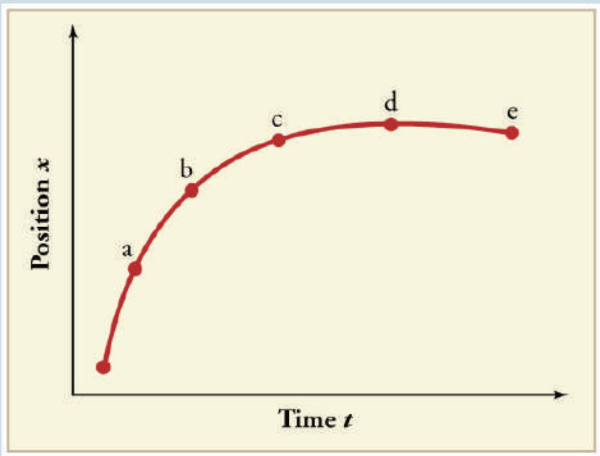


Figure 8.

2: (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in Figure 9 below (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

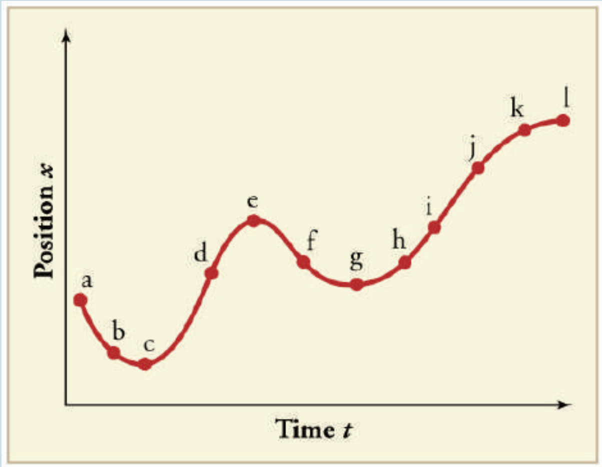


Figure 9.

3: (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure 10 below. (b) Based on the graph, how does acceleration change over time?

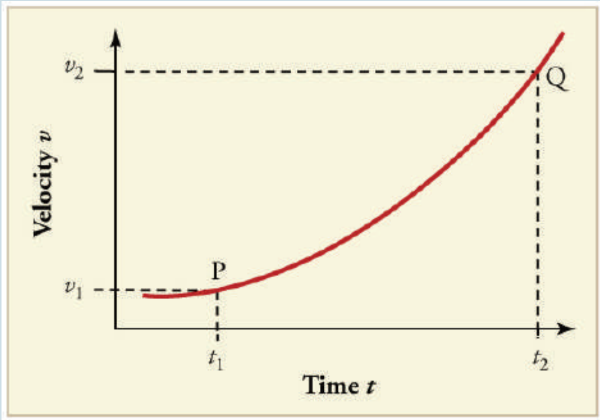


Figure 10.

4: (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 11 below. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

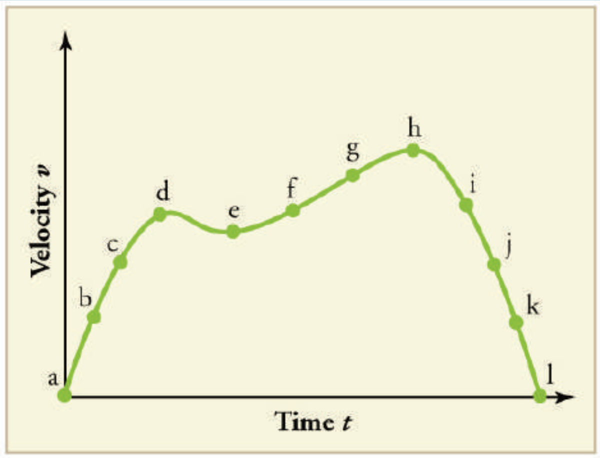


Figure 11.

5: Consider the velocity vs. time graph of a person in an elevator shown in Figure 12. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion you have used earlier in this chapter for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

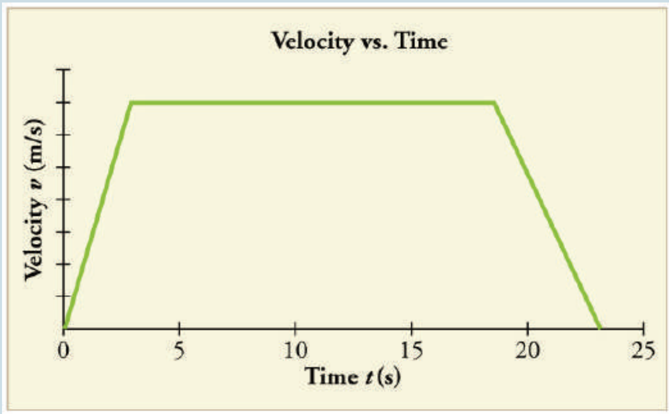


Figure 12.

6: A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

1: (a) By taking the slope of the curve in Figure 13 below, verify that the velocity of the jet car is 115 m/s at t = 20 s. (b) By taking the slope of the curve at any point in Figure 14. verify that the jet car's acceleration is 5.0 m/s^2.

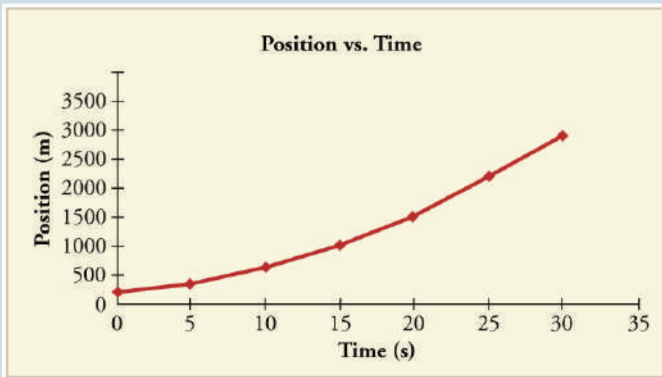


Figure 13.

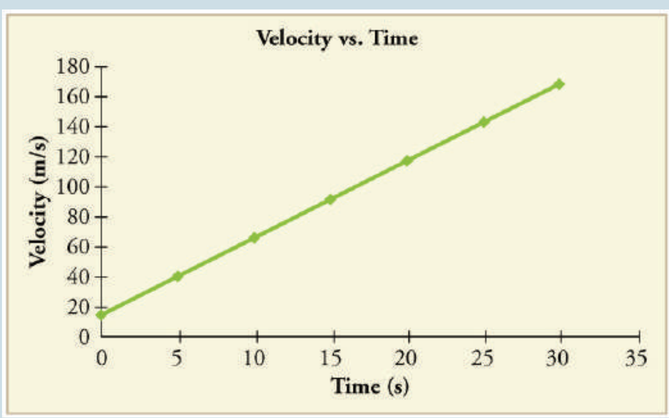


Figure 14.

2: Using approximate values, calculate the slope of the curve in Figure 15 below to verify that the velocity at t = 10.0 s is 0.208 m/s. Assume all values are known to 3 significant figures.

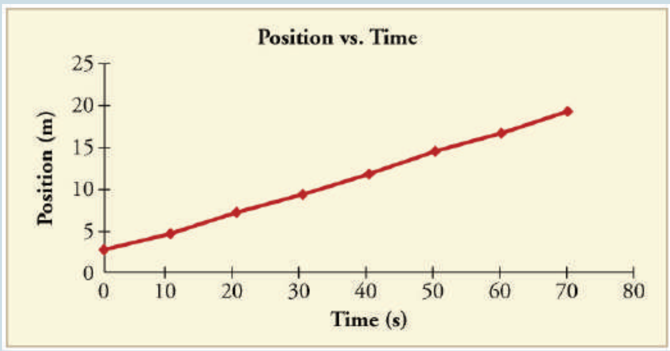


Figure 15.

3: Using approximate values, calculate the slope of the curve in Figure 15 above to verify that the velocity at t = 30.0 s is 0.238 m/s. Assume all values are known to 3 significant figures.

4: By taking the slope of the curve in Figure 16 below, verify that the acceleration is 3.2 m/s^2 at t = 10 s.

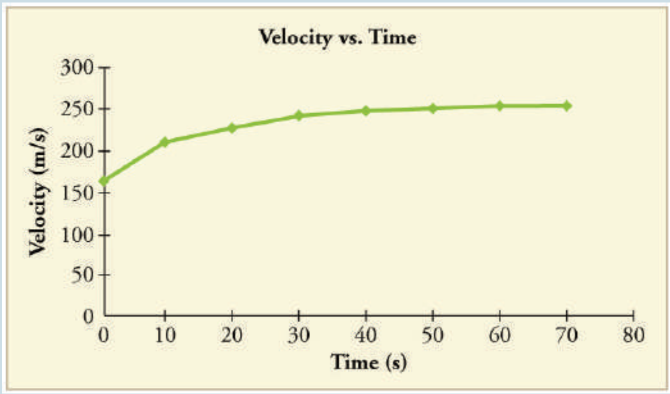


Figure 16.

5: Construct the displacement graph for the subway shuttle train as shown in the previous section, Chapter 2.7 Figure 7(a). Your graph should show the position of the train, in kilometres, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

6: (a) Take the slope of the curve in Figure 17 below to find the jogger's velocity at t = 2.5 s. (b) Repeat at 7.5 s. These values must be consistent with the graph in Figure 18, also below.

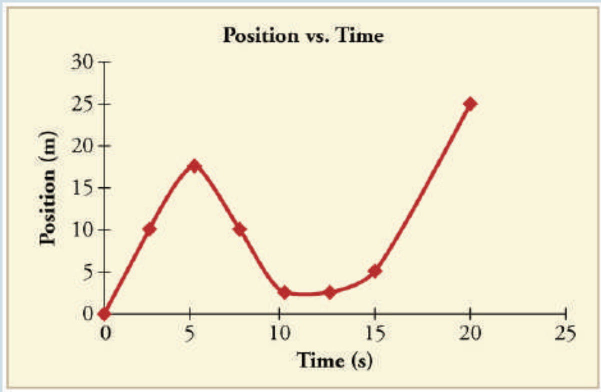


Figure 17.

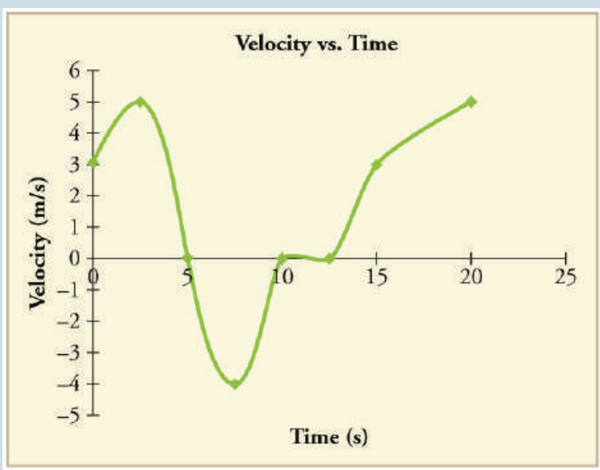


Figure 18.

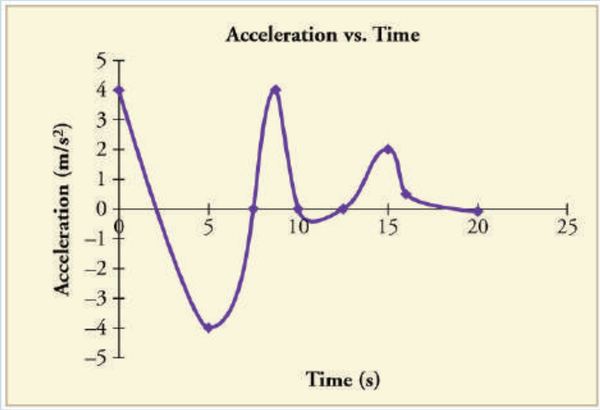


Figure 19.

7: A graph of *v(t)* is shown for a world-class track sprinter in a 100-m race is shown below in Figure 20. What is the runner's (a) average velocity for the first 4 s? (b) instantaneous velocity at t = 5 s? (c) average acceleration between 0 and 4 s? d) acceleration at t = 5 s? (e) time for the race?

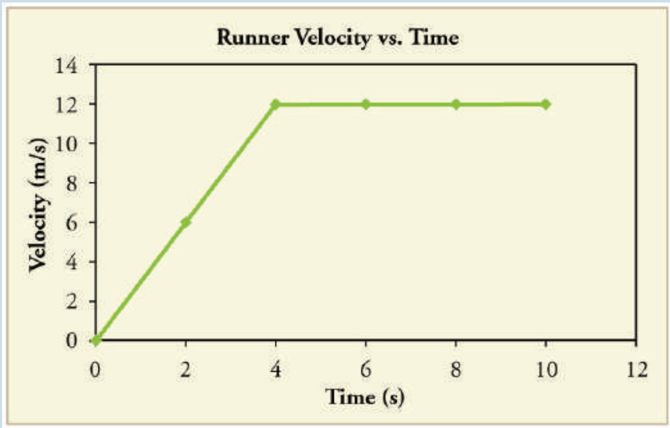


Figure 20.

8: Figure 21 below shows the displacement graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.

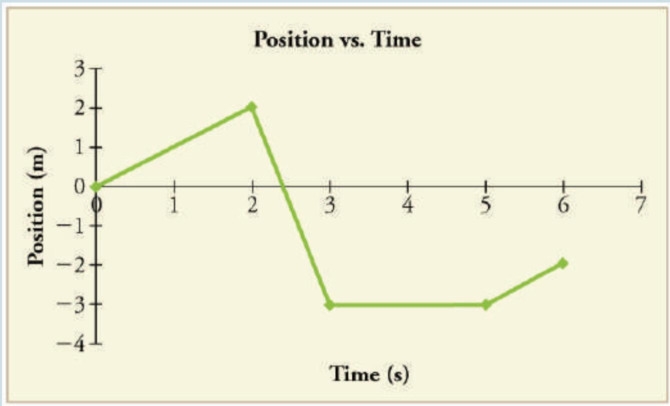


Figure 21.

Glossary

independent variable
: the variable that the dependent variable is measured with respect to; usually plotted along the ***x***-axis

dependent variable
: the variable that is being measured; usually plotted along the ***y***-axis

slope
: the difference in ***y***-value (the rise) divided by the difference in ***x***-value (the run) of two points on a straight line

y-intercept
: the ***y***-value when ***x* = *0***, or when the graph crosses the ***y***-axis

Solutions

Concept Questions

1: (b) t_a 1(c) t_d 1(d) t_e

2: (b) t_a and or t_d 1(c) t_c t_e t_g 1d) t_a t_b t_f

4: (c) t_d t_e t_h

Problems & Exercises

1: (a) 115 m/s (b) 5.0 m/s^2

3: v = (11.7 – 6.95) x 10^3 m / (40.0 x – 20.0 s) = 238 m/s

5:

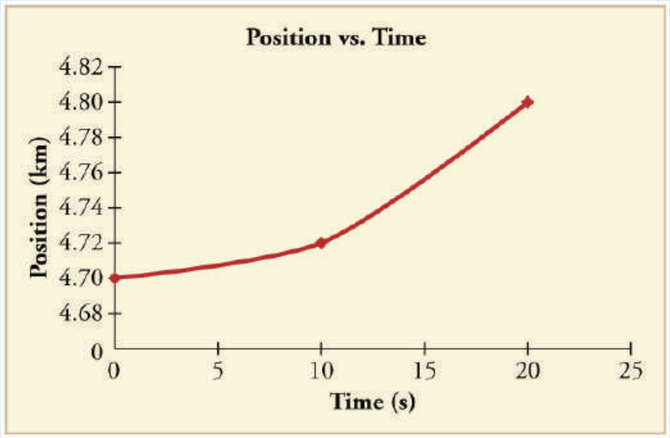


Figure 23.

6: The graphs are hard to read. About a) $v_{2.5}$ = (18 m / 5 s) = + 3.6 m/s b) $v_{7.5}$ = (2 – 18 m) / (10 – 5 s) = – 2.8 m/s Those values are consistent with the graphs.

7: (a) 6 m/s (b) 12 m/s c) 3 m/s ^{2} d) 0 m/s^2 e) 10 s

8: From t = 0 to t = 2 seconds, average velocity = 1 m/ s and the acceleration is zero.

From t = 2 to t = 3 seconds the average velocity is -5 m/ s and the acceleration is zero.

From t = 3 to t = 5.0 seconds the average velocity = 0 m/s and the acceleration = 0 m/s.

From t = 5 to t = 6 seconds, the average velocity is + 1 m/s. It is constant in that time and the acceleration is zero.

The acceleration is zero for all the straight line segments. There was an infinitely large acceleration in the very short time it took the velocities to change.

23. 3.6 Applications to Human Movement Analysis

Measuring kinematic variables in one-dimension can be useful for both athletic assessment of performance and rehabilitation. Let's consider two examples:

1. Sydney is a College sprinter with her eye on a National title. She can maintain a top velocity of 10.89 m/s for most of the 100m sprint. This is comparable, in fact better, to the current National champion at 10.85 m/s. Can you help Sydney beat her opponent and become champion?

As a biomechanist, you may be ask to analyze Sydney's performance and offer advice for improvements. You watch her run 5 race-efforts and notice a trend. Sydney reaches her top speed of 10.89 m/s at the 30-m mark. She manages to maintain that top speed until the end of the 100-m. Her opponent can only reach 10.85 m/s but she reaches her top speed at the 18-m mark. What advice do you have for Sydney and her strength and conditioning coach?

2. Alfred is recovering from a stroke. A stroke is a blockage of blood to parts of the brain, causing the brain tissues to be affected from the lack of oxygen. The right -half of Alfred's body was affected, causing weakness to his right leg. This greatly affected his walking pattern (also called gait pattern). He has recovered enough function for activities of daily living which is encouraging to regain independence. The only obstacle standing in the way between himself and the grocery store is

the ability to cross the street quickly enough to make the green light. How can you help Alfred increase his walking speed?

As a biomechanist, your first job is to assess Alfred's walking speed. You set up a straight 10-m walkway and time how long it takes him to complete the course. Using your knowledge of kinematics, you divide the displacement (10-m) by the time he took (12.3 seconds). You can then compare this value (0.81 m/s) to the required velocity to safely crossed the street (1 m/s).

You have a few options to increase walking velocity. The variables that determine walking velocity include step length and step frequency. Which do you recommend Alfred work on? What strategies would you recommend?

PART IV

CHAPTER 4: LINEAR KINEMATICS IN TWO-DIMENSIONS

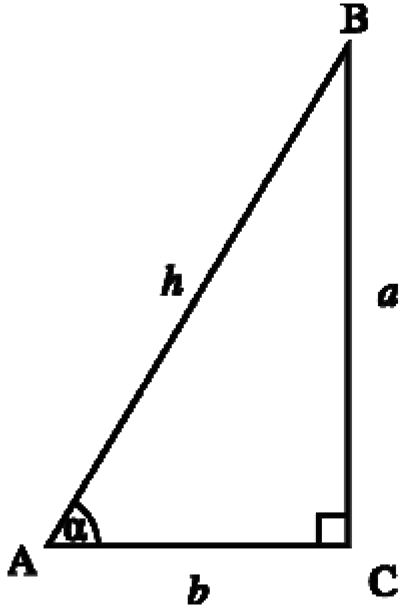
Chapter Objectives

After this chapter, you will be able to:

- Be able to locate coordinates on a cartesian coordinate system and properly report them
- Add and subtract vector variables acting in two-dimensions
- Manipulate equations of uniformly accelerated motion
- Predict the trajectory of a projectile launched in two dimensions

24. 4.0 Introduction - Trigonometry

If a vector is acting in two-dimensions, it can be broken down into a horizontal component and a vertical component. You'll soon realize that a resultant vector broken down into its components (x and y) resembles a right-angle triangle. To be successful in adding and subtracting vectors, you should be comfortable calculating the components of a right-angle triangle. A right-angle triangle, is a triangle that has one 90 degree angle.



We can use the following two concepts to solve the length of the triangle's side and the angle at the corners:

1. **Pythagorean theorem:** The length of the hypothenuse (the longest side of the triangle) squared, it equal to the the sum of both sides squared. Using the figure above, we have the equation:

$$\mathbf{h^2 = a^2 + b^2}$$

2. **Trigonometric ratios:** We will use three of the trigonometric ratios: sin, cos and tan. These are ratios because they are expressed in terms of the length of the sides of a right-angled triangle for a specific angle. Let's take for example, the angle 'alpha' in the figure above. The hypothenuse is 'h', the side opposite to the angle is 'a' and the side adjacent to the angle is 'b'.

The sin of alpha is equal to the length of the opposite side divided by the length of the hypothenuse:

$$\mathbf{\sin \alpha = a/h}$$

The cos of alpha is equal to the length of the adjacent side divided by the length of the hypothenuse:

$$\mathbf{\cos \alpha = b/h}$$

The tan of alpha is equal to the length of the opposite side divided by the length of the adjacent:

$$\mathbf{\tan \alpha = a/b}$$

A common mnemonic for remembering these relationships is SohCahToa, formed from the first letters of "**S**ine is **o**pposite over **h**ypotenuse, **C**osine is **a**djacent over **h**ypotenuse, **T**angent is **o**pposite over **a**djacent."

25. 4.1 Vectors in Two Dimensions

Summary

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

Figure 1. *Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers).*

Two-Dimensional Motion: Running in a City

Suppose you want to run from one point to another in a city with uniform square blocks, as pictured in Figure 2.

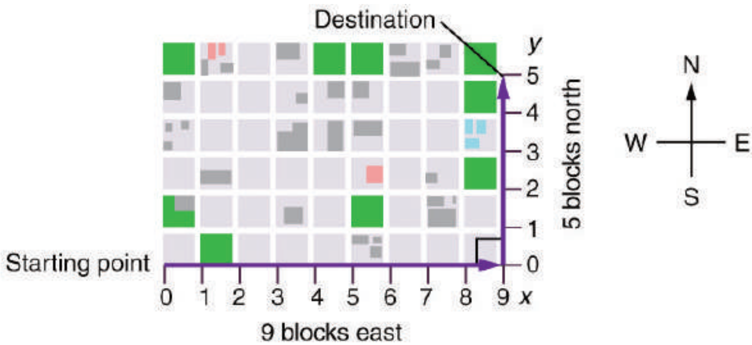


__Figure 2.__ A road runner runs a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You run 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance (displacement)?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2+b^2=c^2$, can be used to find the straight-line distance.

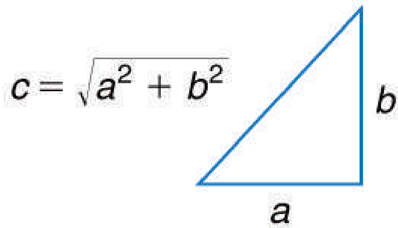


Figure 3. *The Pythagorean theorem relates the length of the legs of a right triangle, labeled* ***a*** *and* ***b****, with the hypotenuse, labeled* ***c****. The relationship is given by:* $a^2+b^2=c^2$*. This can be rewritten, solving for* ***c: c*** *=* $\sqrt{(a^2+b^2)}$*.*

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(\textbf{9 blocks})^2 + (\textbf{5 blocks})^2} = \textbf{10.3 blocks},$ considerably shorter than the 14 blocks you walked.

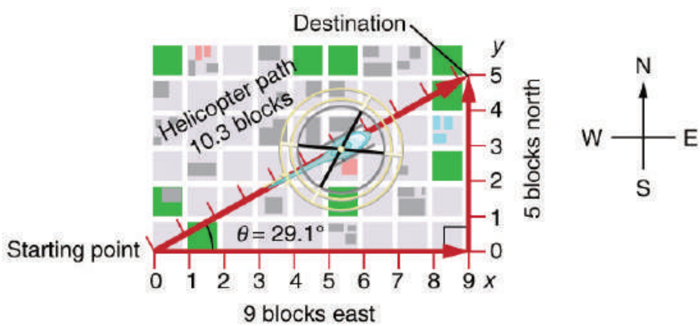


Figure 4. *The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks ran by the pedestrian. All blocks are square and the same size.*

The fact that the straight-line distance (10.3 blocks) in Figure 4 is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in Figure 2 and Figure 4. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion (the **resultant**), one vector shows the horizontal **component** of the motion, and one vector shows the vertical **component** of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in Figure 4. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another later in the book.

The Independence of Perpendicular Motions

The person taking the path shown in Figure 4 runs east and then north (two perpendicular directions). How far he or she runs east is only affected by his or her motion eastward. Similarly, how far he or she runs north is only affected by his or her motion northward.

INDEPENDENCE OF MOTION

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

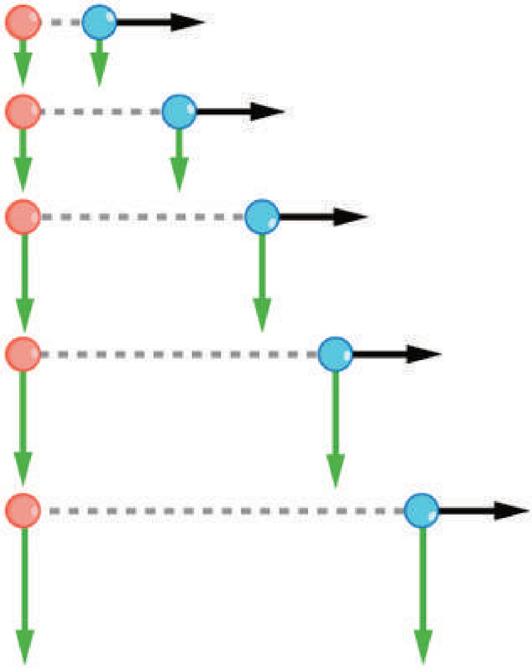


Figure 5. *This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.*

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent.

Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

Glossary

vector

a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

26. 4.2 Vector Addition and Subtraction

Summary

- Define and apply the rules of vector addition and subtraction.

Vectors in One Dimension

A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In the horizontal axis, + corresponds to movement to the right and – corresponds to movement to the left. In the vertical axis, + corresponds to upward movement and – corresponds to downward movement.

If all of the vectors are acting along the horizontal axis (x), vectors can be added or subtracted together like regular numbers. If all of the vectors are acting along the vertical axis (y), vectors can be added or subtracted together like regular numbers. For example, if a person runs 8 m to the right stops and then runs 10 m to the right, their final displacement is (+8m +10m) 18m. If the same person then walked 13 m to the left, the final displacement would be: +18m – 13 m = +5m.

If one of the vectors is acting along the horizontal axis (x) and one is acting along the vertical axis (y), you will have to use a different strategy, suitable for adding or subtracting vectors in two dimensions (x and y).

Vectors in Two Dimensions (One vector acting in y, one in x)

In two dimensions (2-d), we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Figure 1 shows such a *graphical representation of a vector*, using as an example the total displacement for the person running in a city considered in the previous section. The person ran 9 blocks east and 5 blocks north and we want to know his final displacement. We shall use the notation that a symbol with an arrow over it, such as $\vec{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, ***D***, and its direction by ***θ***.

VECTORS IN THIS TEXT

In this text, we will represent a vector with an arrow over a symbol. For example, we will represent the quantity force with the vector $\vec{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as ***F***, and the direction of the variable will be given by an angle ***θ***.

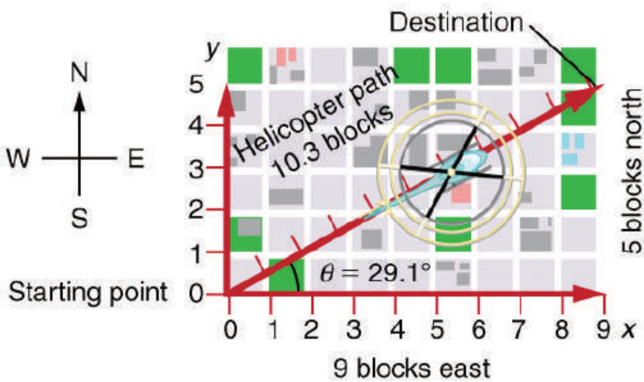


Figure 1. *A person runs 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle* ***29.1°*** *north of east.*

Vector Addition: Tip-to-Tail Method

The **tip-to-tail method** is a graphical way to add two vectors, when one is vertical and the other horizontal. It is described in Figure 2 below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **tip** of a vector is the final, pointed end of the arrow.

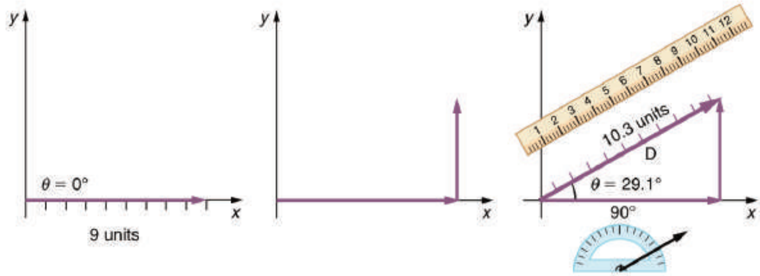


Figure 2. *Tip-to-Tail Method: (a) Draw a vector representing the displacement to the east. (b) At the tip of the first vector, draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or* ***resultant vector D****. The length of the arrow* ***D*** *is calculated using Pythagorean's theorem and measures 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis)* ***θ*** *is calculated withe SOH CAH TOA to be* ***29.1***^{0}.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler.*

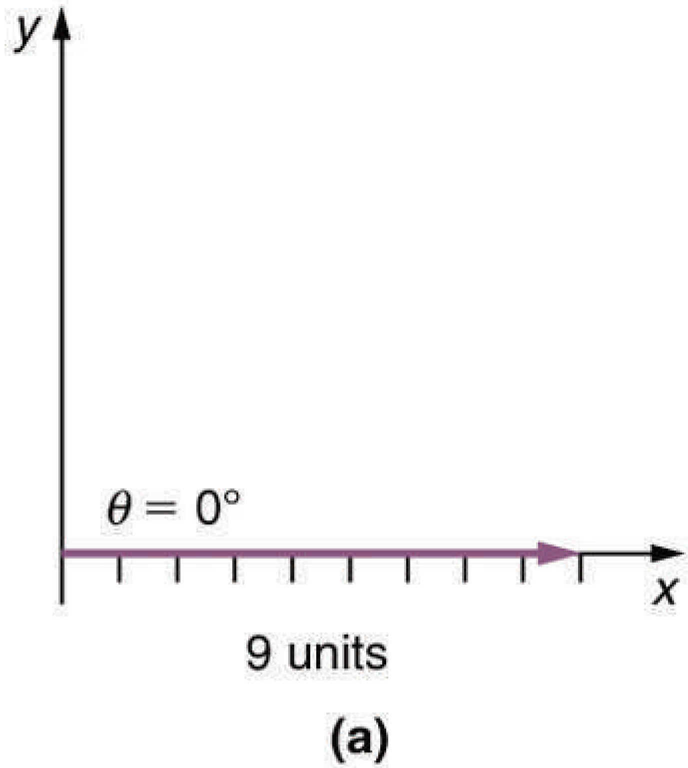


Figure 3.

Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the tip of the first vector*.

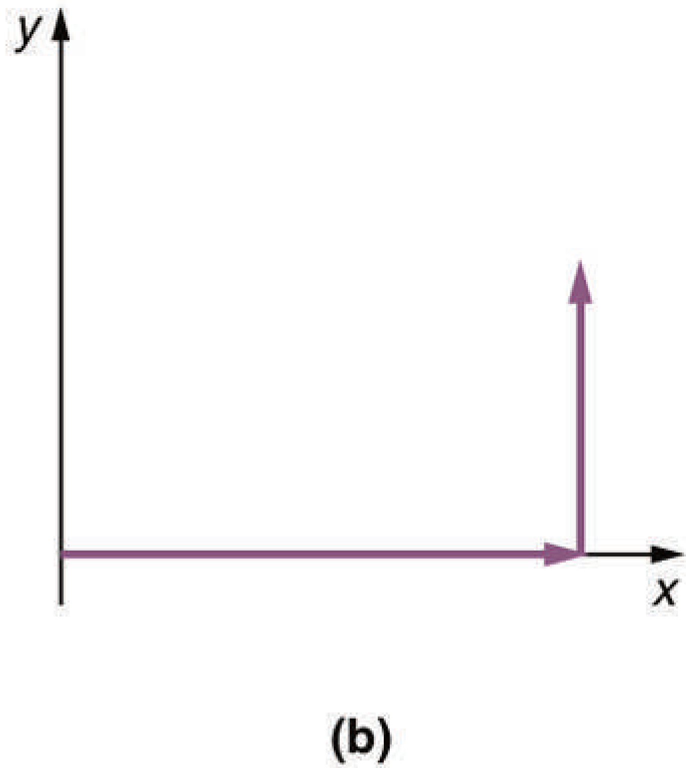


Figure 4.

Step 3. *If there are more than two vectors, start by adding all the horizontal vectors together to have one resultant x vector and add all the vertical vectors together to have one resultant y vector. Use the two resultants in the tip-to-tail method.*

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.

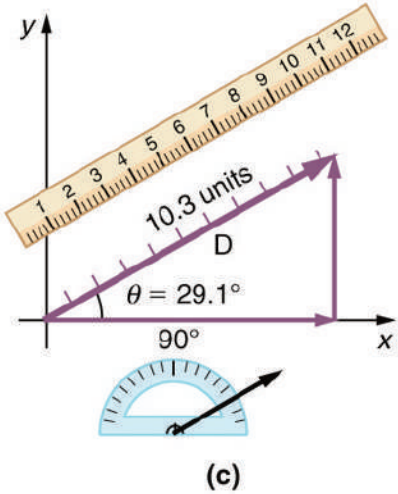


Figure 5.

Step 5. To get the **magnitude** of the resultant, *we will use the Pythagorean theorem to determine the length of the hypotenuse of this newly formed right angle triangle. (D = √x²+y2)*

Step 6. To get the **direction** of the resultant, *we will use trigonometric relationships.*

$$\tan \theta = \text{opposite/adjacent}$$

$$\tan \theta = y/x$$

Finding the vectors components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x– and y*-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction **29.0°** north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. You can draw the components (x and y) of the vector and the resultant (hypothenuse) as a right-angle triangle and use your knowledge of trigonometry to solve for the length of the components (x and y).

Vector in Two-Dimensions (Adding vectors acting at angles)

More realistically, you will have to add vectors that are not acting perfectly along the horizontal axis or the vertical axis. If you have to add vectors that are acting at angles and that are composed of a horizontal component and a vertical component. The preferred strategy in this case, would be to break the vectors down into their horizontal and vertical components. You can then add all of the horizontal components together and all the vertical components together to end up with two vectors acting perfectly along each axis. Use the tip-to-tail method to reconstruct your final results vector. Remember that your final answer should have a magnitude and direction (an angle in this case).

Summary

- The **method of adding vectors** $\vec{A}$ and $\vec{B}$ involves drawing vectors on a graph and adding them using the tip-to-tail method. The resultant vector $\vec{R}$ is defined such that $\vec{A}+\vec{B}=\vec{R}$. The magnitude and direction of $\vec{R}$ are then determined with the Pythagorean theorem and trigonometric ratios (SOH CAH TOA), respectively.
- The **tip-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.

Conceptual Questions

1: Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

2: Give a specific example of a vector, stating its magnitude, units, and direction.

3: What do vectors and scalars have in common? How do they differ?

4: Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is

8.2 km. What is the final displacement of each camper?

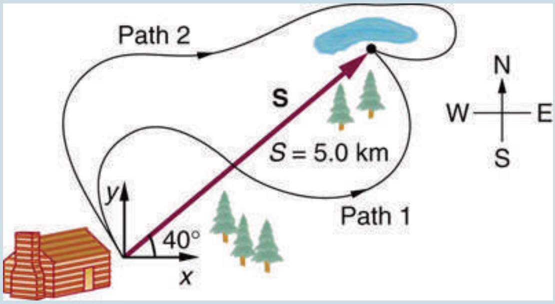


Figure 6.

5: Suppose you take two steps A and B (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point A+B the sum of the lengths of the two steps?

6: Explain why it is not possible to add a scalar to a vector.

7: If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

Problems & Exercises

Solve these problems.

1: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\vec{\text{A }}$ and $\vec{\text{B}}$, as in Figure 7, then this problem asks you to find their sum $\vec{\text{R}}=\vec{\text{A}}+\vec{\text{B}}$.)

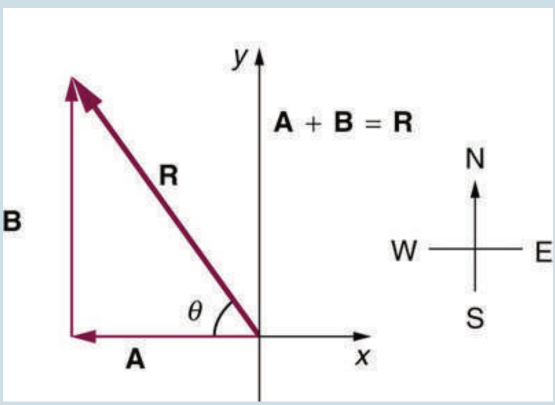


Figure 7. *The two displacements* ***A*** *and* ***B*** *add to give a total displacement* ***R*** *having magnitude* ***R*** *and direction* ***θ***.

2: Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0°

south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position?

Glossary

component (of a 2-d vector)
: a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

commutative
: refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

direction (of a vector)
: the orientation of a vector in space

tip (of a vector)
: the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"

tip-to-tail method
: a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

magnitude (of a vector)
: the length or size of a vector; magnitude is a scalar quantity

resultant
: the sum of two or more vectors

resultant vector
: the vector sum of two or more vectors

scalar
: a quantity with magnitude but no direction

tail
: the start point of a vector; opposite to the head or tip of the arrow

Solutions

Problems & Exercises

2: $\mathbf{19.5\ m, 4.65^{\mathit{o}}}$ south of west

27. 4.3 Processing Data in Biomechanics

In the previous section, we defined distance (length of the path travelled) and displacement (change in position – shortest distance from p_f to p_i). Displacement is the variable most often used in biomechanics. The reason is that the tools used to quantify kinematic variables typically take frames of information. Think of a camera used to assess a squat technique. A researcher would place a camera to film the squat in the sagittal view. Recording the movement with a camera produces a series of still pictures. If the camera records at a frequency rate of 30 Hz, it produces 30 pictures per second. Let's say we are interested in the movement of the barbell. We could place a marker on the end of the bar and record its position relative to a coordinate system in each frame of data (each still picture). To quantify change in position, you can report on the movement between frame 1 and frame 2 but you cannot report what happened between the frames because technically, you don't know what happened between them. You cannot calculate the length of the path travelled (distance) because of this unknown but you can calculate the displacement (p_2-p_1) since you know the position in frame 2 and 1.

Since we don't know what happened between the frames of data, we can only represent the change between the two frames to represent the change in position (displacement), velocity and acceleration. These are the variables most often used in biomechanics.

28. 4.4 Projectile Motion

Summary

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. Since the object or body is under the effects of a constant acceleration (-9.8m/s^2 in the vertical and 0 in the horizontal plane) its trajectory is predictable based on the magnitude and direction of its initial velocity at take-off. The object or body is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

PHET EXPLORATIONS: PROJECTILE MOTION

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

Figure 7. *Projectile Motion*

An interactive or media element has been excluded from this version of the text. You can view it online here: https://pressbooks.bccampus.ca/humanbiomechanics/?p=293

The trajectory of a projectile takes on a parabolic shape. The very top of the trajectory is called the **apex**. If a projectile takes off and lands at the same height, the trajectory is symmetrical. This means that the projectile travels the same distance in both the vertical and horizontal plane on the way up, as on the way down. The time for the projectile to reach the apex, is the same as the time for the projectile to come back to the initial height.

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. Vertical and horizontal motions are independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. Figure 1 illustrates the notation for displacement, where $\vec{\mathbf{d}}$ is defined to be the total displacement and **x** and **y** are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are ***x***, and ***y***. (Note that in the last section we used the notation $\vec{\mathbf{A}}$ to represent a vector with components $\mathbf{A_x}$ and $\mathbf{A_y}$. If we continued this format, we would call displacement $\vec{\mathbf{d}}$ with components $\mathbf{d_x}$ and $\mathbf{d_y}$.

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the *x*– and *y*-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $\mathbf{a_y=-g=-9.81\ m/s^2}$. Because gravity is vertical, $\mathbf{a_x=0}$. Both accelerations are constant, so the following kinematic equations of uniformly accelerated motion can be used.

EQUATIONS OF UNIFORMLY ACCELERATED MOTION
(You can use these when acceleration is a constant value)

$$\boldsymbol{v_f = v_i + at}$$
$$\boldsymbol{v_f^2 = v_i^2 + 2ad}$$
$$\boldsymbol{d = v_i t + 0.5at^2}$$

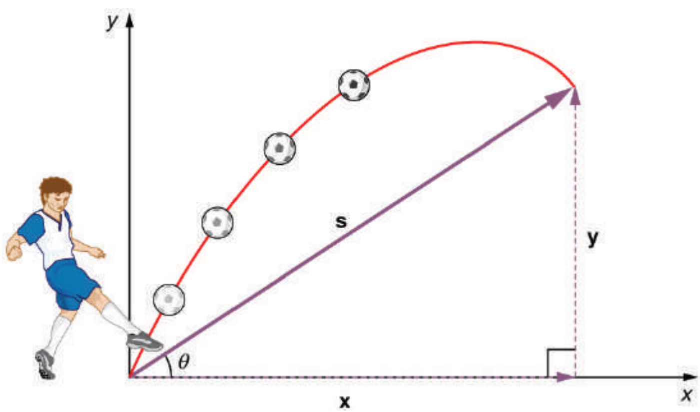


Figure 1. *The total displacement* ***d*** *of a soccer ball at a point along its path. The vector* ***d*** *has components* ***x*** *and* ***y*** *along the horizontal and vertical axes. The initial velocity it marked as s here but typically* ***v****, and it makes an angle* ***θ*** *with the horizontal.*

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. *Resolve or break the initial velocity of the projectile into horizontal and vertical components along the x- and y-axes.* The magnitude of the components of initial velocity $\vec{\textbf{v}}$ along these axes are ***x*** and ***y***. The magnitudes of the components of the velocity in this case $\vec{\textbf{v}}$ are $\boldsymbol{v_x = v \cos \theta}$ and $\boldsymbol{v_y = v \sin \theta}$, where ***v*** is the magnitude of the velocity and $\boldsymbol{\theta}$ is its direction, as shown in Figure 2. Initial values are denoted with a subscript 0 instead of "i".

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations that you should use in the vertical direction include:

$$\boldsymbol{vy_f = vy_i + ayt}$$
$$\boldsymbol{vy_f^2 = vy_i^2 + 2aydy}$$

This is a preference that makes most problems much easier. In theory you can use each equation in either dimensions but you'll just have to trust me on this. Both those equations will allow you to calculate the height of the projectile: $\boldsymbol{vy_f^2 = vy_i^2 + 2aydy}$ and the time it takes for the projectile to reach the apex: $\boldsymbol{vy_f = vy_i + ayt}$.

The kinematic equation that you should use in the horizontal direction is:

$$\boldsymbol{d = v_i t + 0.5at^2}$$

Since the acceleration in the x-axis is 0, the equation can be re-written:

$$\boldsymbol{d_x = vx_i t}$$

This equation will most often allow you to calculate the horizontal range of the projectile.

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time ***t***. But consider this: you are likely interested in how far (d_x) and how high(d_y)

the projectile travelled. If you want to know how far a projectile travelled you are interested in the entire trajectory. If you are interested in how high the projectile traveled, you are interested in the position of the projectile at its highest point which is halfway through a symmetrical trajectory. Which means you want to calculate d_y at the halfway mark (half the total time).

Step 4. *Answer the question asked in the problem.*

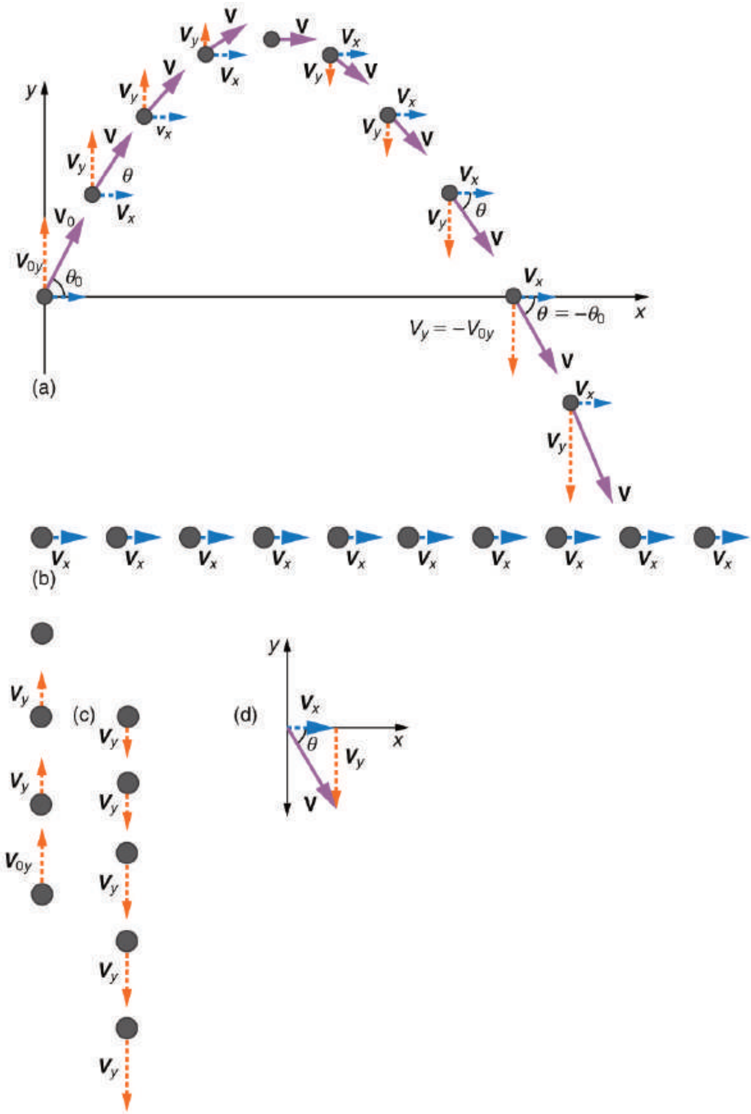


Figure 2. *(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because* $\boldsymbol{a_x=0}$ *and* $\mathbf{v_x}$ *is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in*

the opposite direction to the initial vertical velocity. (d) The x – and y -motions are recombined to give the total velocity at any given point on the trajectory.

DEFINING A COORDINATE SYSTEM

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_i = 0$ and $y_i = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, a_y, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, a_y takes a positive value.

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. On level ground, we define **range** to be the horizontal distance (R) traveled by a projectile. Let us consider projectile range further.

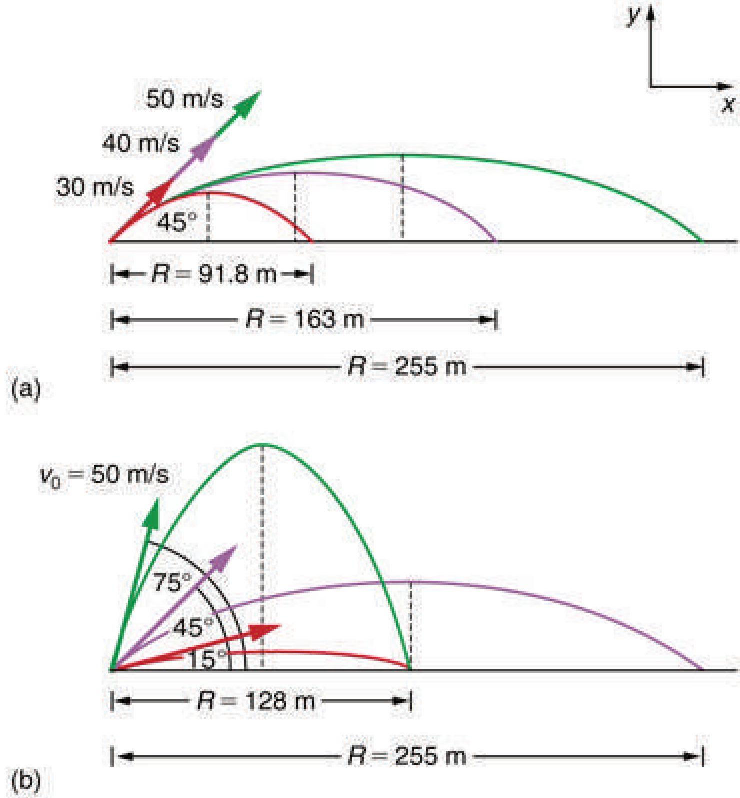


Figure 5. *Trajectories of projectiles on level ground. (a) The greater the initial speed**vi**, the greater the range for a given initial angle. (b) The effect of initial angle **θi** on the range of a projectile with a given initial speed. Note that the range is the same for **15°** and **75°**, although the maximum heights of those paths are different.*

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed $\boldsymbol{v_i}$, the greater the range, as shown in Figure 5(a). The initial angle $\boldsymbol{\theta_i}$ also has a dramatic effect on the range, as illustrated in Figure 5(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\boldsymbol{\theta_i = 45°}$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately **38°**. Interestingly, for every initial angle except **45°** there are two angles that give the same range—the sum of those angles is **90°**.

Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

1. Determine a coordinate system. Then, resolve the initial velocity of the object in the horizontal and vertical components.
2. Note that velocity final (v_{fy} = 0 at the apex for y and v_{fx} = v_{ix} at the end for x) are known. The accelerations in x and y are also known as they are constants (a_x = 0, a_y = -9.81 m/s^2)
3. Calculate the maximum height of the projectile (d_y) and the time it took for the projectile to reach the apex ($t_{1/2}$) using the following equations:

$$\boldsymbol{vy_f = vy_i + ayt}$$
$$\boldsymbol{vy_f^2 = vy_i^2 + 2aydy}$$

4. Double the time found with the equations for y, to account for the entire trajectory (t = 2*$t_{1/2}$).

5. Calculate the maximum range (dx) of the projectile using the following equation:

$$\boldsymbol{d_x = vx_i t}$$

Conceptual Questions

1: Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?

2: Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

3: For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

4: During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

Problems & Exercises

1: A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the *x* and *y* distances from where the projectile was launched to where it lands?

2: A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction on level ground. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c)What maximum height is attained by the ball?

3: A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

4: (a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up 32° ramp at a speed of 40.0 m/s (144 km/h). How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

5: An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

6: A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

7: Verify the ranges for the projectiles in Figure 5(a) for $\theta = 45°$ and the given initial velocities.

8: Verify the ranges shown for the projectiles in Figure 5(b) for an initial velocity of 50 m/s at the given initial angles.

10: In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, *g*. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

11: The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

12: A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of 25° relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?

13: Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be 40° above the horizontal.

14: Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s. Use the maximum range equation here, assuming the launch angle is 45 degrees.

15: The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 7.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

16: In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of 38.0° above the horizontal?

17: A basketball player is running at 5.00 m/s directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

18: A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

Glossary

air resistance
: a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

kinematics
: the study of motion without regard to mass or force

motion
: displacement of an object as a function of time

projectile
: an object that travels through the air and experiences only acceleration due to gravity

projectile motion
: the motion of an object that is subject only to the acceleration of gravity

range
: the maximum horizontal distance that a projectile travels

trajectory
: the path of a projectile through the air

Solutions

Problems & Exercises

1: x = 130 m   y=30.9 m

2: (a) 20 m/s (b) 1.22 seconds (c) 9.8 m

3: (a) 3.50 s (b) $\boldsymbol{28.6\textbf{ m/s}}$ (c) $\boldsymbol{34.3\textbf{ m/s}}$ (d) $\boldsymbol{44.7\textbf{ m/s}}$, $\boldsymbol{50.2^0}$ below horizontal

5: (a) $\mathbf{18.4^0}$ (b) The arrow will go over the branch.

7: $\mathbf{R = \dfrac{\boldsymbol{v_0^2}}{\sin 2\boldsymbol{\theta_0 g}}}$

$\mathbf{For}\ \boldsymbol{\theta} = \mathbf{45^0},\ \mathbf{R} = \dfrac{\boldsymbol{v_0^2}}{\boldsymbol{g}}\ \mathbf{R = 91.8\ m}$ for

$\boldsymbol{v_0} = \mathbf{30\ m/s;\ R = 163\ m}$ for

$\boldsymbol{v_0} = \mathbf{40\ m/s;\ R = 255\ m}$ for

$\boldsymbol{v_0} = \mathbf{50\ m/s.}$

9: $\boldsymbol{1.50\textbf{ m}}$, assuming launch angle of $\mathbf{45^0}$

12: $\boldsymbol{\theta} = \mathbf{6.1^0}$ yes, the ball lands at 5.3 m from the net

13: (a) $\boldsymbol{-0.486\textbf{ m}}$ (b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

14: No, the maximum range (neglecting air resistance) is about 92 m.

16: 15.0 m/s

18: (a) 24.2 m/s (b) The ball travels a total of 57.4 m with the brief gust of wind.

29. 4.5 Problem-Solving Basics

Figure 1. *Problem-solving skills are essential to your success in Biomechanics. (credit: scui3asteveo, Flickr).*

Problem-solving skills are obviously essential to success in a quantitative course such as Biomechanics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, "stopped" means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of biomechanics is to accurately describe human movement. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more biomechanics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in biomechanics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Biomechanics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40 m/s^2 for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$\boldsymbol{v_f = v_i + at = 0 + (0.40\ \mathrm{m/s^2})(100\ \mathrm{s}) = 40\ \mathrm{m/s}.}$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\boldsymbol{\left(\frac{40\ \mathrm{m}}{\mathrm{s}}\right)\left(\frac{3.28\ \mathrm{ft}}{\mathrm{m}}\right)\left(\frac{1\ \mathrm{mi}}{5280\ \mathrm{ft}}\right)\left(\frac{60\ \mathrm{s}}{\mathrm{min}}\right)\left(\frac{60\ \mathrm{min}}{1\ \mathrm{h}}\right) = 89\ \mathrm{mph}}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at $0.40\ m/s^2$, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of $0.40\ m/s^2$ for 100 s (almost two minutes).

Section Summary

- *The six basic problem solving steps for physics are:*

  Step 1. Examine the situation to determine which physical principles are involved.
  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).
  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).
  Step 4. Find an equation or set of equations that can help you solve the problem.
  Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.
  Step 6. Check the answer to see if it is reasonable: Does it make sense?

Conceptual Questions

1: What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

2: What is the last thing you should do when solving a problem? Explain.

PART V

CHAPTER 5: ANGULAR KINEMATICS

Chapter Objectives

After this chapter, you will be able to:

- Observe the kinematics of angular motion.
- Define angular acceleration, velocity and displacement.
- Relate angular motion to linear motion.
- Evaluate problem solving strategies for angular kinematics.
- Understand the application of angular kinematics to biomechanics.

30. 5.0 Introduction

Figure 1. The body of a runner moves linearly through the angular motion each lower limb segment. The thigh rotates about the hip, the shank rotates about the knee and the foot rotates about the ankle to move the body forward. (credit: Photo by Laurine Bailly on Unsplash)

This chapter deals with the simplest form of curved motion, **uniform angular motion**, motion in a circular path at constant speed. Pure *rotational (angular) motion* occurs when points in an object move in circular paths centered on one point (the axis of rotation). Pure *translational (linear) motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

Angular motion is important to biomechanics because most human movements are the result of angular motions of limbs about joint. We thus need to understand how angular motion is measured and described.

Glossary

uniform circular motion
: the motion of an object in a circular path at constant speed

31. 5.1 Angular Position and Displacement

We've previously studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration.Then we dealt with motion of one point in two dimensions. Projectile motion is a special case of two-dimensional kinematics in which the object is projected into the air, while being subject to the gravitational force (constant acceleration). It's trajectory is predictable using equations of uniformly accelerated motion. In this chapter, we consider situations where the objects rotate about a point (the axis of rotation) while traveling in a circular path. We typically consider the movement of an entire segment.

Rigid Bodies

An assumption made in biomechanics is that body segments are rigid bodies. Rigid bodies maintains a constant length. Instead of representing motion relative to a point (ex: center of mass) as we did in linear kinematics, we will represent motion of a rigid body (ex: thigh). This will affect our frame of reference as movement no longer occur in the x and y-axis. That's right, we no longer describe movement in relation to the x and the y axis of a coordinate system but in relation to the axis of rotation.

Angle

An angle is formed at the intersection of two lines, two planes or a line and a plane. Angles are used to define the orientation of these lines or planes relative to each other.

Frame of Reference

Angular motion occurs about an axis of rotation. In the human body, this axis of rotation is a joint and the rigid bodies are the bones rotating about the angle. The axis is always perpendicular to the plane. For example. if we are interested in knee angle in the sagittal plane, we'll be quantifying motion about the mediolateral axis. The frame of reference is no longer a cartesian reference system with two orthogonal axis but a combination of the axis of rotation and a reference axis. In this course, rotation about the axis of rotation in the **clockwise direction will always be negative** and rotation in the **counterclockwise direction will always be positive.**

Do you remember the difference between absolute and relative angles? In absolute angles, the angle of a body segment is reported relative to the horizontal plane. The horizontal plane represents a fixed reference. With relative angles, we measure the angle between two body segments or lines. In this case, both lines are capable of moving.

Angular Position (θ)

Angular position represents the orientation of a line with another line or plane. Angular position is quantified by measuring how far the body is rotated from the reference position. The angular position is denoted by the symbol theta (θ) and can be measured in degrees (°), radians (rads) or revolutions.

Although degrees may be easier for you to interpret, rads play an important role in biomechanics. A radian is the angle you get when making the arc length equal to the radius of the circle. A radian is equal to 180/pi or 57.3 degrees.

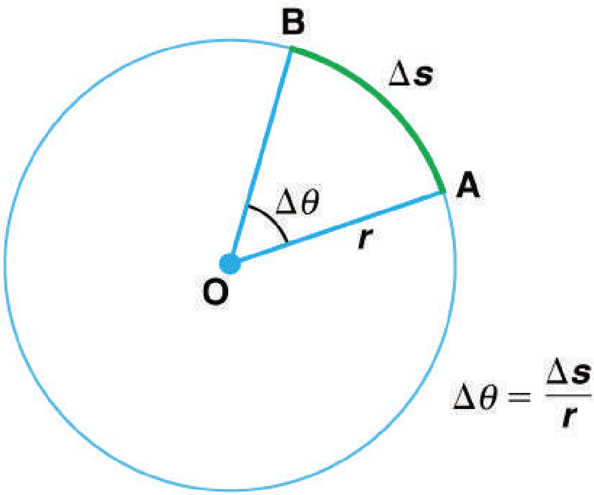


Figure 1. *A radian* ***Δθ*** *is defined as the angle if The arc length* ***Δs*** *had the same length as the radius* ***(r)***.

This result is the basis for defining the units used to measure rotation angles, ***Δθ*** to be **radians**(rad), defined so that

$$\mathbf{2\pi\ rad = 1\ revolution.}$$

A comparison of some useful angles expressed in both degrees and radians is shown in Table 1.

Degree Measures	Radian Measure
30^0	$\frac{\pi}{6}$
60^0	$\frac{\pi}{3}$
90^0	$\frac{\pi}{2}$
120^0	$\frac{2\pi}{3}$
135^0	$\frac{3\pi}{4}$
180^0	π

Table 1. Comparison of Angular Units.

Angular Displacement

Angular displacement is defined as the change from the final position to the initial position ($\Delta\theta = \theta_f - \theta_i$). Angular displacement represents the angle formed between the final position and the initial position of a rotating line. As with linear displacement, angular displacement has a direction associated with it. Rotation in the **clockwise direction is negative** and rotation in the **counterclockwise direction is positive.**

In biomechanics, angular displacement is useful when trying to quantify the range of motion at a joint. If a person starts in full knee extension (position 1) and squats down (position 2), we can measure the range of motion at the knee by measuring the knee angle at both position and using the formula:

$$\textbf{range of motion} = \theta_2 - \theta_1$$

Another use of angular displacement in sport is counting the number of full body rotations. The number of twists or somersault performed in sports like diving, gymnastics or snowboarding for example, determine the level of difficulty of the performance. This aspect is important for the judges who attribute a score to the performance.

The angular displacement of a swing (range of motion) in sports like golf, tennis or hockey, affects the manner in which the ball is hit in these sports. See the relationship between angular displacement of a segment (ex: hockey stick) and the resulting linear displacement of a point (ex: the hockey puck) below:

Angular and Linear Displacement

In angular kinematics we describe the movement of a segment or rigid body as it rotates about a point. This segment could be described as a series of points who must each move linearly to accomplish this change in position. The linear displacement of each point is unique to the point and depends on how far it is located from the axis of rotation. The distance from the axis of rotation to the point of interest is called the radius (r).

Let's consider two points (**a** and **b**) along a segment. Let's say we are interested in the movement of the arm during a jumping jack. We observe the movement in the frontal plane and measure a change in position from 0 degrees to 180 degrees as the participant moves their arm up over their head. Point **a** is located on the elbow and point **b** is located on the wrist. Both point **a** and point **b** must move through 180 degrees. In fact, every point on the arm must move through 180 degrees. But each point on the arm moves a different linear distance to accomplish this angular displacement of 180 degrees. Points closest to the shoulder joint (the point of rotation) don't have to linearly travel as far as points closest to the hands to cover the 180 degrees. Point a (**elbow**) has a smaller linear displacement than point b (**wrist**). Point **a** has a shorter radius than point **b**, thus the linear displacement correlates to how far the point is from the axis of rotation (**r**).

This relationship is expressed with the following equation:

d = rθ

Note that angular position/displacement **MUST be expressed in rads** (not degrees or revolutions) for this relationship to be accurate.

Section Summary

- Uniform circular motion is motion in a circle at constant speed. The rotation angle ***Δθ*** is defined as the ratio of the arc length to the radius of curvature:

$$\boldsymbol{\Delta\theta = \frac{\Delta s}{r},}$$

  where arc length ***Δs*** is distance traveled along a circular path and ***r*** is the radius of curvature of the circular path. The quantity ***Δθ*** is measured in units of radians (rad), for which

$$\mathbf{2\pi\ rad = 360^0 = 1\ revolution.}$$

- The conversion between radians and degrees is **1 rad = 57.3°**.
- Angular displacement defines the movement of a segment as represents the change in angular position.
- Linear displacement of any point along a segment that is rotation can be calculated with: **d = rθ** as long as angular position is expressed in rads.

Glossary

arc length
: Δs, the distance traveled by an object along a circular path.

rotation angle
: the ratio of the arc length to the radius of curvature on a circular path:

$$\Delta\theta = \frac{\Delta s}{r}$$

radius of curvature
: radius of a circular path

radians
: a unit of angle measurement

32. 5.2 Angular Velocity

Angular Velocity

How fast is an object rotating? We define **angular velocity $\boldsymbol{\omega}$** as the rate of change of angular displacement. In symbols, this is

$$\boldsymbol{\omega} = \frac{\boldsymbol{\Delta\theta}}{\boldsymbol{\Delta t}},$$

where an angular rotation $\boldsymbol{\Delta\theta}$ takes place in a time $\boldsymbol{\Delta t}$. The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are degrees per second (°/s), radians per second (rad/s) or revolutions per minute (rpm) where applicable.

Angular velocity is a vector quantity. Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise.

Angular velocity is used in two ways in biomechanics. We are either interests in the **average angular velocity** or the **instantaneous angular velocity**. Average angular velocity tells us how long it takes for something to rotate through a certain angular displacement. Instantaneous angular velocity tells us how fast something is spinning at a specific instant in time. The average angular velocity of a tennis player's swing might determine whether or not she contacts the ball but it is the racket's instantaneous velocity at ball contact that determines how fast and how far the ball will go. In sports where whole body rotations are important (diving, gymnastics), angular velocity is an important determinant of whether or not the athlete will complete a certain number of twists or somersaults before landing.

Angular and Linear Velocity

In several sports, especially in those where an implement is used as an extension of the athlete's limbs (golf, tennis, lacrosse..), the relationship between angular and linear velocity become important. The advantage of using implements is that they amplify the movement (displacement) of our limbs. Take a tennis ball and throw it as far as you can. Now hit that same ball with a tennis racket. Which goes the furthest? The racket enable faster linear velocities because they increase the distance from the point of contact (your hand vs the tennis racket) from the axis of rotation (shoulder joint). The relationship between linear variables, angular variables and the radius discussed in the previous section is important here as well.

The first relationship in $\boldsymbol{v = r\omega}$ or $\boldsymbol{\omega = \mathrm{v} / \mathrm{r}}$ states that the linear velocity $\boldsymbol{v}$ is proportional to the distance from the centre of rotation, thus, it is largest for the point furthest away from the point of rotation. The second relationship states that the faster an object rotates ($\boldsymbol{\omega}$), the faster the linear velocity of a point on the object ($\boldsymbol{v}$). Note that in order to use this equation, **angular velocity must be expressed in rads/s.**

Both $\boldsymbol{\omega}$ and $\boldsymbol{v}$ have directions (hence they are angular and linear *velocities*, respectively). Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise. Linear velocity is tangent to the path.

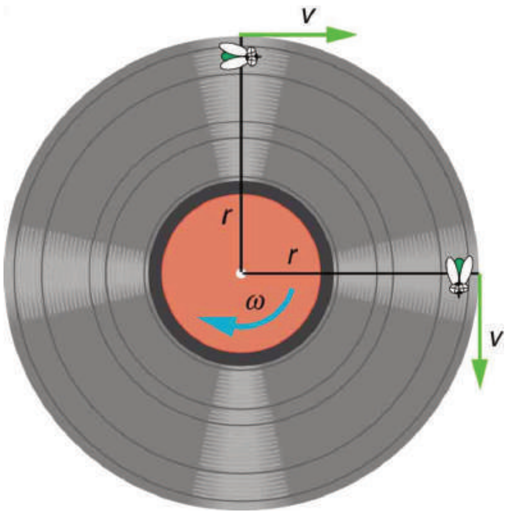


Figure 4. *As an object moves in a circle, here a fly on the edge of an old-fashioned vinyl record, its instantaneous velocity is always tangent to the circle. The direction of the angular velocity is clockwise in this case.*

Applying the relationship between linear and angular velocity to sport

Think of your favourite sports and the equipment required to play. If the sport you are thinking about includes the use of sticks, clubs or rackets you are likely already familiar with the relationship between linear and angular velocities. The linear velocity of a point farther from the axis of rotation is faster if the angular velocity is the same. This linear velocity gets passed along to the ball (or projectile) through the conservation of momentum which will be discussed later. In golf for example, we have two types of clubs: the woods and irons. The woods are the longest clubs and are used to impart faster velocity to the ball as the player drives the ball as far as possible. The irons are shorter clubs, used for closer shots.

You may not always be switching between a long and a shorter implement to affect the linear velocity of the projectile but you can also change the axis of rotation to shorten the radius. Let's say a swing typically originates in the shoulder. By rotating about the wrist you are shortening the radius.

Perhaps you could also move your grip on the apparatus to shorten or lengthen the radius. This is seen commonly in baseball as the players choke up on the bat.

PHET EXPLORATIONS: LADYBUG REVOLUTION

Figure 6. *Ladybug Revolution*

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

- Angular velocity **ω** is the rate of change of an angle,

$$\boldsymbol{\omega} = \frac{\boldsymbol{\Delta\theta}}{\boldsymbol{\Delta t}},$$

where a rotation $\boldsymbol{\Delta\theta}$ takes place in a time $\boldsymbol{\Delta t}$. The units of angular velocity are radians per second (rad/s). Linear velocity $\boldsymbol{v}$ and angular velocity $\boldsymbol{\omega}$ are related by

$$\boldsymbol{v = r\omega \text{ or } \omega = \frac{v}{r}}.$$

- The relationship between linear and angular velocities is expressed in the following equation: **$\boldsymbol{v = r\omega}$ (angular velocity must be expressed in rads/s)**

Problems & Exercises

1: A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher's hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?

2: In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?

3: Integrated Concepts

When kicking a football, the kicker rotates his leg about the hip joint.

(a) If the velocity of the tip of the kicker's shoe is 35.0

m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity?

(b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s?

(c) Find the maximum range of the football, neglecting air resistance. Remember there is an equation for maximum range. It happens when the launch angle is 45 degrees above horizontal.

angular velocity
: **ω**, the rate of change of the angle with which an object moves on a circular path

Solutions

Problems & Exercises

1: 117 rad/second

3: (a) 33.3 rad/s (b) 500 N (c) 40.8 m

33. 5.3 Angular Acceleration

Angular acceleration is denoted by the Greek letter alpha (α). Angular acceleration represents the time rate of change in angular velocity. Another way to think about this is how quickly something is speeding up or slowing down.

acceleration (α) = Δω/Δt

The units are rads/s^2 or degrees/s^2. When velocity is increasing, the acceleration is in the same direction of rotation to increase the velocity. When the velocity is decreasing, there has to be acceleration in the opposite direction of travel acting as a break to decrease the velocity.

Acceleration has a direction. If the object is moving in the counterclockwise direction (+) and gaining velocity, acceleration is positive. If velocity is decreased, acceleration is negative. If the object is moving in the clockwise direction (-) and gaining velocity, acceleration is negative. If velocity is decreased, acceleration is positive.

Example 1: Calculating the Angular Acceleration and Deceleration of a Bike Wheel

Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the angular acceleration in **rad/s^2**. (b) If she now slams on the brakes, causing an angular acceleration of **-87.3 rad/s^2**, how long does it take the wheel to stop?

Strategy for (a)

The angular acceleration can be found directly from its definition in $\boldsymbol{\alpha = \frac{\Delta \omega}{\Delta t}}$ because the final angular velocity and time are given. We see that **Δω** is 250 rpm and **Δ*t*** is 5.00 s.

Solution for (a)

Entering known information into the definition of angular acceleration, we get

$$\begin{aligned} \boldsymbol{\alpha} &= \boldsymbol{\frac{\Delta \omega}{\Delta t}} \\ &= \mathbf{\frac{250\ rpm}{5.00\ s.}} \end{aligned}$$

Because **Δω** is in revolutions per minute (rpm) and we want the standard units of **rad/s^2** for angular acceleration, we need to convert **Δω** from rpm to rad/s:

$$\begin{aligned} \boldsymbol{\Delta \omega} &= \mathbf{250 \frac{rev}{min} \cdot \frac{2\pi\ rad}{rev} \cdot \frac{1\ min}{60\ sec}} \\ &= \mathbf{26.2\ rads.} \end{aligned}$$

Entering this quantity into the expression for $\boldsymbol{\alpha}$, we get

$$\begin{aligned} \boldsymbol{\alpha} &= \frac{\boldsymbol{\Delta\omega}}{\boldsymbol{\Delta t}} \\ &= \frac{\mathbf{26.2\ rad/s}}{\mathbf{5.00\ s}} \\ &= \mathbf{5.24\ rad/s.^2} \end{aligned}$$

Strategy for (b)

In this part, we know the angular acceleration and the initial angular velocity. We can find the stoppage time by using the definition of angular acceleration and solving for **Δ*t***, yielding

$$\boldsymbol{\Delta t} = \frac{\boldsymbol{\Delta\omega}}{\boldsymbol{\alpha}}.$$

Solution for (b)

Here the angular velocity decreases from **26.2 rad/s** (250 rpm) to zero, so that **Δω** is **-26.2 rad/s**, and $\boldsymbol{\alpha}$ is given to be **-87.3 rad/s^2**. Thus,

$$\begin{aligned} \boldsymbol{\Delta t} &= \frac{\mathbf{-26.2\ rad/s}}{\mathbf{-87.3\ rad/s^2}} \\ &= \mathbf{0.300\ s.} \end{aligned}$$

Discussion

Note that the angular acceleration as the girl spins the wheel is small and positive; it takes 5 s to produce an appreciable angular velocity. When she hits the brake, the angular acceleration is large and negative. The angular velocity quickly goes to zero. In both cases, the relationships are analogous to what happens with linear motion. For example, there is a large deceleration when you crash into a brick wall—the velocity change is large in a short time interval.

If the bicycle in the preceding example had been on its wheels instead of upside-down, it would first have accelerated along the ground and then come to a stop. This connection between circular motion and linear motion needs to be explored. For example, it would be useful to know how linear and angular acceleration are related. In circular motion, linear acceleration is *tangent* to the circle at the point of interest, as seen in Figure 2. Thus, linear acceleration is called tangential acceleration $\boldsymbol{a_t}$.

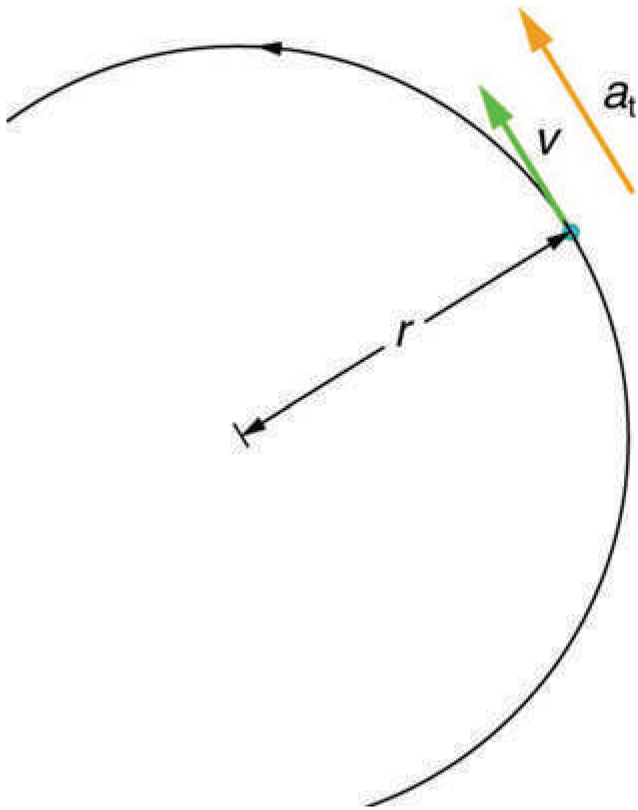


Figure 2. *In circular motion, linear acceleration* ***a****, occurs as the magnitude of the velocity changes:* ***a*** *is tangent to the motion. In the context of circular motion, linear acceleration is also called tangential acceleration* $\boldsymbol{a_t}$.

Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction. We know that in circular motion centripetal acceleration, $\boldsymbol{a_c}$, refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration, as seen in Figure 3. Thus, $\boldsymbol{a_t}$ and $\boldsymbol{a_c}$ are perpendicular and independent of one another. Tangential acceleration $\boldsymbol{a_t}$ is directly related to the angular acceleration α

and is linked to an increase or decrease in the velocity, but not its direction.

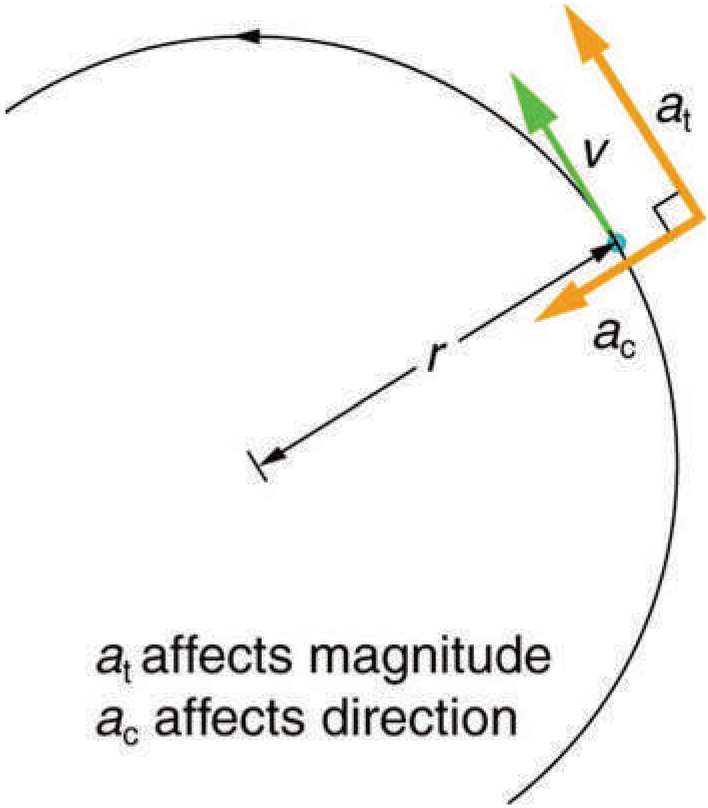


__Figure 3.__ Centripetal acceleration $\boldsymbol{a_c}$ *occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus perpendicular to each other.*

Now we can find the exact relationship between linear acceleration $\boldsymbol{a_t}$ and angular acceleration α. Because linear acceleration is proportional to a change in the magnitude of the velocity, it is defined to be

$$a_{\mathrm{t}} = \frac{\Delta v}{\Delta t}.$$

For circular motion, note that $v = r\omega$, so that

$$a_{\mathrm{t}} = \frac{\Delta(r\omega)}{\Delta t}.$$

The radius r is constant for circular motion, and so $\Delta(r\omega) = r(\Delta\omega)$. Thus,

$$a_{\mathrm{t}} = r\frac{\Delta\omega}{\Delta t}.$$

By definition, $\alpha = \frac{\Delta\omega}{\Delta t}$. Thus,

$$a_{\mathrm{t}} = r\alpha,$$

or

$$\alpha = \frac{a_{\mathrm{t}}}{r}.$$

These equations mean that linear acceleration and angular acceleration are directly proportional. The greater the angular acceleration is, the larger the linear (tangential) acceleration is, and vice versa. For example, the greater the angular acceleration of a bike's drive wheels, the greater the acceleration of the bike. The radius also matters. For example, the smaller a wheel, the smaller its linear acceleration for a given angular acceleration α.

So far, we have defined three rotational quantities— θ, ω, and α. These quantities are analogous to the translational quantities x, v, and a. Table 1 displays rotational quantities, the analogous translational quantities, and the relationships between them.

Rotational	Translational	Relationship
$\boldsymbol{\theta}$	$\boldsymbol{x}$	$\boldsymbol{\theta}=\frac{\boldsymbol{x}}{\boldsymbol{r}}$
$\boldsymbol{\omega}$	$\boldsymbol{v}$	$\boldsymbol{\omega}=\frac{\boldsymbol{v}}{\boldsymbol{r}}$
$\boldsymbol{\alpha}$	$\boldsymbol{a}$	$\boldsymbol{\alpha}=\frac{\boldsymbol{a}_{\mathbf{t}}}{\boldsymbol{r}}$

Table 1. Rotational and Translational Quantities.

Section Summary

- Uniform circular motion is the motion with a constant angular velocity $\boldsymbol{\omega}=\frac{\boldsymbol{\Delta\theta}}{\boldsymbol{\Delta t}}$.
- In non-uniform circular motion, the velocity changes with time and the rate of change of angular velocity (i.e. angular acceleration) is $\boldsymbol{\alpha}=\frac{\boldsymbol{\Delta\omega}}{\boldsymbol{\Delta t}}$.
- Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction, given as $\boldsymbol{a}_{\mathbf{t}}=\frac{\boldsymbol{\Delta v}}{\boldsymbol{\Delta t}}$.
- For circular motion, note that ***v*=*rω***, so that
$$\boldsymbol{a}_{\mathbf{t}}=\frac{\boldsymbol{\Delta(r\omega)}}{\boldsymbol{\Delta t}}.$$
- The radius r is constant for circular motion, and so **Δ(*r*ω)=*r*Δω**. Thus,
$$\boldsymbol{a}_{\mathbf{t}}=\boldsymbol{r}\frac{\boldsymbol{\Delta\omega}}{\boldsymbol{\Delta t}}.$$

- By definition, $\Delta\omega/\Delta t = \alpha$. Thus,

$$a_{\mathrm{t}} = r\alpha$$

or

$$\alpha = \frac{a_{\mathrm{t}}}{r}.$$

Problems & Exercises

4: Unreasonable Results

You are told that a basketball player spins the ball with an angular acceleration of 100 rad/s^2. (a) What is the ball's final angular velocity if the ball starts from rest and the acceleration lasts 2.00 s? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

Glossary

angular acceleration
: the rate of change of angular velocity with time

change in angular velocity
: the difference between final and initial values of angular velocity

tangential acceleration
: the acceleration in a direction tangent to the circle at the point of interest in circular motion

Solutions

Problems & Exercises

4:

34. 5.4 Linear Accelerations

When a limb is rotating at an increasing angular velocity, the tangential (linear) velocity of a point on the limb is increasing as well. The angular and linear accelerations of a point are related but unlike velocity, there are **two** linear accelerations to consider on rotation objects: **tangential** and **centripetal acceleration**.

Tangential Acceleration

The linear acceleration tangent to the circular path of a point on a rotating segment is called: tangential acceleration. It represents the chance in tangential (linear) velocity discussed in the previous section. Tangential acceleration is related to angular acceleration in the following way:

$$\mathbf{a_t = \alpha r}$$

Keep in mind that in order for this equation to be true, angular acceleration must be expressed in rads/s^2.

Tangent: a line is tangent to a circle if the line intersects the circle at just one point. A line from this point to the centre of the circle (radius) is perpendicular to the tangent line.

Centripetal Acceleration

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

Figure 1 shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration** (a_c); centripetal means “toward the center” or “center seeking.”

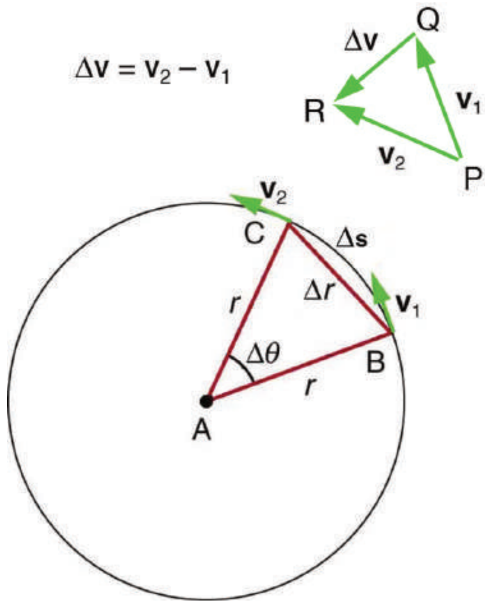


Figure 1. *The directions of the velocity of an object at two different points are shown, and the change in velocity* ***Δv*** *is seen to point directly toward the center of curvature. (See small inset.) Because* ***ac = Δv/Δt****, the acceleration is also toward the center; ac is called centripetal acceleration. (Because* ***Δθ*** *is very small, the arc length* ***Δs*** *is equal to the chord length* ***Δr*** *for small time differences.)*

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii $\boldsymbol{r}$ and $\boldsymbol{\Delta s}$ are similar. Both the triangles ABC and PQR are isosceles triangles (two equal sides). The two equal sides of the velocity vector triangle are the speeds $\boldsymbol{v_1 = v_2 = v}$. Using the properties of two similar triangles, we obtain

$$\frac{\boldsymbol{\Delta v}}{\boldsymbol{v}} = \frac{\boldsymbol{\Delta s}}{\boldsymbol{r}}.$$

Acceleration is **$\Delta v/\Delta t$**, and so we first solve this expression for **Δv**:

$$\boldsymbol{\Delta v = \frac{v}{r}\Delta s.}$$

Then we divide this by **Δt**, yielding

$$\boldsymbol{\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}.}$$

Finally, noting that **$\Delta v/\Delta t = a_c$** and that **$\Delta s/\Delta t = v$**, the linear or tangential speed, we see that the magnitude of the centripetal acceleration is

$$\boldsymbol{a_{\mathrm{c}} = \frac{v^2}{r},}$$

which is the acceleration of an object in a circle of radius ***r*** at a speed ***v***. So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that **a_c** is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that **a_c** is greater for tighter turns, as you have probably noticed.

It is also useful to express **a_c** in terms of angular velocity. Substituting **$v = r\omega$** into the above expression, we find **$a_c = (r\omega)^2/r = r\omega^2$**. We can express the magnitude of centripetal acceleration using either of two equations:

$$\boldsymbol{a_{\mathrm{c}} = \frac{v^2}{r};\ a_{\mathrm{c}} = r\omega^2.}$$

Recall that the direction of **a_c** is toward the center. You may use whichever expression is more convenient, as illustrated in examples below.

Example 1: How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See Figure 2(a).

Strategy

Because ***v*** and ***r*** are given, the first expression in $\boldsymbol{a_{\mathrm{c}} = \frac{v^2}{r};\ a_{\mathrm{c}} = r\omega^2}$ is the most convenient to use.

Solution

Entering the given values of **v = 25.0 m/s** and **r = 500 m** into the first expression for $\boldsymbol{a_c}$ gives

$$\boldsymbol{a_{\mathrm{c}} = \frac{v^2}{r} = \frac{(25.0\ \mathrm{m/s})^2}{500\ \mathrm{m}} = 1.25\ \mathrm{m/s^2}}.$$

Discussion

To compare this with the acceleration due to gravity (**g = 9.80 m/s^2**), we take the ratio of **a_c /g = (1.25 m/s^2)/(9.80m/s^2) = 0.128**. Thus, **a_c = 0.128 g** and is noticeable especially if you were not wearing a seat belt.

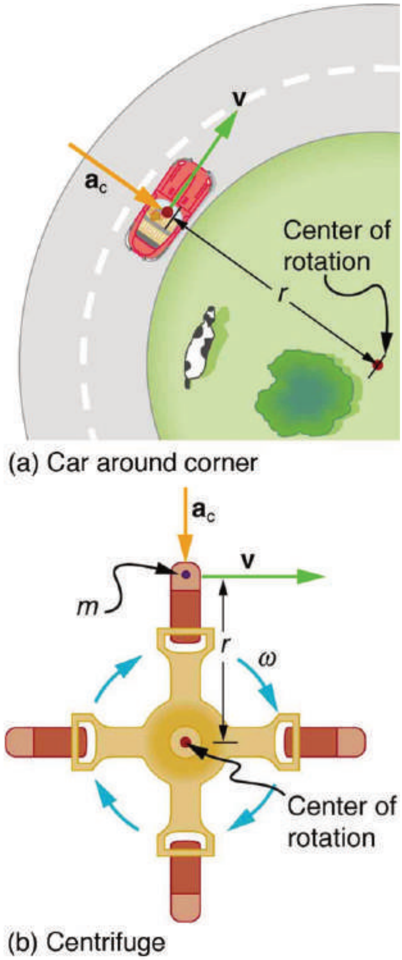


Figure 2. *(a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this centripetal acceleration is found in Example 1. (b) A particle of mass in a centrifuge is rotating at constant angular velocity . It must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in Example 2.*

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration.

Section Summary

- Centripetal acceleration $\boldsymbol{a_c}$ is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity $\boldsymbol{v}$ and has the magnitude

$$\boldsymbol{a_c = \frac{v^2}{r}; \; a_c = r\omega^2.}$$

- The unit of centripetal acceleration is **m/s^2**.

Conceptual Questions

1: Can centripetal acceleration change the speed of circular motion? Explain.

Problems & Exercises

1: A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?

2: Olympic ice skaters are able to spin at about 5 rev/s.

(a) What is their angular velocity in radians per second?

(b) What is the centripetal acceleration of the skater's nose if it is 0.120 m from the axis of rotation?

(c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius?

(d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.

3: Unreasonable Results

A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass.

(a) What is the magnitude of the centripetal acceleration of the child at the low point?

(b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent?

Glossary

centripetal acceleration
: the acceleration of an object moving in a circle, directed toward the center

ultracentrifuge
: a centrifuge optimized for spinning a rotor at very high speeds

Solutions

Problems & Exercises

2: (a) $\mathbf{31.4\ rad/s}$ (b) $\mathbf{118\ m/s}$ (c) $\mathbf{384\ m/s}$

(d) The centripetal acceleration felt by Olympic skaters is 12 times larger than the acceleration due to gravity. That's quite a lot of acceleration in itself. The centripetal acceleration felt by Button's nose was 39.2 times larger than the acceleration due to gravity. It is no wonder that he ruptured small blood vessels in his spins.

3: (a) $\mathbf{40.5\ m/s^2}$ (b) $\boldsymbol{905\textbf{ N}}$ (c) The force in part (b) is very large. The acceleration in part (a) is too much, about 4 g. (d) The speed of the swing is too large. At the given velocity at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction.

PART VI

CHAPTER 6: LINEAR KINETICS

Chapter Objectives

After this chapter, you will be able to:

- Define force, mass and inertia
- Understand Newton's first law of motion.
- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.
- Understand Newton's third law of motion.
- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.
- Define impulse.
- Describe the principle of conservation of momentum.
- Describe an elastic collision of two objects in one dimension.

- Determine the final velocities in an elastic collision given masses and initial velocities.

35. 6.0 Introduction

Figure 1. *Newton's laws of motion describe the motion of the dolphin's path. (credit: Jin Jang)*

Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Kinetics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of kinetics. These laws provide an example of the breadth and simplicity of principles under which nature functions.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe and describe motion.

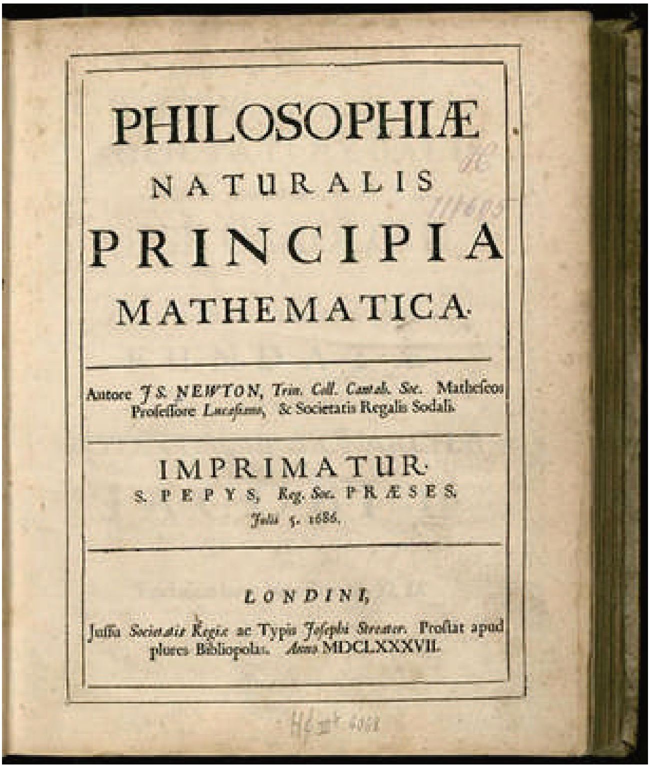


Figure 2. Isaac Newton's monumental work, Philosophiae Naturalis Principia Mathematica, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l'Université de Strasbourg)

36. 6.1 Development of Force Concept

Summary

- Define force.

Kinetics is the study of the effects of forces on objects. **Dynamics** is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a baseball pitcher exerts a strong force on a the ball that is thrown towards the hitter. The earth pulls the high jumper back down towards the landing mat. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in Figure 1, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in Figure 2(a) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods.

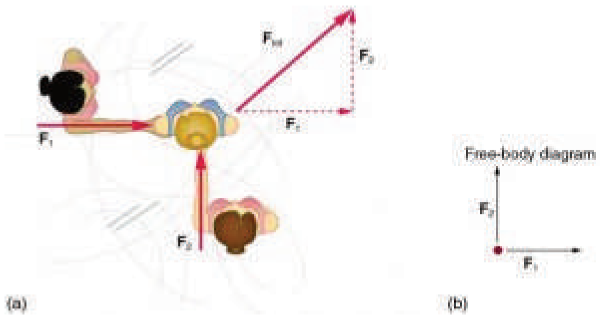


Figure 1. *Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.*

Figure 1(b) is our first example of a **free-body diagram**, which is a technique used to illustrate all the **external forces** acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in Figure 2, and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. Some alternative definitions of force will be given later in this chapter.

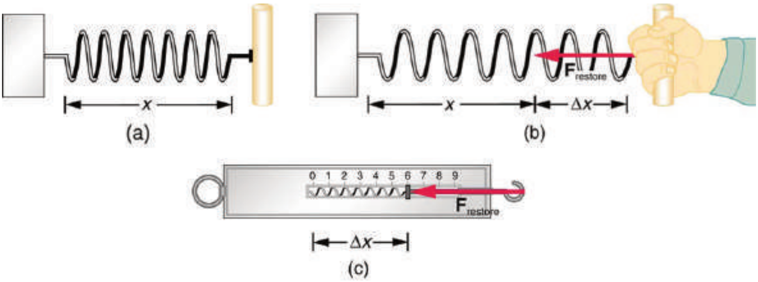


Figure 2. *The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length* ***x*** *when undistorted. (b) When stretched a distance* ***Δx****, the spring exerts a restoring force,* ***F*restore***, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force* ***F*restore** *is exerted on whatever is attached to the hook. Here* ***F*restore** *has a magnitude of 6 units in the force standard being employed.*

TAKE-HOME EXPERIMENT: FORCE STANDARDS

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

Section Summary

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

Conceptual Questions

1: Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

2: What properties do forces have that allow us to classify them as vectors?

Glossary

dynamics
: the study of how forces affect the motion of objects and systems

external force
: a force acting on an object or system that originates outside of the object or system

free-body diagram
: a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

force
: a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

37. 6.2 Newton’s First Law of Motion: Inertia

Summary

- Define mass and inertia.
- Understand Newton’s first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton’s first law of motion** states, however, is the following:

NEWTON’S FIRST LAW OF MOTION

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb “remains.” We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton’s first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered.

Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of "stuff" (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Check Your Understanding

1: Which has more mass: a kilogram of cotton balls or a kilogram of gold?

Section Summary

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.

- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

Conceptual Questions

1: How are inertia and mass related?

2: What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

Glossary

inertia
: the tendency of an object to remain at rest or remain in motion

law of inertia
: see Newton's first law of motion

mass
: the quantity of matter in a substance; measured in kilograms

Newton's first law of motion
: a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

Solutions

Check Your Understanding

1: They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

38. 6.3 Newton's Second Law of Motion: Concept of a System

Summary

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

Newton's second law of motion is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For example, in Figure 1(a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at Figure 1(a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external*. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.

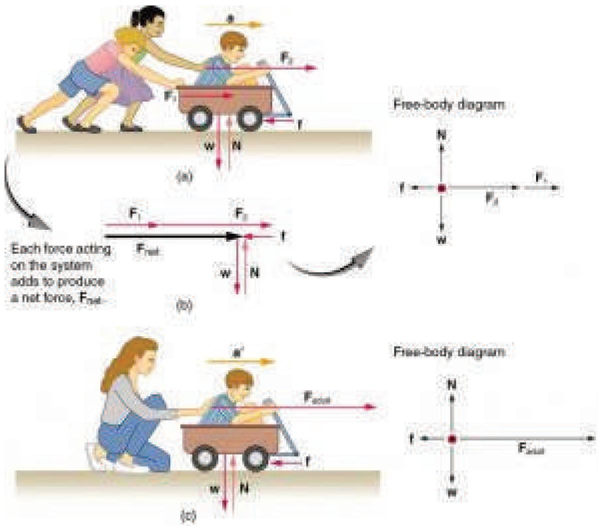


Figure 1. *Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight* ***w*** *of the system and the support of the ground* ***N*** *are also shown for completeness and are assumed to cancel. The vector* ***f*** *represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force,* $\mathbf{F}_{\mathbf{net}}$*. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration (*$\mathbf{a'}>\mathbf{a}$*) when an adult pushes the child.*

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in Figure 1. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight $\boldsymbol{w}$ and the support of the ground $\boldsymbol{N}$, and the horizontal force $\boldsymbol{f}$ represents the force of friction. These will be discussed in more detail in later sections. For now, we will define friction as a force that opposes the motion past each other of objects that are touching. Figure 1(b) shows how vectors representing the external forces add together to produce a net force, $\boldsymbol{F}_{\mathbf{net}}$.

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

$$\mathbf{a} \propto \boldsymbol{F}_{\mathbf{net}} ,$$

where the symbol $\propto$ means "proportional to," and $\boldsymbol{F}_{\mathbf{net}}$ is the net external force. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors. This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in Figure 2, the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

$$\mathbf{a} \propto \frac{\mathbf{1}}{\boldsymbol{m}}$$

where $\boldsymbol{m}$ is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.

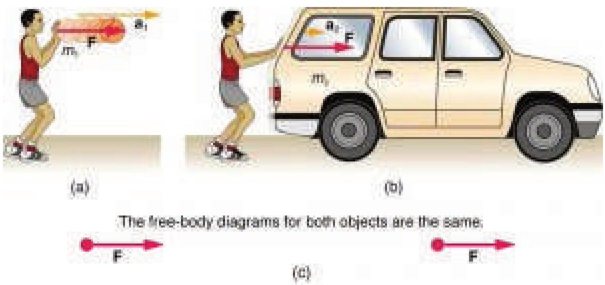


Figure 2. *The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.*

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

NEWTON'S SECOND LAW OF MOTION

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_{\mathbf{net}}}{\boldsymbol{m}}.$$

This is often written in the more familiar form

$$\vec{\mathbf{F}}_{\mathbf{net}} = \boldsymbol{m}\vec{\mathbf{a}}.$$

When only the magnitude of force and acceleration are considered, this equation is simply

$$\boldsymbol{F}_{\mathbf{net}} = \boldsymbol{ma}.$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

Units of Force

$\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of **1 m/s^2**. That is, $\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$,

$$\mathbf{1\ N = 1\ kg \cdot m/s^2}.$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight** ***w***. Weight can be denoted as a vector $\vec{\mathbf{w}}$ because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as ***w***. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration ***g***. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass ***m*** falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude ***w***. Newton's second law states that the magnitude of the net external force on an object is $F_{net} = ma$.

Since the object experiences only the downward force of gravity, $\boldsymbol{F_{\text{net}}} = \boldsymbol{w}$. We know that the acceleration of an object due to gravity is $\boldsymbol{g}$, or $\boldsymbol{a} = \boldsymbol{g}$. Substituting these into Newton's second law gives

WEIGHT

This is the equation for *weight*—the gravitational force on a mass $\boldsymbol{m}$:

$$\boldsymbol{w = mg.}$$

Since $\boldsymbol{g} = \mathbf{9.80\ m/s^2}$ on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$\boldsymbol{w = mg = (1.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 9.8\,\mathrm{N}.}$$

Recall that $\boldsymbol{g}$ can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity $\boldsymbol{g}$ varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only $\mathbf{1.67\ m/s^2}$. A

1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

COMMON MISCONCEPTIONS: MASS VS. WEIGHT

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the "slug" in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (***m***) multiplied by the acceleration due to gravity (***g***). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is **1.67 m/s^2** (which is much less than the acceleration due to gravity on Earth, **9.80 m/s^2**). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you "weigh" much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are "losing weight," they really mean that they are losing "mass" (which in turn causes them to weigh less).

TAKE-HOME EXPERIMENT: MASS AND WEIGHT

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same "mass" on Earth as on the Moon?

Example 1: What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration**?**

Figure 3. *The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?*

Strategy

Since $\boldsymbol{F}_{\textbf{net}}$ and $\boldsymbol{m}$ are given, the acceleration can be calculated directly from Newton's second law as stated in $\boldsymbol{F}_{\textbf{net}} = \boldsymbol{m}\ \boldsymbol{a}$.

Solution

The magnitude of the acceleration $\boldsymbol{a}$ is

$\boldsymbol{a} = \dfrac{\boldsymbol{F}_{\textbf{net}}}{\boldsymbol{m}}$. Entering known values gives

$$\boldsymbol{a} = \frac{\textbf{51 N}}{\textbf{24 kg}}$$

Substituting the units **kg·m/s²** for N yields

$$\boldsymbol{a} = \frac{\textbf{51 kg}\cdot\textbf{m/s}^{\textbf{2}}}{\textbf{24 kg}} = \textbf{2.1 m/s}^{\textbf{2}}.$$

Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

Section Summary

- Acceleration, $\boldsymbol{a}$, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is

$$\mathbf{a} = \frac{\boldsymbol{F}_{\mathbf{net}}}{\boldsymbol{m}}.$$

- This is often written in the more familiar form: $\boldsymbol{F}_{\mathbf{net}} = \boldsymbol{m}\,\boldsymbol{a}$.
- The weight $\boldsymbol{w}$ of an object is defined as the force of gravity acting on an object of mass $\boldsymbol{m}$. The object experiences an acceleration due to gravity $\boldsymbol{g}$:

$$\boldsymbol{w} = \boldsymbol{m}\,\boldsymbol{g}.$$

- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

Conceptual Questions

1: Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

2: Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

3: Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.

4: Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

5: A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

6: A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

7: (a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

8: If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

9: If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

10: The gravitational force on the basketball in Figure 2 is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

Problems & Exercises

You may assume data taken from illustrations is accurate to three digits.

1: A 63.0-kg sprinter starts a race with an acceleration of 4.20 m/s^2.What is the net external force on him?

2: If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

3: A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

4: Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

Glossary

acceleration
: the rate at which an object's velocity changes over a period of time

free-fall
: a situation in which the only force acting on an object is the force due to gravity

friction

a force past each other of objects that are touching; examples include rough surfaces and air resistance

net external force

the vector sum of all external forces acting on an object or system; causes a mass to accelerate

Newton's second law of motion

the net external force F_{net} on an object with mass m is proportional to and in the same direction as the acceleration of the object, a, and inversely proportional to the mass; defined mathematically as $\mathbf{a} = \frac{\boldsymbol{F}_{\mathbf{net}}}{\boldsymbol{m}}$

system

defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

weight

the force w due to gravity acting on an object of mass m; defined mathematically as: $w = mg$, where g is the magnitude and direction of the acceleration due to gravity

Solutions

Problems & Exercises

1: $\mathbf{265\ N}$

3: $\mathbf{13.3\ m/s}$

4: (a) The system is the child in the wagon plus the wagon.

(b)

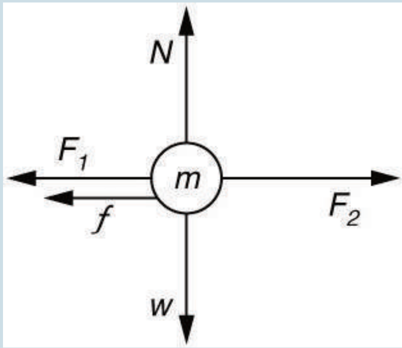


Figure 8.

(c) $\boldsymbol{a = 0.130\ \mathrm{m/s^2}}$ in the direction of the second child's push. (d) $\boldsymbol{a = 0.00\ \mathrm{m/s^2}}$

39. 6.4 Newton's Third Law of Motion: Symmetry in Forces

Summary

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

There is a passage in the musical *Man of la Mancha* that relates to Newton's third law of motion. Sancho, in describing a fight with his wife to Don Quixote, says, "Of course I hit her back, Your Grace, but she's a lot harder than me and you know what they say, 'Whether the stone hits the pitcher or the pitcher hits the stone, it's going to be bad for the pitcher.'" This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in **Newton's third law of motion**.

NEWTON'S THIRD LAW OF MOTION

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton's third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 1. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $\mathbf{F}_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $\mathbf{F}_{\text{wall on feet}}$. In contrast, the force $\mathbf{F}_{\text{feet on wall}}$

acts on the wall and not on our system of interest. Thus $\boldsymbol{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\boldsymbol{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.

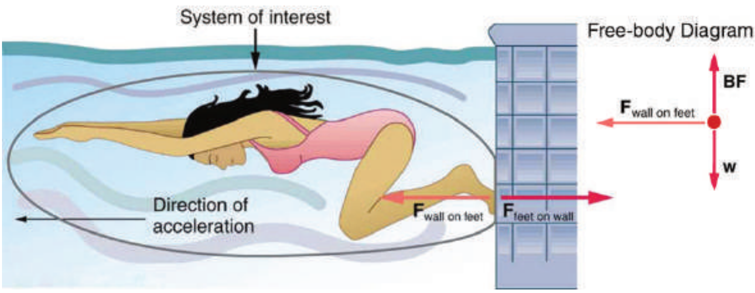


Figure 1. *When the swimmer exerts a force* $\mathbf{F}_{\text{feet on wall}}$ *on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to* $\mathbf{F}_{\text{feet on wall}}$. *This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force* $\mathbf{F}_{\text{wall on feet}}$ *on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that* $\mathbf{F}_{\text{feet on wall}}$ *does not act on this system (the swimmer) and, thus, does not cancel* $\mathbf{F}_{\text{wall on feet}}$. *Thus the free-body diagram shows only* $F_{\text{wall on feet}}$, ***w****, the gravitational force, and* ***BF****, the buoyant force of the water supporting the swimmer's weight. The vertical forces* ***w*** *and* ***BF*** *cancel since there is no vertical motion.*

Other examples of Newton's third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. A professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

Example 1: Getting Up To Speed: Choosing the Correct System

A biomechanics professor pushes a cart of demonstration equipment to a lecture hall, as seen in Figure 2. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

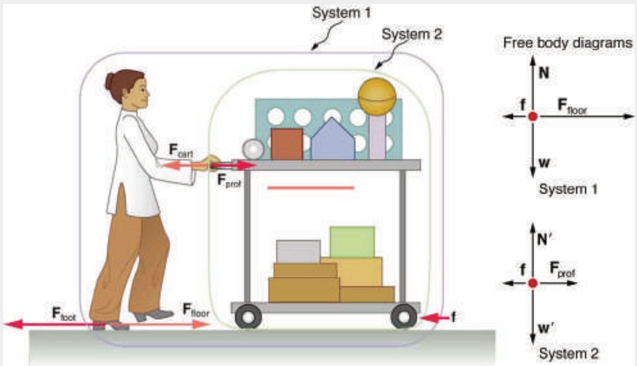


Figure 2. *A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for **f**, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for Example 2, since it asks for the acceleration of the entire group of objects. Only* $\mathbf{F}_{\text{floor}}$ *and **f** are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for this example so that* $\mathbf{F}_{\text{prof}}$ *will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.*

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in Figure 2. The professor pushes backward with a force $\mathbf{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\mathbf{F}_{\text{floor}}$ of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted, ***f*** opposes the motion and is thus in the opposite direction of $\boldsymbol{F}_{\mathbf{floor}}$. Note that we do not include the forces $\boldsymbol{F}_{\mathbf{prof}}$ or $\boldsymbol{F}_{\mathbf{cart}}$ because these are internal forces, and we do not include $\boldsymbol{F}_{\mathbf{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{\mathbf{net}}}{\boldsymbol{m}}.$$

The net external force on System 1 is deduced from Figure 2 and the discussion above to be

$$\mathbf{F}_{\mathbf{net}} = \mathbf{F}_{\mathbf{floor}} - \mathbf{f} = \mathbf{150\ N} - \mathbf{24.0\ N} = \mathbf{126\ N}.$$

The mass of System 1 is

$$\boldsymbol{m} = (\mathbf{65.0} + \mathbf{12.0} + \mathbf{7.0})\ \mathbf{kg} = \mathbf{84\ kg}.$$

These values of $\boldsymbol{F}_{\mathbf{net}}$ and $\boldsymbol{m}$ produce an acceleration of

$$\boldsymbol{a} \quad \frac{\boldsymbol{F}_{\mathbf{net}}}{\boldsymbol{m}}$$

$$\boldsymbol{a} \quad \frac{\mathbf{126\ N}}{\mathbf{84\ kg}} = \mathbf{1.5\ m/\ s^2}$$

Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example 2: Force of the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in Figure 2 using data from the previous example if needed.

Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in Figure 2), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, $\boldsymbol{F}_{\textbf{prof}}$, is an external force acting on System 2. $\boldsymbol{F}_{\textbf{prof}}$ was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

Solution

Newton's second law can be used to find $\boldsymbol{F}_{\textbf{prof}}$. Starting with

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{\textbf{net}}}{\boldsymbol{m}}$$

and noting that the magnitude of the net external force on System 2 is

$$\boldsymbol{F}_{\textbf{net}} = \boldsymbol{F}_{\textbf{prof}} - \boldsymbol{f},$$

we solve for $\boldsymbol{F}_{\textbf{prof}}$, the desired quantity:

$$\boldsymbol{F}_{\textbf{prof}} = \boldsymbol{F}_{\textbf{net}} + \boldsymbol{f}.$$

The value of $\boldsymbol{f}$ is given, so we must calculate net $\boldsymbol{F}_{\textbf{net}}$. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$\boldsymbol{F}_{\textbf{net}} = \boldsymbol{ma},$$

where the mass of System 2 is 19.0 kg ($\boldsymbol{m}$= 12.0 kg + 7.0 kg) and its acceleration was found to be $\boldsymbol{a} = \mathbf{1.5\ m/s^2}$ in the previous example. Thus,

$$\boldsymbol{F}_{\textbf{net}} = \boldsymbol{ma},$$

$$\boldsymbol{F}_{\textbf{net}} = \mathbf{(19.0\ kg)(1.5\ m/s^2) = 29\ N.}$$

Now we can find the desired force:

$$\boldsymbol{F}_{\textbf{prof}} = \boldsymbol{F}_{\textbf{net}} + \boldsymbol{f},$$

$$\boldsymbol{F}_{\textbf{prof}} = \mathbf{29\ N + 24.0\ N = 53\ N.}$$

Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

Section Summary

- **Newton's third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.

Conceptual Questions

1: Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?

2: An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

Problems & Exercises

1: A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at 1.20 m/s^2 backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and **write the net force equation.**

Glossary

Newton's third law of motion
whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude

and opposite in direction to the force that the first body exerts

Solutions

Problems & Exercises

1: net force on the losing player = m a = (90.0 kg)(1.20 m/s^2) = 108 N. The pushing backwards force is 800 N but the net force is only 108 Ns so that means friction = (800-108) = 692 N. Without friction the net force would be 800 N and the player would accelerate very quickly backwards. b) net force on the winning player = m a = (110.0 kg)(1.20 m/s^2) = 132 N. He is exerting a force of 800 N so the friction force = ?

40. 6.5 Normal, Tension, and Other Examples of Forces

Summary

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in Figure 1(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in Figure 1(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

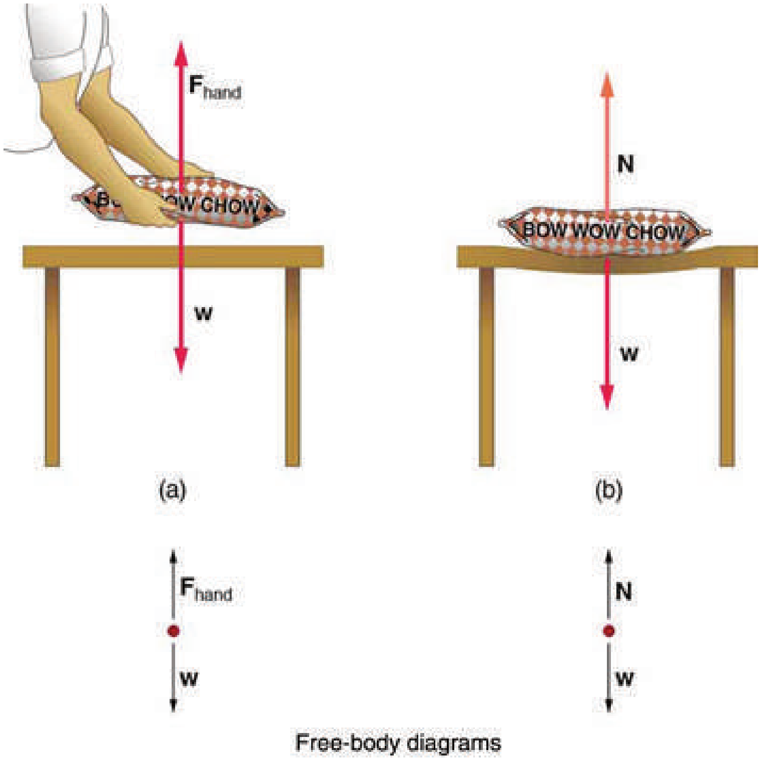


Figure 1. *(a) The person holding the bag of dog food must supply an upward force* $\boldsymbol{F}_{\textbf{hand}}$ *equal in magnitude and opposite in direction to the weight of the food* ***w****. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force* ***N*** *equal in magnitude and opposite in direction to the weight of the load.*

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol ***N***. (This is not the unit for force N.) The word normal means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

COMMON MISCONCEPTIONS: NORMAL FORCE (N) VS. NEWTON (N)

In this section we have introduced the quantity normal force, which is represented by the variable ***N***. Note: In biomechanics, the normal force (**N**) is sometimes referred to as the Ground Reaction Force (**GRF**). The **N** should not be confused with the symbol for the newton, which is also represented by the letter **N**. These symbols are particularly important to distinguish because the units of a normal force (***N***) happen to be newtons (**N**). For example, the normal force ***N*** that the floor exerts on a chair might be ***N* = 100 N**. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (***W***) and the unit watts (**W**).

Example 1: Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in Figure 2. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N?

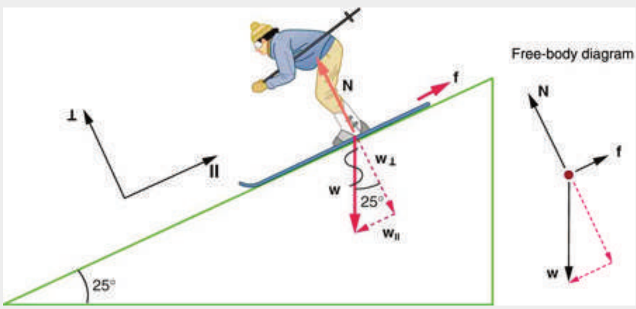


Figure 2. *Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).* ***N*** *is perpendicular to the slope and* ***f*** *is parallel to the slope, but* ***w*** *has components along both axes.* ***N*** *is equal in magnitude to the weight component into the slope (along the perpendicular axis) because there is no motion perpendicular to the slope, but* ***f*** *is less than the component of the weight parallel to the slope or* $\mathbf{w}_{||}$*, so that there is a down slope acceleration (along the parallel axis).*

Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating *two* connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols $\perp$ and $\|$ to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled ***w***, and ***N*** in Figure 2. ***N*** is always perpendicular to the slope, and ***f*** is parallel to it. But ***w*** is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining $\boldsymbol{w}_{\|}$ to be the component of weight parallel to the slope and $\boldsymbol{w}_{\perp}$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is

$$w_{\parallel} = w \sin(25^{o}) = mg \sin(25^{0}),$$

and the magnitude of the component of the weight perpendicular to the slope is

$$w_{\perp} = w \cos(25^{0}) = mg \cos(25^{0}).$$

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope $w_{\parallel}$ and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_{\parallel} = \frac{F_{\mathrm{net}\parallel}}{m}$$

where $F_{\mathrm{net}\parallel} = w_{\parallel} = mg \sin(25^{0}),$ assuming no friction for this part, so that

$$a_{\parallel} = \frac{F_{\mathrm{net}\parallel}}{m} = \frac{mg \sin(25^{0})}{m} = g \sin(25^{0})$$

$$(9.80\ \mathrm{m/s^2})(0.4226) = 4.14\ \mathrm{m/s^2}$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$F_{\mathrm{net}\parallel} = w_{\parallel} - f,$$

and substituting this into Newton's second law,

$\boldsymbol{a_{\parallel} = \frac{F_{\text{net}}}{m}}$, gives

$$\boldsymbol{a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} = \frac{w_{\parallel} - f}{m} = \frac{mg\sin(25^{0}) - f}{m}}.$$

We substitute known values to obtain

$$\boldsymbol{a_{\parallel} = \frac{(60.0\ \text{kg})(9.80\ \text{m/s}^2)(0.4226) - 45.0\ \text{N}}{60.0\ \text{kg}}},$$

which yields

$$\boldsymbol{a_{\parallel} = 3.39\ \text{m/s}^2},$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is ***a* = *g* sin *θ***, *regardless of mass*. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

RESOLVING WEIGHT INTO COMPONENTS

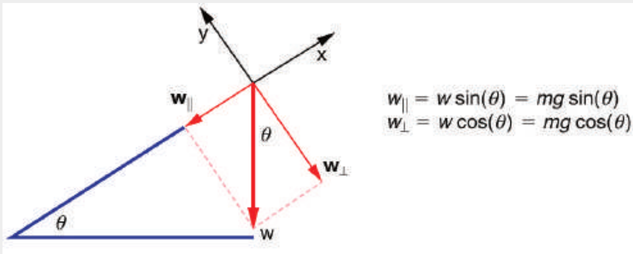


Figure 3. *An object rests on an incline that makes an angle **θ** with the horizontal.*

When an object rests on an incline that makes an angle ***θ*** with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, $\mathbf{w}_{\perp}$, and a force acting parallel to the plane, $\mathbf{w}_{||}$. The perpendicular force of weight, $\mathbf{w}_{\perp}$, is typically equal in magnitude and opposite in direction to the normal force, ***N***. The force acting parallel to the plane, $\mathbf{w}_{||}$, causes the object to accelerate down the incline. The force of friction, ***f***, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle $\boldsymbol{\theta}$ to the horizontal, then the magnitudes of the weight components are

$$\boldsymbol{w_{\parallel} = w\,\textbf{sin}\,(\theta) = mg\,\textbf{sin}\,(\theta)}$$

and

$$\boldsymbol{w_{\perp} = w\,\textbf{cos}\,(\theta) = mg\,\textbf{cos}\,(\theta).}$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the $\boldsymbol{\theta}$ of the incline is the same as the angle formed between $\mathbf{w}$ and $\mathbf{w}_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

\begin{array}{r @{{}={}}l} \boldsymbol{\textbf{cos}(\theta)} & \boldsymbol{\frac{w_{\perp}}{w}} \end{array}

$$\boldsymbol{w_{\perp} = w\,\textbf{cos}\,(\theta) = mg\,\textbf{cos}\,(\theta)}$$

\begin{array}{r @{{}={}}l} \boldsymbol{\textbf{sin}(\theta)} & \boldsymbol{\frac{w_{\parallel}}{w}} \end{array}

\boldsymbol{w_{\parallel}} = \boldsymbol{w\:\textbf{sin}\:(\theta) = mg\:\textbf{sin}\:(\theta)}

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in Figure 4.

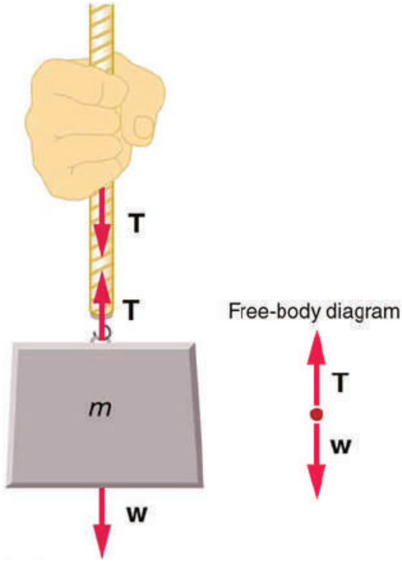


Figure 4. *When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force* ***T****, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.*

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the

5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\boldsymbol{F}_{\text{net}} = \mathbf{0}$. The only external forces acting on the mass are its weight $\boldsymbol{w}$ and the tension $\boldsymbol{T}$ supplied by the rope. Thus,

$$\boldsymbol{F}_{\mathbf{net}} = \boldsymbol{T} - \boldsymbol{w} = \mathbf{0},$$

where $\boldsymbol{T}$ and $\boldsymbol{w}$ are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$\boldsymbol{T} = \boldsymbol{w} = \boldsymbol{mg}$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$\boldsymbol{T} = \boldsymbol{mg} = (\mathbf{5.00\ kg})(\mathbf{9.80\ m/s^2}) = \mathbf{49.0\ N.}$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in Figure 5 (a) and (b).

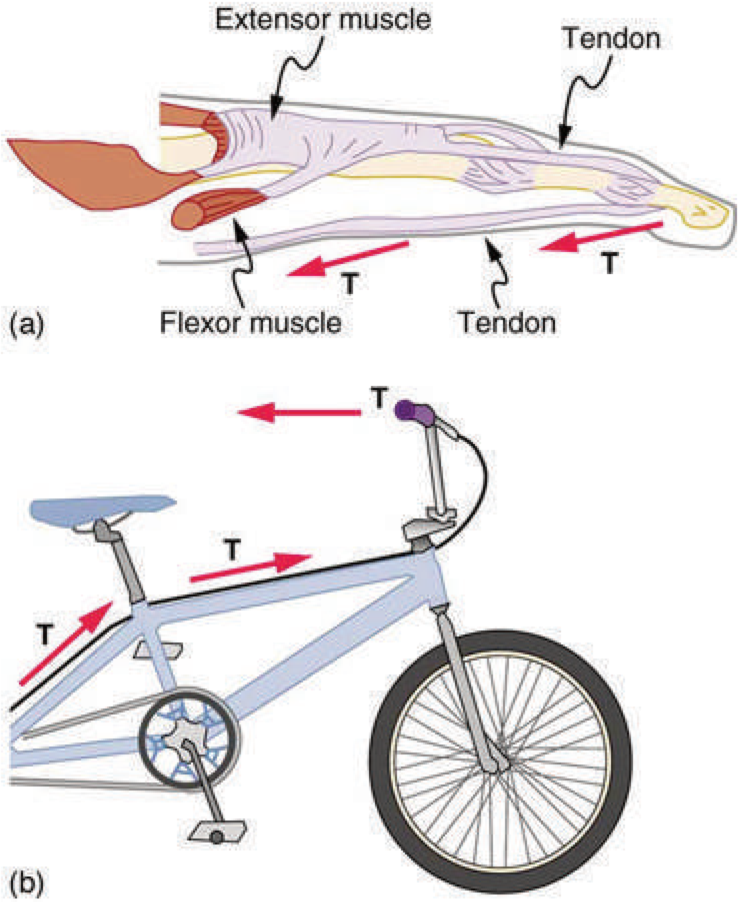


*Figure 5. (a) Tendons in the finger carry force **T** from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension **T** from the handlebars to the brake mechanism. Again, the direction but not the magnitude of **T** is changed.*

Example 2: What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in Figure 6.

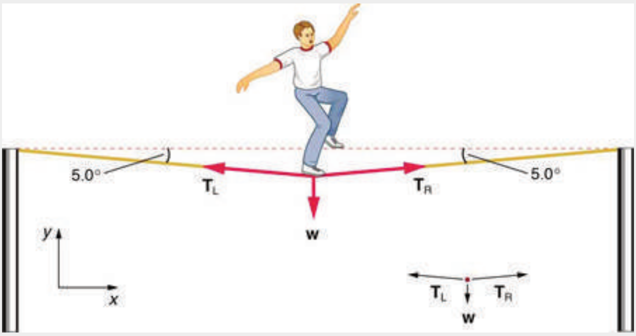


Figure 6. *The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.*

Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight $\boldsymbol{w}$ and the two tensions $\boldsymbol{T}_{\mathbf{L}}$ (left tension) and $\boldsymbol{T}_{\mathbf{R}}$ (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions $\boldsymbol{T}_{\mathbf{L}}$ and $\boldsymbol{T}_{\mathbf{R}}$ must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are $\boldsymbol{T}_{\mathbf{L}}$ and $\boldsymbol{T}_{\mathbf{R}}$. Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the *x*-axis and the vertical the *y*-axis.

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.

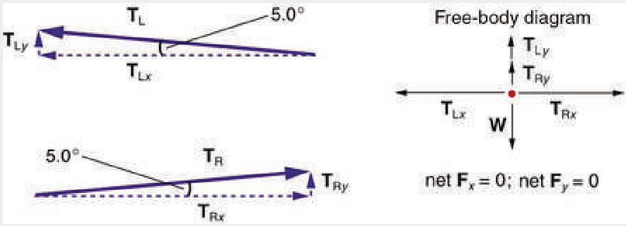


Figure 7. *When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in **T** being much greater than **w**.*

Consider the horizontal components of the forces (denoted with a subscript $\boldsymbol{x}$):

$$\boldsymbol{F}_{\mathbf{net}\boldsymbol{x}} = \boldsymbol{T}_{\mathbf{L}\boldsymbol{x}} - \boldsymbol{T}_{\mathbf{R}\boldsymbol{x}}.$$

The net external horizontal force $\boldsymbol{F}_{\mathbf{net}\boldsymbol{x}} = \mathbf{0}$, since the person is stationary. Thus,

$$\boldsymbol{F}_{\mathbf{net}\boldsymbol{x}} = \mathbf{0} \quad \boldsymbol{T}_{\mathbf{L}\boldsymbol{x}} - \boldsymbol{T}_{\mathbf{R}\boldsymbol{x}}$$

$$\boldsymbol{T}_{\mathbf{L}\boldsymbol{x}} \qquad \boldsymbol{T}_{\mathbf{R}\boldsymbol{x}}$$

Now, observe Figure 7. You can use trigonometry to determine the magnitude of $\boldsymbol{T}_{\mathbf{L}}$ and $\boldsymbol{T}_{\mathbf{R}}$. Notice that:

$$\cos\left(5.0^{\circ}\right) \qquad \frac{T_{\mathrm{L}x}}{T_{\mathrm{L}}}$$

$$T_{\mathrm{L}x} \qquad T_{\mathrm{L}}\cos\left(5.0^{\circ}\right)$$

$$\cos\left(5.0^{\circ}\right) \qquad \frac{T_{\mathrm{R}x}}{T_{\mathrm{R}}}$$

$$T_{\mathrm{R}x} \qquad T_{\mathrm{R}}\cos\left(5.0^{\circ}\right)$$

Equating $T_{\mathrm{L}x}$ and $T_{\mathrm{R}x}$:

$$T_{\mathrm{L}}\cos(5.0^{0}) = T_{\mathrm{R}}\cos(5.0^{0}).$$

Thus,

$$T_{\mathrm{L}} = T_{\mathrm{R}} = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton's second law implies that net $F_{\mathrm{y}} = 0$. Thus, as illustrated in the free-body diagram in Figure 7,

$$F_{\mathrm{net}y} = T_{\mathrm{L}y} + T_{\mathrm{R}y} - w = 0.$$

Observing Figure 7, we can use trigonometry to determine the relationship between $T_{\mathrm{L}y}$, $T_{\mathrm{R}y}$, and T. As we determined from the analysis in the horizontal direction, $T_{\mathrm{L}} = T_{\mathrm{R}} = T$:

$$\sin(5.0^\circ) \qquad \frac{T_{Ly}}{T_L}$$

$$T_{Ly} = T_L \sin(5.0^\circ) \quad T\sin(5.0^\circ)$$

$$\sin(5.0^\circ) \qquad \frac{T_{Ry}}{T_R}$$

$$T_{Ry} = T_R \sin(5.0^\circ) \quad T\sin(5.0^\circ)$$

Now, we can substitute the values for T_{Ly} and T_{Ry}, into the net force equation in the vertical direction:

$$F_{\text{net}y} \qquad T_{Ly} + T_{Ly} - w = 0$$

$$F_{\text{net}y} \qquad T\sin(5.0^\circ) + T\sin(5.0^\circ) - w = 0$$

$$2\,T\sin(5.0^\circ) - w \qquad 0$$

$$2\,T\sin(5.0^\circ) \qquad w$$

and

$$T = \frac{w}{2\sin(5.0^0)} = \frac{mg}{2\sin(5.0^0)},$$

so that

$$T = \frac{(70.0\text{ kg})(9.80\text{ m/s}^2)}{2(0.0872)},$$

and the tension is

$$T = 3900\text{ N}.$$

Discussion

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in Figure 8. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

$$\boldsymbol{T} = \frac{\boldsymbol{w}}{\mathbf{2}\,\mathbf{sin}(\boldsymbol{\theta})}.$$

We can extend this expression to describe the tension ***T*** created when a perpendicular force or $\boldsymbol{F}_{\perp}$ is exerted at the middle of a flexible connector:

$$\boldsymbol{T} = \frac{\boldsymbol{F}_{\perp}}{\mathbf{2}\,\mathbf{sin}(\boldsymbol{\theta})}.$$

Note that **θ** is the angle between the horizontal and the bent connector. In this case, ***T*** becomes very large as ***θ*** approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., **θ = 0** and **sin*θ* = 0**). (See Figure 8.)

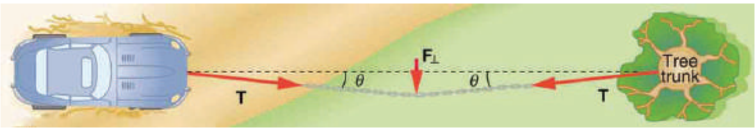


Figure 8. *We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by* $\boldsymbol{T = F_{\perp} / 2 \sin(\theta)}$*; since* $\boldsymbol{\theta}$ *is small,* $\boldsymbol{T}$ *is very large. This situation is analogous to the tightrope walker shown in Figure 6, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where* $\boldsymbol{F_{\perp}}$ *is applied.*

PHET EXPLORATIONS: FORCES IN 1 DIMENSION

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

Figure 10. Forces in 1 Dimension

Section Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, ***N***.
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:
$$\boldsymbol{N = mg.}$$
- When objects rest on an inclined plane that makes an angle $\boldsymbol{\theta}$ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular ($\mathbf{w}_{\perp}$) and parallel ($\mathbf{w}_{||}$) to the surface of the plane. These components can be calculated using:
$$\boldsymbol{w_{||} = w \sin(\theta) = mg \sin(\theta)}$$
$$\boldsymbol{w_{\perp} = w \cos(\theta) = mg \cos(\theta).}$$
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, ***T***.When a rope supports the weight of an object that is at

rest, the tension in the rope is equal to the weight of the object:

$$\boldsymbol{T = mg.}$$

Conceptual Questions

1: If a leg is suspended by a traction setup as shown in Figure 11, what is the tension in the rope?

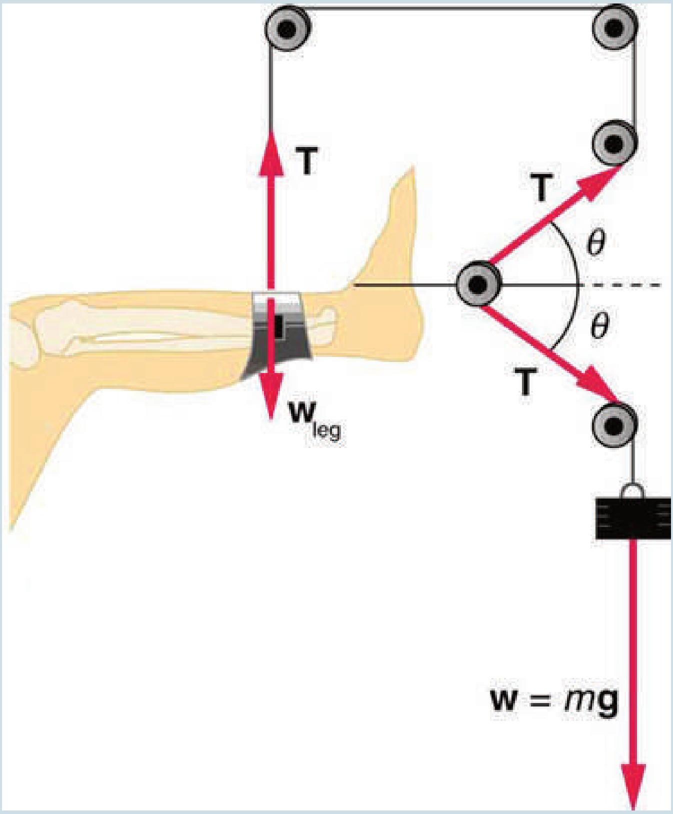


Figure 11. A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force T without changing its magnitude.

2: In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See Figure 11.) (Note that the tibia is the shin bone shown in this image.)

Problems & Exercises

1: Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

2: What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at 7.50 m/s^2? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

3: Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of 1.50 m/s^2?

4: Consider the baby being weighed in Figure 12. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension T_1 in the cord attaching the baby to the scale? (c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

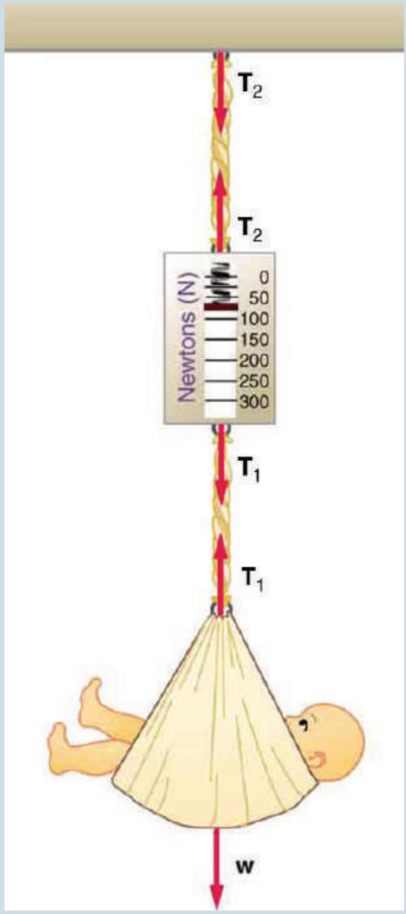


Figure 12. *A baby is weighed using a spring scale.*

Glossary

normal force
: the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests. In Biomechanics, the normal force is often referred to as the ground reaction force (GRF).

tension
: the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

Solutions

Problems & Exercises

1: (a) $\mathbf{0.11\ m/s^2}$ (b) $\mathbf{1.2 \times 10^4\ N}$

41. 6.6 Friction

Summary

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behaviour of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

FRICTION

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

KINETIC FRICTION

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure 1 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

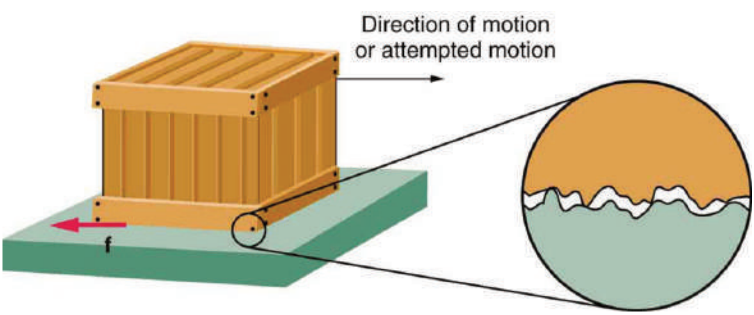


Figure 1. *Frictional forces, such as* ***f****, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.*

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction** $\boldsymbol{f_s}$ is

$$\boldsymbol{f_{\mathrm{s}} \leq \mu_{\mathrm{s}}\, N,}$$

where $\boldsymbol{\mu_s}$ is the coefficient of static friction and $\boldsymbol{N}$ is the magnitude of the normal force (the force perpendicular to the surface).

MAGNITUDE OF STATIC FRICTION

Magnitude of static friction $\boldsymbol{f_s}$ is

$$\boldsymbol{f_{\mathrm{s}} \leq \mu_{\mathrm{s}}\, N,}$$

where $\boldsymbol{\mu_s}$ is the coefficient of static friction and $\boldsymbol{N}$ is the magnitude of the normal force.

The symbol ≤ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\boldsymbol{\mu_s N}$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $\boldsymbol{f_{s(max)}}$, the object will move. Thus

$$\boldsymbol{f_{\mathrm{s(max)}} = \mu_{\mathrm{s}} N.}$$

Once an object is moving, the **magnitude of kinetic friction** $\boldsymbol{f_k}$ is given by

$$\boldsymbol{f_{\mathrm{k}} = \mu_{\mathrm{k}} N,}$$

where $\boldsymbol{\mu_k}$ is the coefficient of kinetic friction. A system in which $\boldsymbol{f_k = \mu_k N}$ is described as a system in which *friction behaves simply*.

> MAGNITUDE OF KINETIC FRICTION
>
> The magnitude of kinetic friction $\boldsymbol{f_k}$ is given by
>
> $$\boldsymbol{f_{\mathrm{k}} = \mu_{\mathrm{k}} N},$$
>
> where $\boldsymbol{\mu_k}$ is the coefficient of kinetic friction.

As seen in Table 1, the coefficients of kinetic friction are less than their static counterparts. That values of $\boldsymbol{\mu}$ in Table 1 are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

System	Static friction, μ_s	Kinetic friction, μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

Table 1. Coefficients of Static and Kinetic Friction.

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $\boldsymbol{W = mg}$ **= (100 kg)(9.80 m/s^2) = 980 N**, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $\boldsymbol{f_{s(max)} = \mu N}$ **= (0.45)(980 N) = 440 N** to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($\boldsymbol{f_K = \mu_K N}$ **= (0.30)(980 N) = 290 N**) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure 2). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

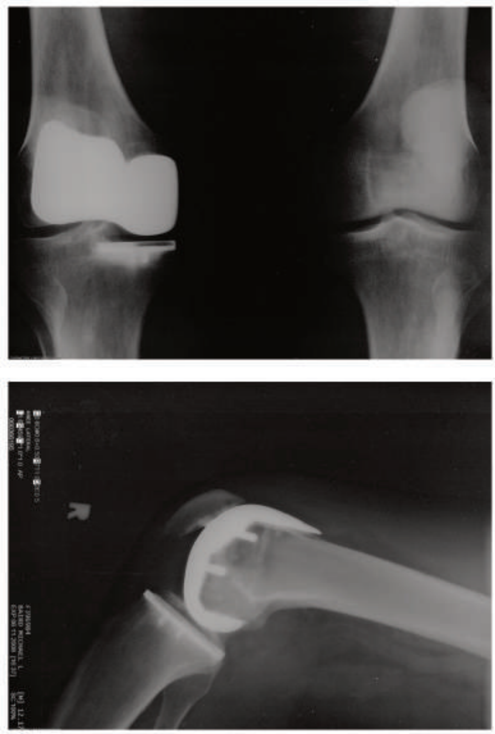


Figure 2. *Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)*

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

Example 1: Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force $\boldsymbol{N}$ as $\boldsymbol{f_k = \mu_k N}$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in Figure 3.)

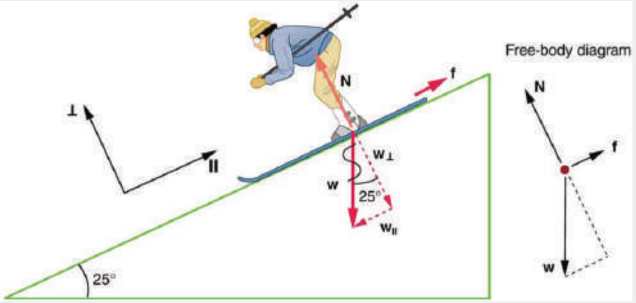


Figure 3. *The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).* ***N*** *(the normal force) is perpendicular to the slope, and* ***f*** *(the friction) is parallel to the slope, but* ***w*** *(the skier's weight) has components along both axes, namely* $\mathbf{w}_{\perp}$ *and* $\mathbf{w}_{//}$*.* ***N*** *is equal in magnitude to* $\mathbf{w}_{\perp}$*, so there is no motion perpendicular to the slope. However,* ***f*** *is less than* $\mathbf{w}_{//}$ *in magnitude, so there is acceleration down the slope (along the x-axis).*

That is,

$$N = w_{\perp} = w\cos 25^0 = mg\cos 25^0.$$

Substituting this into our expression for kinetic friction, we get

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} mg\cos 25^0,$$

which can now be solved for the coefficient of kinetic friction μ_{k}.

Solution

Solving for μ_{k} gives

$$\mu_{\mathbf{k}} = \frac{f_{\mathbf{k}}}{N} = \frac{f_{\mathbf{k}}}{w\cos 25^0} = \frac{f_{\mathbf{k}}}{mg\cos 25^0}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_{\mathbf{k}} = \frac{45.0\ \mathrm{N}}{(62\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in Table 1 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_k = \mu_k mg \cos\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

PHET EXPLORATIONS: FORCES

AND MOTION

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

Figure 6. Forces and Motion

Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force $\boldsymbol{N}$ pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction $\boldsymbol{f_s}$ between systems stationary relative to one another is given by

$$\boldsymbol{f_\mathrm{s}} \leq \boldsymbol{\mu}\mathbf{N},$$

where μ_s is the coefficient of static friction, which depends on both of the materials.

- The kinetic friction force f_k between systems moving relative to one another is given by

$$f_k = \mu_k N,$$

where $\mu_k$is the coefficient of kinetic friction, which also depends on both materials.

Conceptual Questions

1: Define normal force. What is its relationship to friction when friction behaves simply?

Problems & Exercises

1: (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

2: A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

3: Consider the 65.0-kg ice skater being pushed by two others shown in Figure 7. (a) Find the direction and magnitude of F_{tot}, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of F_{tot}? (c) What is her acceleration assuming she is already moving in the direction of F_{tot}? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)

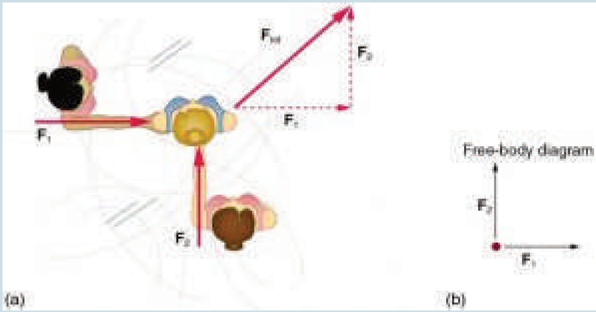


Figure 7.

4: Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g \sin \theta$. (Note that this acceleration is independent of mass.)

5: Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_K = \mu_K N$) is $a = g (\sin \theta - \mu_K \cos \theta)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small ($\mu_K = 0$).

6: Calculate the deceleration of a snow boarder going up a 5.0°, slope assuming the coefficient of friction for waxed wood on wet snow. The result of Exercise 9 may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in Chapter 4.6 Problem-Solving Strategies.

7: (a) Calculate the acceleration of a skier heading down a 10.0° slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of Exercise 9 to be useful. Explicitly show how you follow the steps in the Chapter 4.6 Problem-Solving Strategies.

8: Consider the 52.0-kg mountain climber in Figure 8. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?

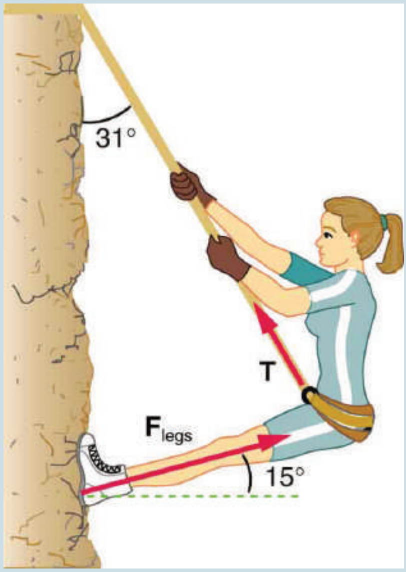


Figure 8. *Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.*

9: A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in Figure 9(a). (a) Calculate the minimum force *F* he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

10: Repeat Exercise 18 with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in Figure 9(b).

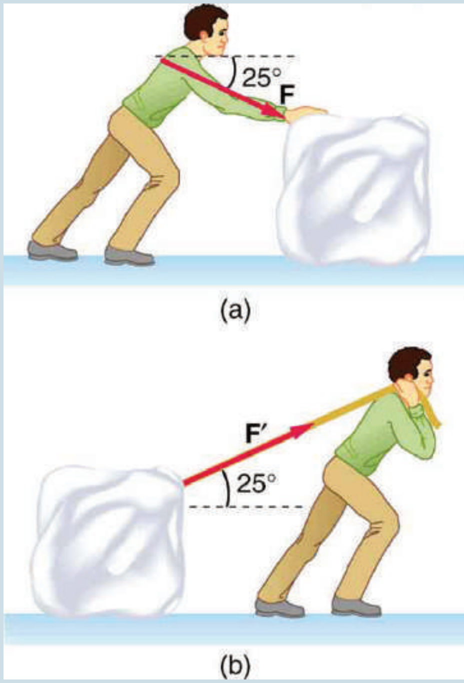


Figure 9. *Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?*

Glossary

friction
: a force that opposes relative motion or attempts at motion between systems in contact

kinetic friction
: a force that opposes the motion of two systems that are in contact and moving relative to one another

static friction
: a force that opposes the motion of two systems that are in contact and are not moving relative to one another

magnitude of static friction
: $f_s \leq \mu_s N$, where μ_s is the coefficient of static friction and N is the magnitude of the normal force

magnitude of kinetic friction
: $f_k = \mu_k N$, where μ_k is the coefficient of kinetic friction

Solutions

Problems & Exercises

2: (a)

$\mathbf{3.29\ m/s^2}$\$(b)\$$\mathbf{3.52\ m/s^2}$\$(c)\$$\mathbf{980\ N;\ 945\ N}$

6: $\mathbf{1.83\ m/s^2}$

9: (a) $\mathbf{51.0\ N}$ (b) $\mathbf{0.720\ m/s^2}$

42. 6.7 Problem-Solving Strategies

Summary

- Understand and apply a problem-solving procedure to solve problems using Newton's laws of motion.

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is* *particularly important to draw a careful sketch of the situation*. Such a sketch is shown in Figure 1(a). Then, as in Figure 1(b), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).

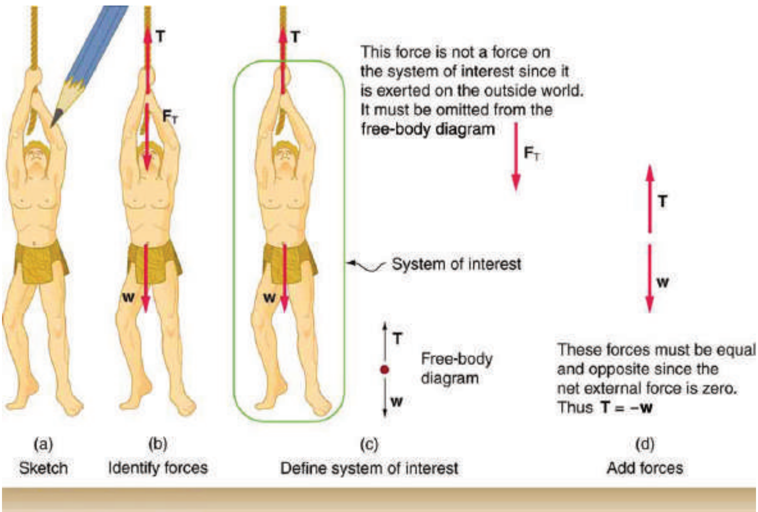


Figure 1. *(a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces.* ***T*** *is the tension in the vine above Tarzan,* $\mathbf{F_T}$ *is the force he exerts on the vine, and* ***w*** *is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram.* $\mathbf{F_T}$ *is no longer shown, because it is not a force acting on the system of interest; rather,* $\mathbf{F_T}$ *acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that* ***T = -w****, if Tarzan is stationary.*

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest*. This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See Figure 1(c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a **free-body diagram**. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. Figure 1(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem*. This is done in Figure 1(d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

Applying Newton's Second Law

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation: $\boldsymbol{F}_{\textbf{net}} = \boldsymbol{ma}$. For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$\boldsymbol{F}_{\textbf{net } x} = \boldsymbol{ma},$$
$$\boldsymbol{F}_{\textbf{net } y} = \mathbf{0}.$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable*. In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

Section Summary

- To solve problems involving Newton's laws of motion, follow the procedure described:

  1. Draw a sketch of the problem.
  2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
  3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the *x*-direction) then $\boldsymbol{F_{\text{net}x} = 0}$. If the object does accelerate in that direction, $\boldsymbol{F_{\text{net}x} = ma}$.
  4. Check your answer. Is the answer reasonable? Are the units correct?

Problems & Exercises

1: Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

2: When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

3: (a) Find the magnitudes of the forces F_1 and F_2 that add to give the total force F_{tot} shown in Figure 2. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of F_1 and F_2. (c) Find the direction and magnitude of some other pair of vectors that add to give F_{tot}. Draw these to scale on the same drawing used in part (b) or a similar picture.

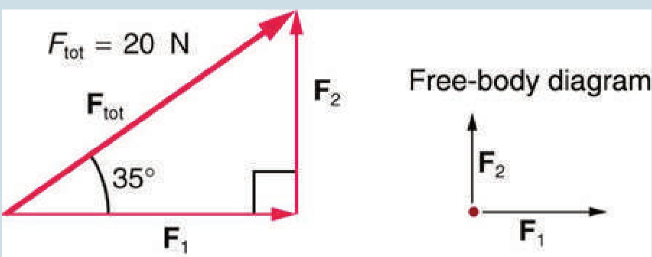


Figure 2.

4: Two children pull a third child on a snow saucer sled exerting forces F_1 and F_2 as shown from above in Figure 3. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of F_1 and F_2.

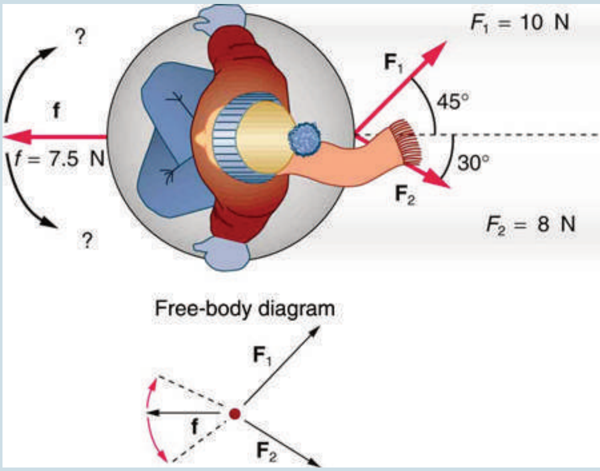


Figure 3. *An overhead view of the horizontal forces acting on a child's snow saucer sled.*

5: Figure 6 shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.

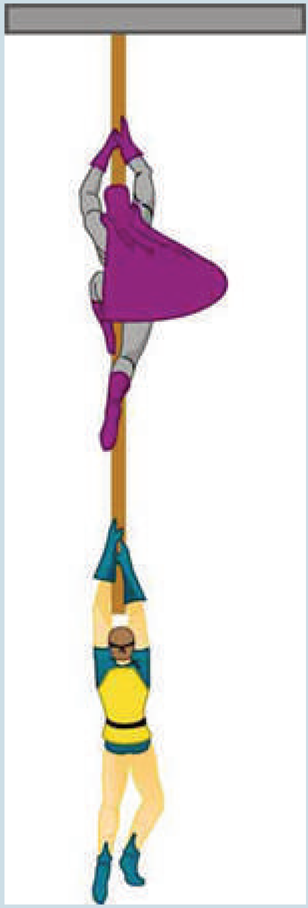


Figure 6. *Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope?*

6: A nurse pushes a cart by exerting a force on the handle at a downward angle $35.0°$ below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N. (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?

7: Construct Your Own Problem Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.

Solutions

Problems & Exercises

1:

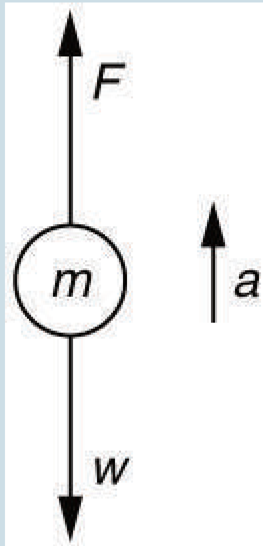


Figure 8.

1. Use Newton's laws of motion
2. Given :

$\boldsymbol{a = 4.00g = (4.00)(9.80\ m/s^2) = 39.2\ m/s^2\ ;\ m = 70.0\ kg}$,

Find: $\boldsymbol{F}$.

3. $\sum \boldsymbol{F} = +\boldsymbol{F} - \boldsymbol{w} = \boldsymbol{ma}$, so that

$$\boldsymbol{F = ma + w = ma + mg = m(a + g)}.$$

$\boldsymbol{F = (70.0\ kg)[(39.2\ m/s^2) + (9.80\ m/s^2)] = 3.43 \times 10^3\ N}$.

The force exerted by the high-jumper is actually down on the ground, but $\boldsymbol{F}$ is up from the ground and makes him jump.

4. This result is reasonable, since it is quite possible for a person to exert a force of the

magnitude of $\mathbf{10^3\ N}$.

43. 6.8 Further Applications of Newton's Laws of Motion

Summary

- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

Integrating Concepts: Newton's Laws of Motion and Kinematics

Biomechanics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

Problem-Solving Strategy

Step 1. *Identify which physical principles are involved*. Listing the givens and the quantities to be calculated will allow you to identify the principles involved.

Step 2. *Solve the problem using strategies outlined in the text*. If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

Example 4: What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

Strategy

1. *To solve an integrated concept problem,* we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics.* Part (b) deals with force, a topic of *dynamics* found in this

chapter.

2. The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

Solution for (a)

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is **Δ*v* = 8.00 m/s**. We are given the elapsed time, and so **Δ*t* = 2.50 s**. The unknown is acceleration, which can be found from its definition:

$$\boldsymbol{a} = \frac{\boldsymbol{\Delta v}}{\boldsymbol{\Delta t}}.$$

Substituting the known values yields

$$\boldsymbol{a} = \frac{\mathbf{8.00\ m/s}}{\mathbf{2.50\ s}} = \mathbf{3.20\ m/s^2}.$$

Discussion for (a)

This is an attainable acceleration for an athlete in good condition.

Solution for (b)

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

$$\boldsymbol{F}_{\mathbf{net}} = \boldsymbol{ma}.$$

Substituting the known values of ***m*** and ***a*** gives

$$\boldsymbol{F}_{\mathbf{net}} = \mathbf{(70.0\ kg)(3.20\ m/s^2) = 224}\boldsymbol{N}$$

Discussion for (b)

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.

- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $\mathbf{F_{net}} = \boldsymbol{ma}$ or $\mathbf{F_{net}} = \mathbf{0}$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

Problems & Exercises

1: Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in Figure 4. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

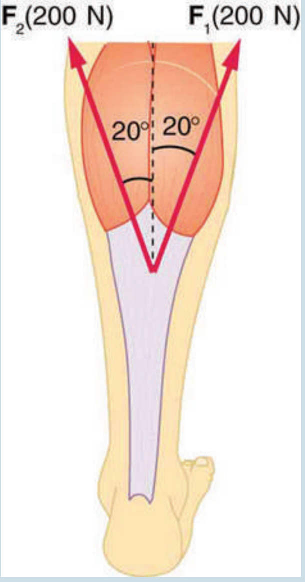


Figure 4. *Achilles tendon*

2: A 76.0-kg person is being pulled away from a burning building as shown in Figure 5. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

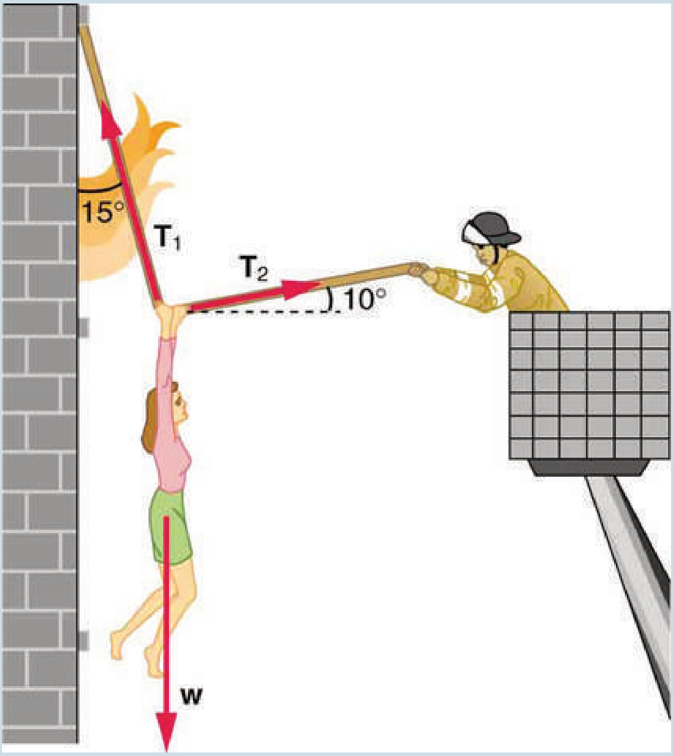


Figure 5. *The force **T_2** needed to hold steady the person being rescued from the fire is less than her weight and less than the force **T_1** in the other rope, since the more vertical rope supports a greater part of her weight (a vertical force)*

3: Integrated Concepts When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?   Hint: Find his acceleration first then use kinematics.

4: Integrated Concepts A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

Solutions

Problems & Exercises

2: T_1 = 736 N T_2 = 194 N   as net force is 0 N so using magnitudes only

T1 cos 15^{o} + T2 sin 10^{o} = Weight = mg   and   T1 sin 15^{o} = T2 cos10^{o}

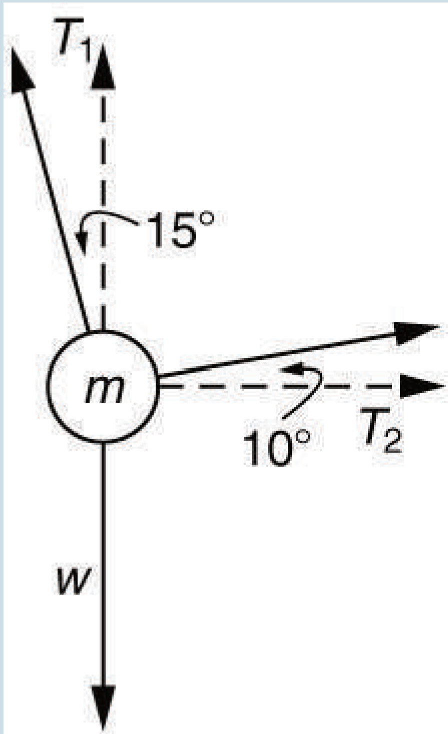


Figure 6.

3: (a) 7.43 m/s (b) 2.97 m as the acceleration is 650 N / 70.0 kg = 9.29 m/s^2

4: (a) 4.20 m/s 7 (b) 29.4 m/s^2 7 (c) 4.31 x 10^3 N

44. 6.9 Introduction to momentum

Figure 1. *Each rugby player has great momentum, which will affect the outcome of their collisions with each other and the ground. (credit: ozzzie, Flickr)*

We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature, and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.

45. 6.10 Linear Momentum and Force

Summary

- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}.$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum $\vec{\mathbf{p}}$ is a vector having the same direction as the velocity $\vec{\mathbf{v}}$. The SI unit for momentum is **kg · m/s**.

LINEAR MOMENTUM

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}.$$

Example 1: Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p. (As usual, a symbol that is in italics is a magnitude, whereas one that has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

$$\boldsymbol{p = mv}$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

$$\mathbf{p_{player} = (110\,kg)(8.00\,m/s) = 880\,kg{\cdot}\,m/s}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$$\mathbf{p_{ball} = (0.410\,kg)(25.0\,m/s) = 10.3\,kg{\cdot}\,m/s}$$

The ratio of the player's momentum to that of the ball is

$$\mathbf{\frac{p_{player}}{p_{ball}} = \frac{880}{10.3} = 85.9.}$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\boldsymbol{\Delta}\vec{\mathbf{p}}}{\boldsymbol{\Delta t}},$$

where $\vec{\mathbf{F}}_{\mathbf{net}}$ is the net external force, $\boldsymbol{\Delta}\vec{\mathbf{p}}$ is the change in momentum, and $\boldsymbol{\Delta t}$ is the change in time.

NEWTON'S SECOND LAW OF MOTION IN TERMS OF MOMENTUM

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\boldsymbol{\Delta}\vec{\mathbf{p}}}{\boldsymbol{\Delta t}}$$

MAKING CONNECTIONS: FORCE AND MOMENTUM

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum.

This statement of Newton's second law of motion includes the more familiar $\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta\vec{\mathbf{p}}$ is given by

$$\Delta\vec{\mathbf{p}} = \Delta(m\vec{\mathbf{v}}).$$

If the mass of the system is constant, then

$$\Delta(m\vec{\mathbf{v}}) = m\Delta\vec{\mathbf{v}}.$$

So that for constant mass, Newton's second law of motion becomes

$$\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{m\Delta\vec{\mathbf{v}}}{\Delta t}.$$

Because $\dfrac{\Delta\vec{\mathbf{v}}}{\Delta t} = \vec{\mathbf{a}},$ we get the familiar equation

$$\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$$

when the mass of the system is constant.

Example 2: Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\boldsymbol{F}_{\mathbf{net}} = \frac{\boldsymbol{\Delta p}}{\boldsymbol{\Delta t}}.$$

As noted above, when mass is constant, the change in momentum is given by

$$\boldsymbol{\Delta p} = \boldsymbol{m\Delta v} = \boldsymbol{m}(\boldsymbol{v}_{\mathbf{f}} - \boldsymbol{v}_{\mathbf{i}}).$$

In this example, the velocity just after impact and the change in time are given; thus, once $\boldsymbol{\Delta p}$ is calculated, $\boldsymbol{F}_{\mathbf{net}} = \dfrac{\boldsymbol{\Delta p}}{\boldsymbol{\Delta t}}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned} \boldsymbol{\Delta p} &= \boldsymbol{m(v_{\mathbf{f}} - v_{\mathbf{i}})} \\ &= \mathbf{(0.057\ kg)(58\ m/s - 0\ m/s)} \\ &= \mathbf{3.306\ kg\cdot m/s \approx 3.3\ kg\cdot m/s} \end{aligned}$$

Now the magnitude of the net external force can determined by using $\boldsymbol{F}_{\mathbf{net}} = \dfrac{\boldsymbol{\Delta p}}{\boldsymbol{\Delta t}}$:

$$\begin{aligned} \boldsymbol{F}_{\mathbf{net}} &= \frac{\boldsymbol{\Delta p}}{\boldsymbol{\Delta t}} = \frac{\mathbf{3.306\ kg\cdot m/s}}{\mathbf{5.0 \times 10^{-3}\ s}} \\ &= \mathbf{661\ N \approx 660\ N,} \end{aligned}$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $\boldsymbol{F}_{\mathbf{net}} = \boldsymbol{ma}$, but one additional step would be required compared with the strategy used in this example.

Section Summary

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\vec{\mathbf{p}}$ is defined to be

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}},$$

  where m is the mass of the system and $\vec{\mathbf{v}}$ is its velocity.
- The SI unit for momentum is **kg · m/s**.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

$$\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t},$$

  $\vec{\mathbf{F}}_{\mathbf{net}}$ is the net external force, $\Delta\vec{\mathbf{p}}$ is the change in momentum, and Δt is the change time.

Conceptual Questions

1: An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

2: An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

3: Professional Application

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

4: How can a small force impart the same momentum to an object as a large force?

linear momentum
: the product of mass and velocity

second law of motion
: physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

46. 6.11 Impulse

Summary

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The effect of a force on an object depends on how long it acts, as well as how great the force is. For example, in the tennis swing, very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\boldsymbol{\Delta}\vec{\mathbf{p}}$.

By rearranging the equation $\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\boldsymbol{\Delta}\vec{\mathbf{p}}}{\boldsymbol{\Delta t}}$ to be

$$\boldsymbol{\Delta}\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\mathbf{net}}\boldsymbol{\Delta t},$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\vec{\mathbf{F}}_{\mathbf{net}}\boldsymbol{\Delta t}$ is given the name **impulse**. Impulse is the same as the change in momentum.

IMPULSE: CHANGE IN MOMENTUM

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\boldsymbol{\Delta}\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\mathbf{net}}\,\boldsymbol{\Delta t}$$

The quantity $\vec{\mathbf{F}}_{\mathbf{net}}\,\boldsymbol{\Delta t}$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

Example 1: Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of $30°$ from the perpendicular, and bounces off at an angle of $30°$ from perpendicular to the wall.

(a) Determine the direction of the force on the wall due to each ball.

(b) Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the *x*-axis to be normal to the wall and to be positive in the initial direction of motion. Choose the *y*-axis to be along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

Solution for (a)

The first ball bounces directly into the wall and exerts a force on it in the **+*x*** direction. Therefore the wall exerts a force on the ball in the **–*x*** direction. The second ball continues with the same momentum component in the ***y*** direction, but reverses its *x*-component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the **–*x*** direction, so the force of the wall on each ball is along the **–*x*** direction.

Strategy for (b)

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

Solution for (b)

Let ***u*** be the speed of each ball before and after collision with the wall, and ***m*** the mass of each ball. Choose the *x*-axis and *y*-axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

$$\boldsymbol{p_{\mathrm{xi}} = mu;\ p_{\mathrm{yi}} = 0}$$
$$\boldsymbol{p_{\mathrm{xf}} = -mu;\ p_{\mathrm{yf}} = 0}$$

Impulse is the change in momentum vector. Therefore the *x*-component of impulse is equal to **-2*mu*** and the *y*-component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

$$\boldsymbol{p_{\mathrm{xi}} = mu\cos 30^0;\ p_{\mathrm{yi}} = -mu\sin 30^0}$$
$$\boldsymbol{p_{\mathrm{xf}} = -mu\cos 30^0;\ p_{\mathrm{yf}} = -mu\sin 30^0}$$

It should be noted here that while $\boldsymbol{p_x}$ changes sign after the collision, $\boldsymbol{p_y}$ does not. Therefore the *x*-component of impulse is equal to **-2*mu* cos 30°** and the *y*-component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

$$\boldsymbol{\frac{2mu}{2mu\cos 30^0} = \frac{2}{\sqrt{3}} = 1.155.}$$

Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative *x*-direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive *x*-direction.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt. *Forces are usually not constant*. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. Figure 1 shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2. That area is equal to the area inside the rectangle bounded by F_{eff}, t_1, and t_2. Thus the impulses and their effects are the same for both the actual and effective forces.

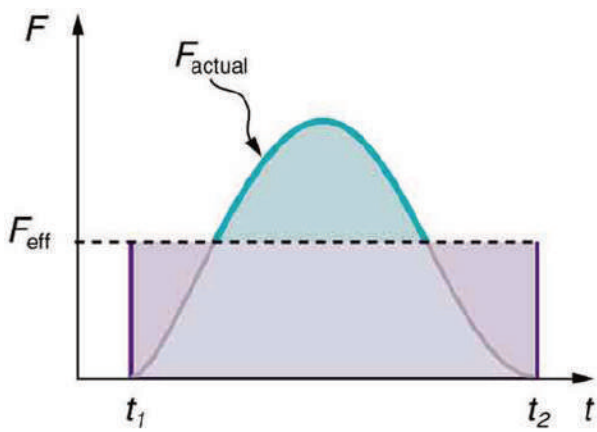


Figure 1. *A graph of force versus time with time along the **x**-axis and force along the **y**-axis for an actual force and an equivalent effective force. The areas under the two curves are equal.*

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—HAND MOVEMENT AND IMPULSE

Try catching a ball while “giving” with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

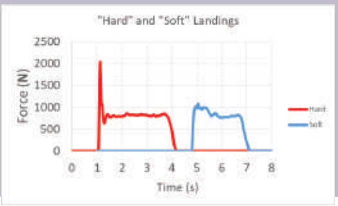
Everyday Examples: Landing after a Jump

You naturally tend to bend your knees when landing after a jump, rather than keep your knees locked and your legs rigid. The reason is that rigid legs bring you to an abrupt stop, but bending your knees allows you to spread the landing out over a longer time which reduces the average and peak force applied to your legs.

Stiff and bent-leg landings that produced the force vs. time data shown below.

The force vs. time graphs show the normal force applied to the person landing on one foot after stepping off from a 0.1 **m** height as seen in the previous GIF. The graph on the left was the more rigid leg landing (it didn't feel good) and the graph on the right was a bent-knee landing.

Notice that the stiff-legged “hard” landing nearly doubled the peak force applied to the body.



Force vs. time data for a stiff-legged landing (red) and crouching landing (blue).

Reinforcement Activity

Throw an egg or a water balloon up into the air and catch it. Did you move your hands with the object as you caught it on the return, or did you hold your hands remain still as it arrived?

Adults have learned from experience how to reduce the force on objects as we control their motion. In my experience, toddlers do not apply this technique.

Section Summary

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:
$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\mathbf{net}}\Delta t.$$
- Forces are usually not constant over a period of time.

Conceptual Questions

1: Professional Application

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

2: While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

3: Professional Application

Tennis racquets have "sweet spots." If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

Problems & Exercises

1: A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

2: Professional Application

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

3: Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of 1.76×10^4 N.

4: Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

5: A ball with an initial velocity of 10 m/s moves at an angle 60° above the *+x*-direction. The ball hits a vertical wall and bounces off so that it is moving 60° above the *-x*-direction with the same speed. What is the impulse delivered by the wall?

6: When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

7: A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55° above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

Glossary

change in momentum
: the difference between the final and initial momentum; the mass times the change in velocity

impulse
: the average net external force times the time it acts; equal to the change in momentum

Solutions

Problems & Exercises

1: (a) $\mathbf{2.40 \times 10^3\ N}$ toward the leg (b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton’s third law) because the change in momentum and the time interval are the same.

4:

$$\mathbf{p} = \boldsymbol{m}\mathbf{v} \Rightarrow \boldsymbol{p}^2 = \boldsymbol{m}^2\boldsymbol{v}^2 \Rightarrow \frac{\boldsymbol{p}^2}{\boldsymbol{m}} = \boldsymbol{m}\boldsymbol{v}^2$$

$$\Rightarrow \frac{\boldsymbol{p}^2}{\mathbf{2}\boldsymbol{m}} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{m}\boldsymbol{v}^2 = \mathbf{KE}$$

$$\mathbf{KE} = \frac{\boldsymbol{p}^2}{\mathbf{2}\boldsymbol{m}}$$

6: $\mathbf{60.0\ g}$

47. 6.12 Conservation of Momentum

Summary

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in previous chapters where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in Figure 1. Both cars are coasting in the same direction when the lead car (labeled m_2) is bumped by the trailing car (labeled m_1). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

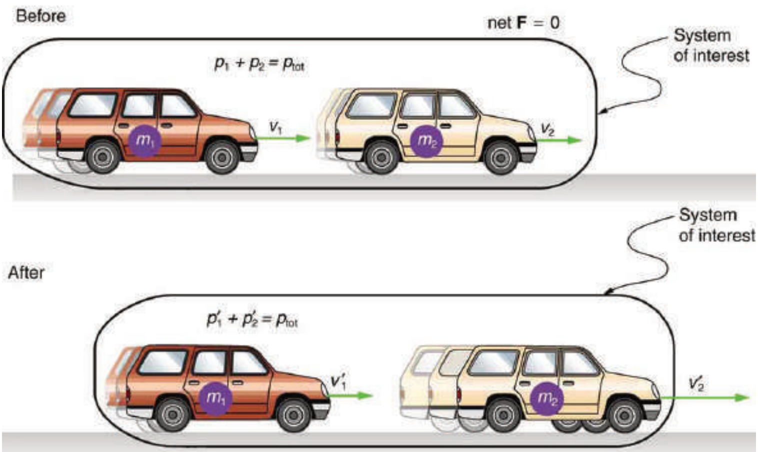


Figure 1. *A car of mass* $\mathbf{m_1}$ *moving with a velocity of* $\mathbf{v_1}$ *bumps into another car of mass* $\mathbf{m_2}$*and velocity* $\mathbf{v_2}$ *that it is following. As a result, the first car slows down to a velocity of* $\mathbf{v'_1}$*and the second speeds up to a velocity of* $\mathbf{v'_2}$*. The momentum of each car is changed, but the total momentum* $\mathbf{p_{tot}}$*of the two cars is the same before and after the collision (if you assume friction is negligible).*

Using the definition of impulse, the change in momentum of car 1 is given by

$$\mathbf{\Delta\vec{p}_1} = \mathbf{\vec{F}_1\Delta} \boldsymbol{t},$$

where $\vec{\mathbf{F}}_{\mathbf{1}}$ is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

$$\boldsymbol{\Delta}\vec{\mathbf{p}}_{\mathbf{2}} = \vec{\mathbf{F}}_{\mathbf{2}}\boldsymbol{\Delta t},$$

where $\vec{\mathbf{F}}_{\mathbf{2}}$ is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law that $\vec{\mathbf{F}}_{\mathbf{1}} = -\vec{\mathbf{F}}_{\mathbf{2}}$, and so

$$\boldsymbol{\Delta}\vec{\mathbf{p}}_{\mathbf{2}} = -\vec{\mathbf{F}}_{\mathbf{1}}\boldsymbol{\Delta t} = -\boldsymbol{\Delta}\vec{\mathbf{p}}_{\mathbf{1}}.$$

Thus, the changes in momentum are equal and opposite, and

$$\boldsymbol{\Delta}\vec{\mathbf{p}}_{\mathbf{1}} + \boldsymbol{\Delta}\vec{\mathbf{p}}_{\mathbf{2}} = \mathbf{0}.$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$\vec{\mathbf{p}}_{\mathbf{1}} + \vec{\mathbf{p}}_{\mathbf{2}} = \mathbf{constant},$$

$$\vec{\mathbf{p}}_{\mathbf{1}} + \vec{\mathbf{p}}_{\mathbf{2}} = \vec{\mathbf{p}}'_{\mathbf{1}} + \vec{\mathbf{p}}'_{\mathbf{2}},$$

where $\vec{\mathbf{p}}'_{\mathbf{1}}$ and $\vec{\mathbf{p}}'_{\mathbf{2}}$ are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

$$\vec{\mathbf{p}}_{\mathbf{tot}} = \mathbf{constant},$$

or

$$\vec{\mathbf{p}}_{\mathbf{tot}} = \vec{\mathbf{p}}'_{\mathbf{tot}},$$

where $\vec{\mathbf{p}}_{\mathbf{tot}}$ is the total momentum (the sum of the momenta of the individual objects in the system) and $\vec{\mathbf{p}}'_{\mathbf{tot}}$ is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero $\mathbf{(\vec{F}_{net} = 0)}$.

CONSERVATION OF MOMENTUM PRINCIPLE

$$\vec{\mathbf{p}}_{\mathbf{tot}} = \mathbf{constant}$$
$$\vec{\mathbf{p}}_{\mathbf{tot}} = \vec{\mathbf{p}}'_{\mathbf{tot}} \ \mathbf{(isolated\ system)}$$

ISOLATED SYSTEM

An isolated system is defined to be one for which the net external force is zero $\mathbf{(\vec{F}_{net} = 0)}$.

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum, $\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\Delta \vec{\mathbf{p}}_{\mathbf{tot}}}{\Delta t}$. For an isolated system, $\vec{\mathbf{F}}_{\mathbf{net}} = \mathbf{0}$; thus, $\Delta \vec{\mathbf{p}}_{\mathbf{tot}} = \mathbf{0}$, and $\vec{\mathbf{p}}_{\mathbf{tot}}$ is constant.

We have noted that the three length dimensions in nature—*x*, *y*, and *z*—are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See Figure 2.) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

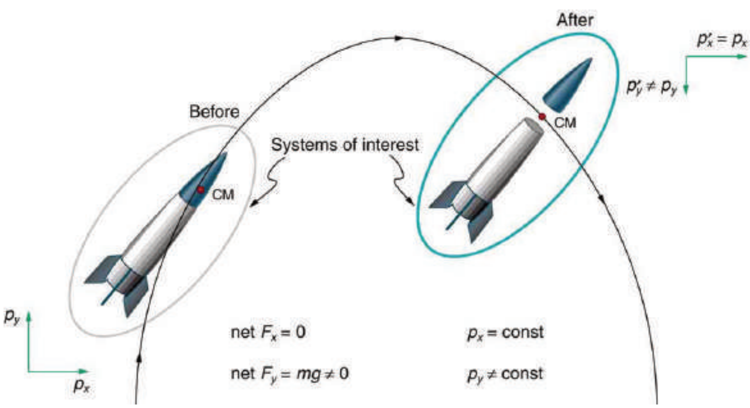


Figure 2. *The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force* **$F_{x\text{-net}}$** *is still zero. The vertical component of the momentum is not conserved, because the net vertical force* **$F_{y\text{-net}}$** *is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.*

Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—DROP OF TENNIS BALL AND A BASEBALL

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

MAKING CONNECTIONS:

TAKE-HOME INVESTIGATION—TWO TENNIS BALLS IN A BALLISTIC TRAJECTORY

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

Section Summary

- The conservation of momentum principle is written
$$\vec{\mathbf{p}}_{\mathbf{tot}} = \mathbf{constant}$$
or
$$\vec{\mathbf{p}}_{\mathbf{tot}} = \vec{\mathbf{p}}'_{\mathbf{tot}}\ \mathbf{(isolated\ system)},$$
$\vec{\mathbf{p}}_{\mathbf{tot}}$ is the initial total momentum and $\vec{\mathbf{p}}'_{\mathbf{tot}}$ is the total momentum some time later.
- An isolated system is defined to be one for which the net external force is zero $\mathbf{(\vec{F}_{net} = 0)}$.
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal

direction because horizontal forces are zero.

- Conservation of momentum applies only when the net external force is zero.

Conceptual Questions

1: Professional Application

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

2: Under what circumstances is momentum conserved?

3: Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

4: Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

conservation of momentum principle
: when the net external force is zero, the total momentum of the system is conserved or constant

isolated system
: a system in which the net external force is zero

48. 6.13 The Impulse-Momentum Theorem

When thinking about how to reduce forces during collisions we intuitively know that increasing the duration of the collision is helpful. The combination of the force and collision duration is known as the impulse. The impulse can be calculated by multiplying the average net force ($\boldsymbol{F_{ave}}$) by the duration of the collision ($\boldsymbol{\Delta t}$). (*Alternatively, the impulse is equal to the area underneath the force vs. time curve for the collision such as those in the previous example*). The impulse-momentum theorem states that *the impulse applied to an object will be equal to the change in its momentum*.

$$\boldsymbol{\Delta\vec{\mathbf{t}}\mathbf{F} = m(v_f) - m(v_i)}$$

Notice that we have calculated the change in momentum as the initial momentum ($\boldsymbol{m_i v_i}$) subtracted from the final momentum ($\boldsymbol{m_f v_f}$). If the mass of the object doesn't change during the collision, then the initial and final mass are the same. In this case we call it *m* and factor it out on the right side of the equation:

\boldsymbol{\Delta{\vec{\textbf{t}}}\textbf{F}}=\boldsymbol{ m({v_f} - {v_i})}

Now we see that the impulse-momentum theorem shows us how a small net force applied over a long time can be used to produce the same velocity change as a large net force applied over a short time.

Everyday Example: Landing

A person jumping from a height of 5 **m**, or about 20 **ft**, hits the ground with a speed of nearly 10 **m/s,** or about 22 **mph** (we'll learn how to figure that out later). Let's calculate the average force applied to a 100 **kg** person during such a landing if the collision with the ground lasts 1/10 of a second. We start with the impulse-momentum theorem.

$$\boldsymbol{\Delta{\vec{\textbf{t}}}\textbf{F}}=\boldsymbol{m({v_f} - {v_i})}$$

We want force, so let's divide over the collision duration:

$$\boldsymbol{{F}}=\boldsymbol{(m({v_f} - {v_i}))/\Delta{\vec{\textbf{t}}}}$$

Remembering that direction is important when working with forces and velocities, we need to define some directions. Let's make downward negative so the initial velocity is -10 **m/s**. The final velocity is 0 **m/s** because the person comes to rest on the ground during landing. The stated collision duration was 0.1 **s**, so we are ready to calculate the average net force:

$$\boldsymbol{{F}}=\boldsymbol{(100kg({0 m/s} - {-10 m/s}))/{0.1s} = 10, 000N}$$

We see that the net force is positive, meaning that it points upward because we chose downward as the negative direction. This makes sense because the ground pushes up on the person to provide the impulse to stop the persons downward motion.

Finally, we need to remember that we have calculated the average net force, which how much the forces are out of balance. This person has a weight of about 1,000 **N** (100 **kg** x 9.8 **m/s/s** = 1000 **N**). Weight acts downward, so to get the required 10,000 **N** of net force upward there must actually be a 11,000 **N** applied upward on their feet, with 1000 **N** of that being cancelled out by their weight.

Spreading the force out over a longer time would reduce the average force (and peak force) applied to the person. For example, the the collision were made to last 5/10 of a second instead of 1/10 of a second, the net force would be five times smaller:

\boldsymbol{{F}}=\boldsymbol{(100kg({0 m/s} - {-10 m/s}))/{0.5s} = 5, 000N}

And adding the 1000 **N** body weight to get the total force on the feet we get 6,000 **N**.

The people in this video are well practiced at techniques for reducing forces by extending impact time.

A YouTube element has been excluded from this version of the text. You can view it online here: https://pressbooks.bccampus.ca/humanbiomechanics/?p=379

Reinforcement Exercises: Fall time

Apply the impulse-momentum theorem to calculate the fall time for the person who fell from the **5 m** height in the previous example. [Hint: If we ignore air resistance, then the only force on them during the fall is their weight, so that is the net force. You already know the initial velocity at the start of the fall is zero, and the final velocity was given to be 10 m/s.]

It’s important to recognize that we have been applying the impulse-momentum theorem to only one object involved in the collision. We know from the Principle of Momentum Conservation that the total combined momentum change of all objects involved in a collision is zero, so applying the impulse-momentum theorem to all of the objects would just tell us that the total net force on ALL objects during the collision is zero.

49. 6.15 Safety Technology as Related to Impulse

We have developed a qualitative understanding that increasing the time over which an object changes velocity will reduce the size of the force applied to the object. We can extend what we learned to the design of injury prevention technology in a quantitative way using the impulse-momentum relationship:

Force*time = mass*(v_f – v_i)

The left hand side of the previous equation is known as the impulse. We can see during a typical collision, the impulse required is determined by the mass (m) and change in velocity (v_f-v_i). Stopping a larger mass will require a larger impulse, as will causing a greater change to the velocity of any mass. We also see that for a specific mass and change in velocity in a particular situation, the overall impulse will be pre-determined so the average force must go down when the impact time goes up.

Everyday Example: Airbags

Check out this video of crash-testing with and without airbags.

During a car crash the driver's head starts out having the same velocity as the car to having zero velocity.

That can happen on impact with the steering wheel or dashboard, or preferably an airbag. The change in velocity that occurs is set by the initial driving speed and the mass of the head doesn't change during the collision, so according to the previous equation, the impulse experienced by the head is known. When a person's head is stopped by the steering wheel the impulse occurs over a short time and the force is large. When the head is stopped by the airbag the impulse occurs over a longer time and the force is reduced.

Reinforcement Exercises

A crash test dummy with a 5 **kg** head mass is sitting in a car moving at 60 **mph** (27 **m/s**), which is stopped by a slammed into a concrete wall during a crash test.

What is the impulse on the dummy's head?

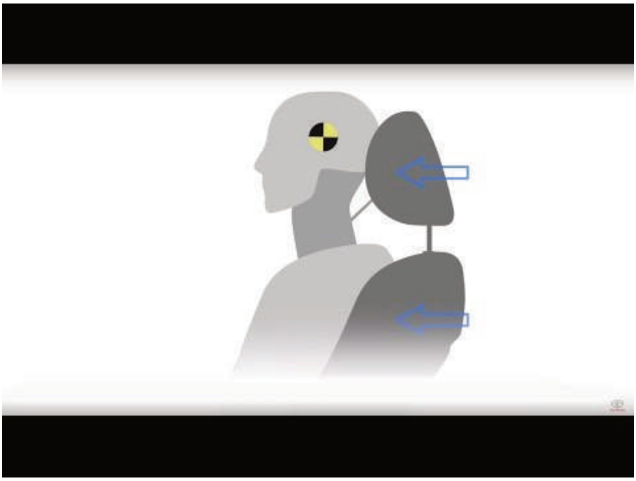
High speed camera footage reveals that a crash-test dummy head comes to rest over roughly 0.05 **s** when impacting the steering wheel.What is the average force applied to the head by the steering wheel?

If the head instead hits an airbag and comes to rest over 0.2 **s**, what is the average force applied?

How many times larger is the force applied when there is no airbag?

Check out this simulation of a lunar lander, which allows you to experience hard and soft landings and how changes in momentum are related to forces applied over time intervals.

Toyota has applied the concepts in this chapter to the development of Active Headrests to reduce whiplash injury.



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Understanding that they cannot control the change in momentum and associated impulse felt by a person during a collision, car manufactures have created seats that reduce the force they can apply to the body without compressing. The time required to compress the seat increases the time it takes the body to change velocity, which reduces the force on the body. More importantly, the compressing seat reduces the distance between the body and the head-rest during a rear-end collision and provides more time over which the headrest can move the head to keep up with the body. Toyota has taken the concept a step further and installed a mechanical linkage between the seat back and the headrest so that the compression causes the headrest to move forward, further increasing its effectiveness.

PART VII

CHAPTER 7: WORK, POWER, AND ENERGY

Chapter Objectives

After this chapter, you will be able to:

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.
- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $PE_g = mgh$.
- Explain the law of the conservation of energy.
- Calculate power by calculating changes in energy over time.
- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

50. 7.0 Introduction

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to running a marathon or to lift a 250 lbs weight. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is conserved.

Conservation of energy (the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

51. 7.1 Work: The Scientific Definition

Summary

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force* *acts*. For one-way motion in one dimension, this is expressed in equation form as

$$\boldsymbol{W} = |\vec{\mathbf{F}}|(\mathbf{cos}\boldsymbol{\theta})|\mathbf{d}|,$$

where ***W*** is work, ***d*** is the displacement of the system, and ***θ*** is the angle between the force vector $\vec{\mathbf{F}}$ and the displacement vector ***d***, as in Figure 1. We can also write this as

$$\boldsymbol{W} = \boldsymbol{Fd}\mathbf{cos}\boldsymbol{\theta}.$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

WHAT IS WORK?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$\boldsymbol{W} = \boldsymbol{Fd}\,\mathbf{cos}\boldsymbol{\theta},$$

where ***W*** is work, ***F*** is the magnitude of the force on the system, ***d*** is the magnitude of the displacement of the system, and ***θ*** is the angle between the force vector $\vec{\mathbf{F}}$ and the displacement vector ***d***.

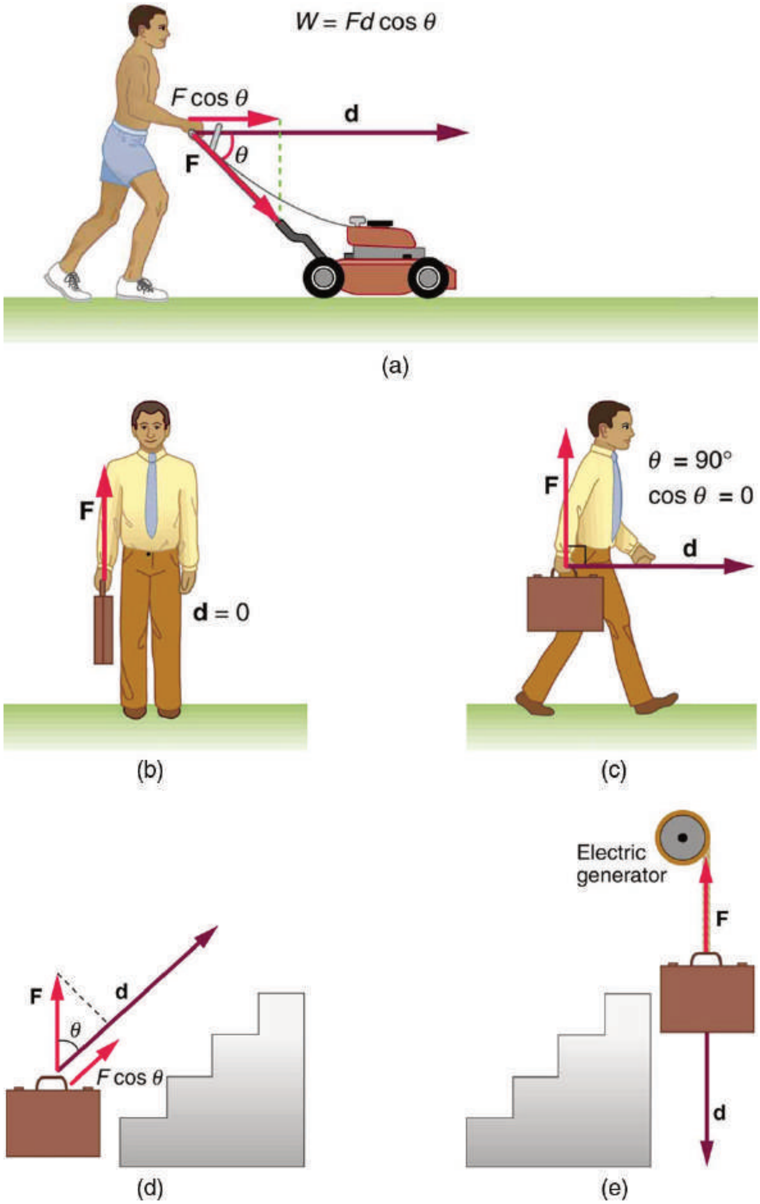

Figure 1. *Examples of work. (a) The work done by the force* ***F*** *on this lawn mower is* ***Fd cosθ****. Note that* ***F cosθ*** *is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred*

*to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force **F** in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because **F** and **d** are in opposite directions.*

To examine what the definition of work means, let us consider the other situations shown in Figure 1. The person holding the briefcase in Figure 1(b) does no work, for example. Here $\boldsymbol{d = 0}$, so $\boldsymbol{W = 0}$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest*. There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in Figure 1(c) does no work on it, because the force is perpendicular to the motion. That is, $\mathbf{\cos 90° = 0}$, and so $\boldsymbol{W = 0}$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in Figure 1(d), work *is* done—energy is transferred to the briefcase. Finally, in Figure 1(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes $\boldsymbol{\theta = 180°}$, and $\mathbf{\cos 180° = -1}$; therefore, $\boldsymbol{W}$ is negative.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and **1 J = 1 N · m = 1 kg m^2/s^2**. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example 1: Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in Figure 1(a) if he exerts a constant force of **75.0 N** at an angle 35° below the horizontal and pushes the mower **25.0 m** on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of **10,000 kJ** (about **2400 kcal**) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1 °C, and is equivalent to **4.184 J**, while one *food calorie* (1 kcal) is equivalent to **4184 J**.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $\boldsymbol{W = Fd \cos \theta}$. The force, angle, and displacement are given, so that only the work ***W*** is unknown.

Solution

The equation for the work is

$$\boldsymbol{W = Fd\,\cos\theta}.$$

Substituting the known values gives

$$\begin{aligned}\boldsymbol{W} &= \mathbf{(75.0\ N)(25.0\ m)\ cos\ (35.0^{0})} \\ &= \mathbf{1536\ J = 1.54 \times 10^{3}\ J}\end{aligned}.$$

Converting the work in joules to kilocalories yields ***W* = (1536 J)(1 kcal/4184 J) = 0.367 kcal**. The ratio of the work done to the daily consumption is

$$\frac{\boldsymbol{W}}{\mathbf{2400\ kcal}} = \mathbf{1.53 \times 10^{-4}}.$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we “work” all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Section Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work ***W*** that a force $\vec{\mathbf{F}}$ does on an object is the

product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,

$$W = Fd\cos\theta.$$

- The SI unit for work and energy is the joule (J), where **$1\ \mathrm{J} = 1\ \mathrm{N \cdot m} = 1\ \mathrm{kg \cdot m^2/s^2}$**.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

Conceptual Questions

1: Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

2: Describe a situation in which a force is exerted for a long time but does no work. Explain.

Problems & Exercises

1: How much work does a supermarket checkout attendant do on a can of soup that is pushed 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

2: A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

3: Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See Figure 2.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed.

a) What is the work done on the crate, b) what is the work done on himself to lift him up and c) what is the total work = a + b? This is the work he does on the crate *and* on his body to get up the ramp.

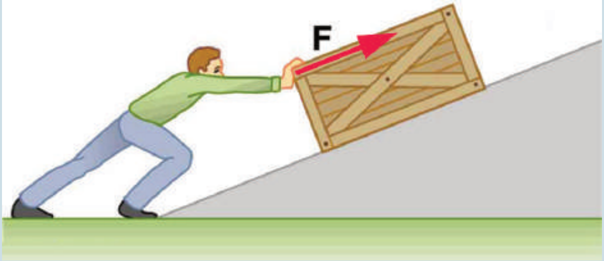

Figure 2. *A man pushes a crate up a ramp.*

4: How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in Figure 3? Assume no friction acts on the wagon.

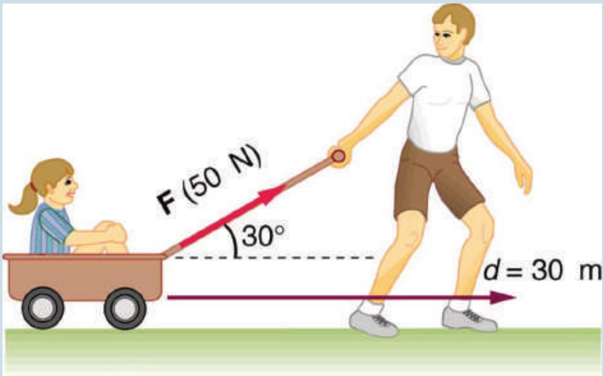

Figure 3. The boy does work on the system of the wagon and the child when he pulls them as shown.

5: A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

6: Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in Figure 4. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

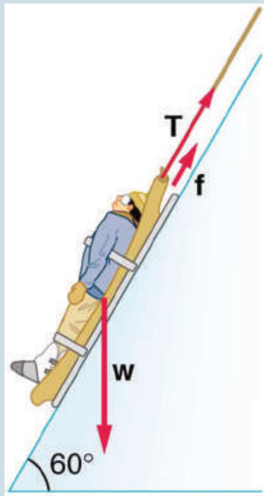

Figure 4. *A rescue sled and victim are lowered down a steep slope.*

Glossary

energy
: the ability to do work

work
: the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

joule

SI unit of work and energy, equal to one newton-meter

Solutions

Problems & Exercises

1: 3.00 J = 7.17 x 10^{-4} kcal

3: **a)** work = force x distance = 2000 J as angle between force and 4 m is 0 degrees b) m g h where h = 4m sin 20.0 = 1140 c) 3140 J in total

5: (a) -700 J (b) 0 J (c) 700 J (d) 38.6 N (e) 0 J

52. 7.2 Kinetic Energy and the Work-Energy Theorem

Summary

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in Chapter 7.1 Figure 1(a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in Chapter 7.1 Figure 1(d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in Chapter 7.1 Figure 1(e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force $\boldsymbol{F_{\text{net}}}$. In equation form, this is $\boldsymbol{W_{\text{net}} = F_{\text{net}} d \cos \theta}$ where $\boldsymbol{\theta}$ is the angle between the force vector and the displacement vector.

Figure 1(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $\boldsymbol{F \cos \theta}$ vs. $\boldsymbol{d}$ graph. In this case, $\boldsymbol{F \cos \theta}$ is constant. You can see that the area under the graph is $\boldsymbol{Fd \cos \theta}$, or the work done. Figure 1(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $\boldsymbol{(F \cos \theta)_{i(\text{ave})}}$. The work done is $\boldsymbol{(F \cos \theta)_{i(\text{ave})} d_{\text{i}}}$ for each strip, and the total work done is the sum of the $\boldsymbol{W_{\text{i}}}$. Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

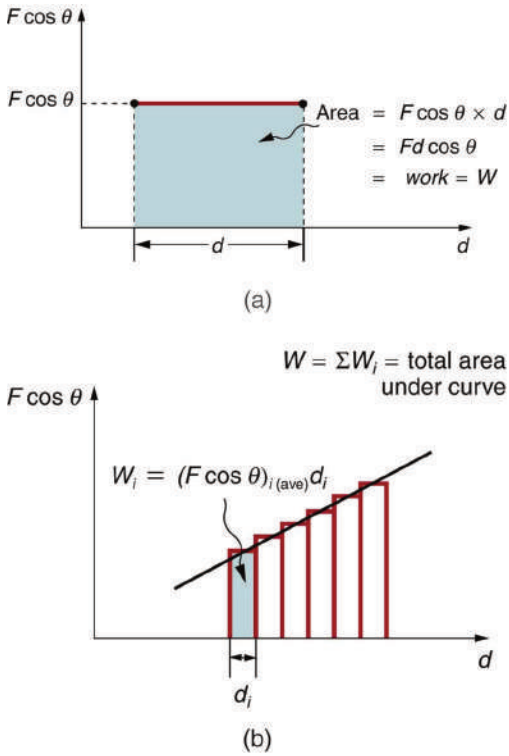

Figure 1. *(a) A graph of **F cos θ** vs.**d**, when **F cos θ** is constant. The area under the curve represents the work done by the force. (b) A graph of **F cos θ** vs. **d** in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.*

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure 2.

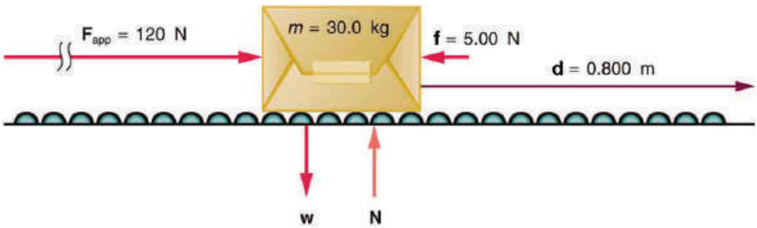

Figure 2. *A package on a roller belt is pushed horizontally through a distance* ***d***.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force Fapp and the horizontal friction force ***f***. Thus, as expected, the net force is parallel to the displacement, so that $\boldsymbol{\theta} = 0°$ and $\cos \boldsymbol{\theta} = 1$, and the net work is given by

$$\boldsymbol{W}_{\mathbf{net}} = \boldsymbol{F}_{\mathbf{net}}\boldsymbol{d}.$$

The effect of the net force $\boldsymbol{F}_{\mathbf{net}}$ is to accelerate the package from $\boldsymbol{v}_0$ to $\boldsymbol{v}$. The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See Example 1.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $\boldsymbol{F}_{\mathbf{net}} = \boldsymbol{ma}$ from Newton's second law gives

$$\boldsymbol{W}_{\mathbf{net}} = \boldsymbol{mad}.$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $\boldsymbol{d} = \boldsymbol{x} - \boldsymbol{x}_0$ and use the equation studied in Chapter 2.5 Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance d if the acceleration has the constant value a; namely, $v^2 = v_0^2 + 2ad$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - v_0^2}{2d}$. When a is substituted into the preceding expression for W_{net}, we obtain

$$W_{\text{net}} = m\left(\frac{v^2 - v_0^2}{2d}\right)d.$$

The d cancels, and we rearrange this to obtain

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

THE WORK-ENERGY THEOREM

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$\boldsymbol{W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2}.$$

The quantity $\boldsymbol{\frac{1}{2}mv^2}$ in the work-energy theorem is defined to be the translational kinetic energy (KE) of a mass $\boldsymbol{m}$ moving at a speed $\boldsymbol{v}$. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$\mathbf{KE} = \boldsymbol{\frac{1}{2}mv^2},$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in Figure 2, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example 1: Calculating the Kinetic

Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in Figure 2 is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass ***m*** and speed ***v*** are given, the kinetic energy can be calculated from its definition as given in the equation $\mathbf{KE}=\frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

$$\mathbf{KE}=\frac{1}{2}mv^2,$$

Entering known values gives

$$\mathbf{KE=0.5(30.0\ kg)(0.500\ m/s)^2},$$

which yields

$$\mathbf{KE=3.75\ k\cdot m^2/s^2=3.75\ J}.$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example 2: Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in Figure 2 with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See Figure 2.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or F_{net} **= 120 N – 5.00 N = 115 N**. Thus the net work is

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}d = (115\ \text{N})(0.800\ \text{m}) \\ &= 92.0\text{N}\cdot\text{m} = 92.0\ \text{J}. \end{aligned}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$\begin{aligned} W_{\text{app}} &= F_{\text{app}}d\cos(0^\circ) = F_{\text{app}}d \\ &= (120\ \text{N})(0.800\ \text{m}) \\ &= 96.0\ \text{J} \end{aligned}$$

The friction force and displacement are in opposite directions, so that $\theta = 180°$, and the work done by friction is

$$\begin{aligned} \boldsymbol{W}_{\mathbf{fr}} &= \boldsymbol{F}_{\mathbf{fr}}\,\boldsymbol{d}\cos 180^{\circ} = -\boldsymbol{F}_{\mathbf{fr}}\,\boldsymbol{d} \\ &= (-5.00\ \mathrm{N})(0.800\ \mathrm{m}) \\ &= -4.00\ \mathrm{J}. \end{aligned}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$\begin{aligned} \boldsymbol{W}_{\mathbf{gr}} &= 0, \\ \boldsymbol{W}_{\mathbf{N}} &= 0, \\ \boldsymbol{W}_{\mathbf{app}} &= 96.0\ \mathrm{J}, \\ \boldsymbol{W}_{\mathbf{fr}} &= -4.00\ \mathrm{J}. \end{aligned}$$

The total work done as the sum of the work done by each force is then seen to be

$$\boldsymbol{W}_{\mathbf{total}} = \boldsymbol{W}_{\mathbf{gr}} + \boldsymbol{W}_{\mathbf{N}} + \boldsymbol{W}_{\mathbf{app}} + \boldsymbol{W}_{\mathbf{fr}} = 92.0\ \mathrm{J}.$$

Discussion for (b)

The calculated total work $\boldsymbol{W}_{\mathbf{total}}$ as the sum of the work by each force agrees, as expected, with the work $\boldsymbol{W}_{\mathbf{net}}$ done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example 3: Determining Speed

from Work and Energy

Find the speed of the package in Figure 2 at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}mv_0^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0^2.$$

Thus,

$$\frac{1}{2}mv^2 = 92.0\text{ J} + 3.75\text{ J} = 95.75\text{ J}.$$

Solving for the final speed as requested and entering known values gives

$$\begin{aligned} v &= \sqrt{\frac{2(95.75\text{ J})}{m}} = \sqrt{\frac{191.5\text{ kg}\cdot\text{m}^2/\text{s}^2}{30.0\text{ kg}}} \\ &= 2.53\text{ m/s}. \end{aligned}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Example 4: Work and Energy Can Reveal Distance, Too

How far does the package in Figure 2 coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta = 180°$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{fr} = -95.75\ \mathrm{J}$. Furthermore, $W_{fr} = fd' \cos\theta = -fd'$, where d' is the distance it takes to stop. Thus,

$$d' = -\frac{W_{\mathrm{fr}}}{f} = -\frac{-95.75\ \mathrm{J}}{5.00\ \mathrm{N}},$$

and so

$$d' = 19.2\ \mathrm{m}.$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

Section Summary

- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\mathbf{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy,

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Conceptual Questions

1: The person in Figure 3 does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?

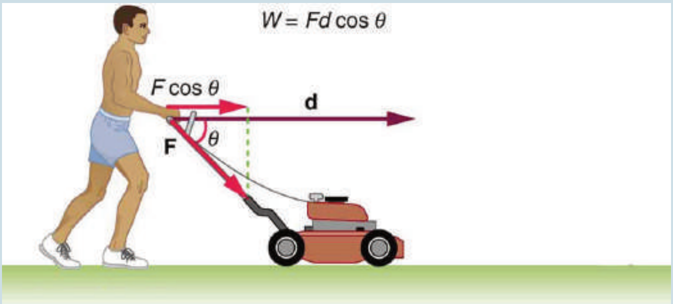

Figure 3.

2: Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

3: When solving for speed in Example 3, we kept only the positive root. Why?

Problems & Exercises

1: (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

2: Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

3: Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

Glossary

net work
: work done by the net force, or vector sum of all the forces, acting on an object

work-energy theorem
: the result, based on Newton’s laws, that the net work done on an object is equal to its change in kinetic energy

kinetic energy
: the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

Solutions

Problems & Exercises

3: net force = 72 N so the person force = 72 + 30 = 102 N net force = change in kinetic energy = 1/2 m (v final)^2 – 1/2 m (v initial)^2

53. 7.3 Gravitational Potential Energy

Summary

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $\mathbf{PE_g} = mgh$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in Figure 1. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then

W = Fd = mgh. We define this to be the **gravitational potential energy** (**PE_g**)put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the **PE_g** gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to ***mgh*** on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of **PE_g** to **KE** without explicitly considering the intermediate step of work. (See Example 2.) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.

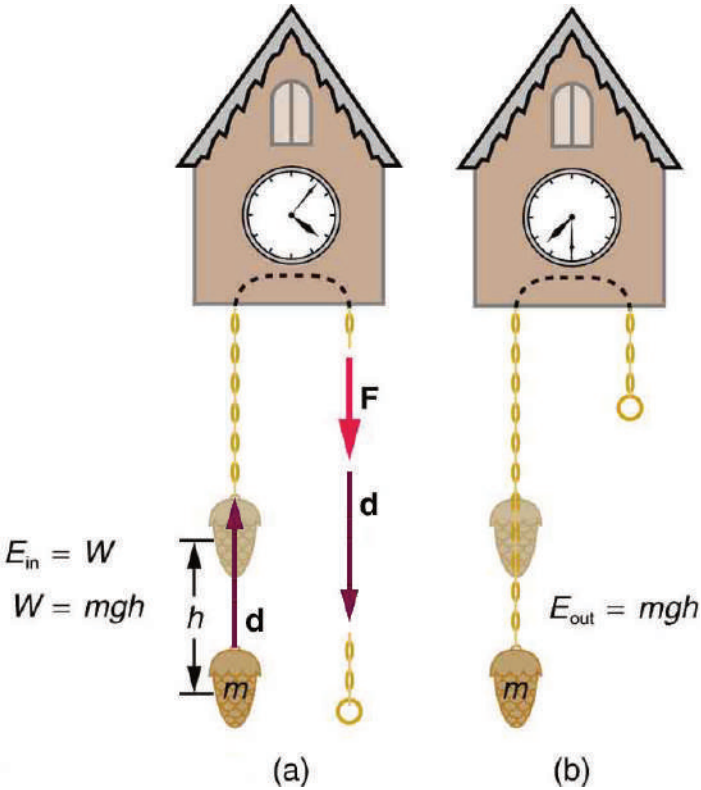

Figure 1. *(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.*

More precisely, we define the *change* in gravitational potential energy $\Delta \mathbf{PE_g}$ to be

$$\Delta \mathbf{PE_g} = \boldsymbol{mgh},$$

where, for simplicity, we denote the change in height by ***h*** rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$\begin{aligned} mgh &= (0.500\ \text{kg})(9.80\ \text{m/s})(21.00\ \text{m}) \\ &= 4.90\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 4.90\ \text{J.} \end{aligned}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta PE_g = mgh$ applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See Figure 2.) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.

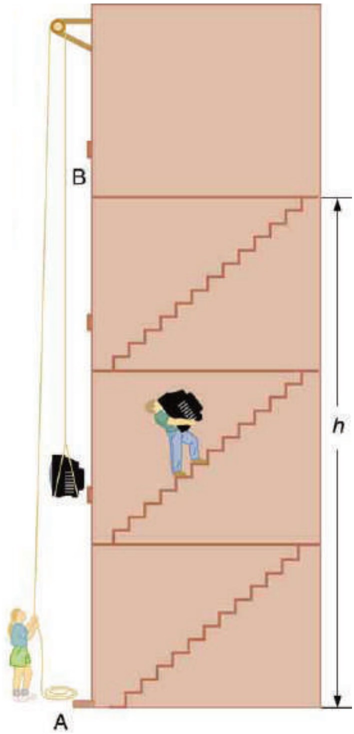

Figure 2. *The change in gravitational potential energy* ***(ΔPE_g)*** *between points A and B is independent of the path.* ***$\Delta PE_g = mgh$*** *for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.*

Example 1: The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial $\mathbf{PE_g}$ is transformed into **KE** as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

$$\boldsymbol{W = Fd\cos\theta = -Fd,}$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions ($\mathbf{\cos\theta = \cos 180° = 1}$). The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height $\boldsymbol{h}$:

$$\mathbf{KE = -\Delta PE_g = -\boldsymbol{mgh},}$$

The distance ***d*** that the person's knees bend is much smaller than the height ***h*** of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work ***W*** done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$\boldsymbol{W = -\mathbf{KE} = mgh.}$$

Combining this equation with the expression for ***W*** gives

$$\boldsymbol{-Fd = mgh.}$$

Recalling that ***h*** is negative because the person fell *down*, the force on the knee joints is given by

$$\boldsymbol{F = -\frac{mgh}{d} = -\frac{(60.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(-3.00\ \mathrm{m})}{5.00 \times 10^{-3}\ \mathrm{m}} = 3.53 \times 10^{5}\ \mathrm{N}.}$$

Discussion

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See Figure 3.)

Figure 3. *The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)*

Example 2: Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in Figure 4 if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

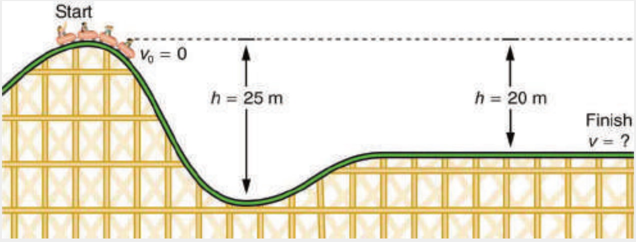

Figure 4. *The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all* $\boldsymbol{\Delta PE_g}$ *is converted to* ***KE****.*

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance ***h*** equals the *gain* in kinetic energy. This can be written in equation form as **-ΔPEg = ΔKE**. Using the equations for $\mathbf{PE_g}$ and **KE**, we can solve for the final speed ***v***, which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\boldsymbol{\Delta \mathrm{KE} = \frac{1}{2}mv^2}$. The equation for change in potential energy states that $\boldsymbol{\Delta \mathrm{PE_g} = mgh}$. Since ***h*** is negative in this case, we will rewrite this as $\boldsymbol{\Delta \mathrm{PE_g} = -mg|h|}$ to show the minus sign clearly. Thus,

$$\boldsymbol{-\Delta \mathrm{PE_g} = \Delta \mathrm{KE}}$$

becomes

$$\boldsymbol{mg|h| = \frac{1}{2}mv^2}.$$

Solving for ***v***, we find that mass cancels and that

$$\boldsymbol{v = \sqrt{2g|h|}}.$$

Substituting known values,

$$\begin{aligned} \boldsymbol{v} &= \boldsymbol{\sqrt{2(9.80\ \mathrm{m/s})(20.0\ \mathrm{m})}} \\ &= \boldsymbol{19.8\ \mathrm{m/s}.} \end{aligned}$$

Solution for (b)

Again $\mathbf{-\Delta PE_g = \Delta KE}$. In this case there is initial kinetic energy, so $\Delta\mathrm{KE} = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2$. Thus,

$$mg|h| = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2.$$

Rearranging gives

$$\frac{1}{2}mv^2 = mg|h| + \frac{1}{2}m{v_0}^2.$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g|h| + {v_0}^2}.$$

This equation is very similar to the kinematics equation $v = \sqrt{{v_0}^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$\begin{aligned} v &= \sqrt{2(9.80\ \mathrm{m/s^2})(20.0\ \mathrm{m}) + (5.00\ \mathrm{m/s})^2} \\ &= 20.4\ \mathrm{m/s}. \end{aligned}$$

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in Chapter 2.7 Falling Objects that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—CONVERTING POTENTIAL TO

KINETIC ENERGY

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see Figure 5). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.

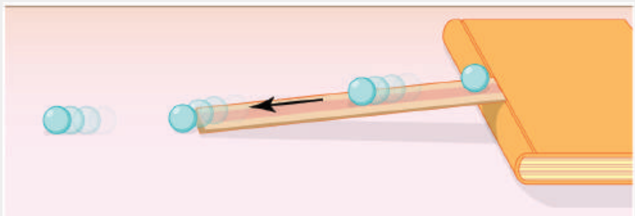

Figure 5. *A marble rolls down a ruler, and its speed on the level surface is measured.*

Section Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, $\mathbf{\Delta PE_g}$, is $\mathbf{\Delta PEg} = \boldsymbol{mgh}$, with $\boldsymbol{h}$ being the increase in height and $\boldsymbol{g}$ the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, $\mathbf{\Delta PE_g}$, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\mathbf{\Delta KE = -\Delta PE_g}$.

Conceptual Questions

1: In Example 2, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that it had the same final speed. Explain in terms of conservation of energy.

2: Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

Problems & Exercises

1: In Example 2, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta PE >> KE_i$. Confirm this statement by taking the ratio of **ΔPE to KE_i**. (Note that mass cancels.)

2: In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30° slope neglecting friction. Hint: you have to use trigonometry to find the height of the hill first. (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

Glossary

gravitational potential energy
: the energy an object has due to its position in a gravitational field

Solutions

Problems & Exercises

2: $h = 35.0\text{ m} = (70\text{ m})\sin 30.0^{\circ}$ $mgh = 1/2\, m v^2$ so $v = 26.2\text{ m/s}$

54. 7.6 Conservation of Energy

Summary

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy **(KE + PE)** and energy transferred via work done by nonconservative forces **(W_{nc})**. But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called **other energy (OE)**. Then we can state the conservation of energy in equation form as

$$\mathbf{KE_i + PE_i} + \boldsymbol{W}_{\mathbf{nc}} + \mathbf{OE_i = KE_f + PE_f + OE_f}.$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is **KE**, work done by a conservative force is represented by **PE**, work done by nonconservative forces is $\boldsymbol{W}_{\mathbf{nc}}$, and all other energies are included as **OE**. This equation applies to all previous examples; in those situations **OE** was constant, and so it subtracted out and was not directly considered.

MAKING CONNECTIONS: USEFULNESS OF THE ENERGY CONSERVATION PRINCIPLE

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does **OE** play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of **OE**).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Table 1 gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency** ***Eff*** of an energy conversion process is defined as

$$\textbf{Efficiency}(\boldsymbol{Eff}) = \frac{\textbf{useful energy or work output}}{\textbf{total energy input}} = \frac{\boldsymbol{W}_{\textbf{out}}}{\boldsymbol{E}_{\textbf{in}}}.$$

Table 2 lists some efficiencies of mechanical devices and human activities.

Activity/device	Efficiency (%)[1]
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

Table 2. Efficiency of the Human Body and Mechanical Devices.

Section Summary

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $\mathbf{KE_i + PE_i + \mathit{W}nc + OE_i = KE_f + PE_f + OE_f}$, where **OE** is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to

convert all the energy of a system to work.

- The efficiency ***Eff*** of a machine or human is defined to be $Eff = \frac{W_{\text{out}}}{E_{\text{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

Conceptual Questions

1: Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

2: List the energy conversions that occur when riding a bicycle.

Problems & Exercises

1: Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

Footnotes

1. 1 Representative values

Glossary

law of conservation of energy
: the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

electrical energy
: the energy carried by a flow of charge

chemical energy
: the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy
: the energy carried by electromagnetic waves

nuclear energy
: energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy
: the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

efficiency
: a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

Solutions

Problems & Exercises

2: Equating $\mathbf{\Delta PE_g}$ and $\mathbf{\Delta KE,}$ we obtain

$$\boldsymbol{v = \sqrt{2gh + v_0^2} = \sqrt{2(9.80\ \mathrm{m/s^2})(20.0\ \mathrm{m}) + (15.0\ \mathrm{m/s})^2} = 24.8\ \mathrm{m/s}}$$

55. 7.7 Power

Summary

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent or a dragster roaring away from the starting line.

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (***P***) as the rate at which work is done.

POWER

Power is the rate at which work is done.

$$\boldsymbol{P = \frac{W}{t}}$$

The SI unit for power is the **watt** (**W**), where 1 watt equals 1 joule/second (**1 W = 1 J/s**).

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example 1: Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See Figure 2.)

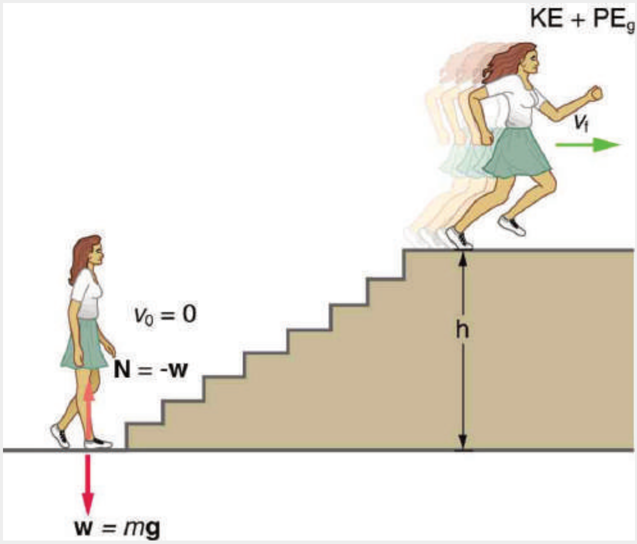

Figure 2. *When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.*

Strategy and Concept

The work going into mechanical energy is ***W*** **= KE + PE**. At the bottom of the stairs, we take both **KE** and **PE$_{gas}$** initially zero; thus,

$$\boldsymbol{W = \mathrm{KE_f} + \mathrm{PE_g} = \frac{1}{2}m{v_f}^2 + mgh},$$

where ***h*** is the vertical height of the stairs. Because all terms are given, we can calculate ***W*** and then divide it by time to get power.

Solution

Substituting the expression for ***W*** into the definition of power given in the previous equation, ***P* = *W*/*t*** yields

$$\boldsymbol{P = \frac{W}{t} = \frac{\frac{1}{2}mv_{\mathrm{f}}{}^{2} + mgh}{t}}.$$

Entering known values yields

$$\begin{aligned} \boldsymbol{P} &= \frac{\mathbf{0.5(60.0\ kg)(2.00\ m/s)^2 + (60.0\ kg)(9.80\ m/s^2)(3.00\ m)}}{\mathbf{3.50\ s}} \\ &= \frac{\mathbf{120\ J + 1764\ J}}{\mathbf{3.50\ s}} \\ &= \mathbf{538\ W.} \end{aligned}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** (**1 hp = 746 W**)! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—MEASURE YOUR POWER RATING

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Section Summary

- Power is the rate at which work is done, or in equation form, for the average power $\boldsymbol{P}$ for work $\boldsymbol{W}$ done over a time $\boldsymbol{t}$, $\boldsymbol{P = W/t}$.
- The SI unit for power is the watt (W), where **1 W = 1 J/s**.

Problems & Exercises

1: A person in good physical condition can put out 100 W of useful power for several hours at a stretth, perhaps by pedalling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

2: (a) What is the average useful power output of a person who does 6.00×10^6 J of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

Glossary

power
: the rate at which work is done

watt
: (W) SI unit of power, with **1 W = 1 J/s**

kilowatt-hour
: (**kW • h**) unit used primarily for electrical energy provided by electric utility companies

Solutions

Problems & Exercises

1: (a) 40 (b) 8 million

2: (a) 208 W (b) 141 s

56. 7.8 Work, Energy, and Power in Human Physiology

Summary

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See Figure 1.) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.

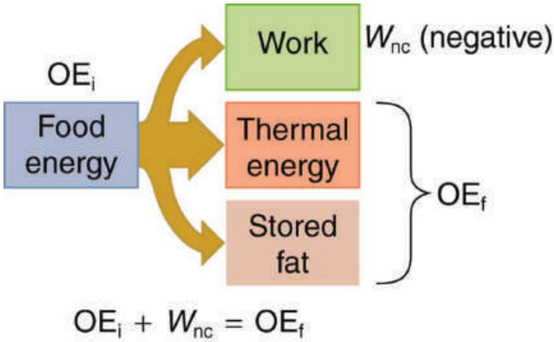

Figure 1. *Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.*

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate** (BMR) and is divided among various systems in the body, as shown in Table 4. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

Table 4. Basal Metabolic Rates (BMR).

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See Figure 2.) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. Table 5 shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy **(KE + PE)** of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as Example 1 illustrates.

Example 1: Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

Table 5 states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

$$\mathbf{Time} = \frac{\mathbf{energy}}{\left(\frac{\mathbf{energy}}{\mathbf{time}}\right)} = \frac{\mathbf{1000\ kJ}}{\mathbf{400\ W}} = \mathbf{2500\ s = 42\ min.}$$

Discussion

If this person uses more energy than he or she consumes, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

$$\mathbf{Fat\ loss} = \mathbf{(1000\ kJ)\left(\frac{1.0\ g\ fat}{39\ kJ}\right) = 26\ g,}$$

assuming the energy content of fat to be 39 kJ/g.

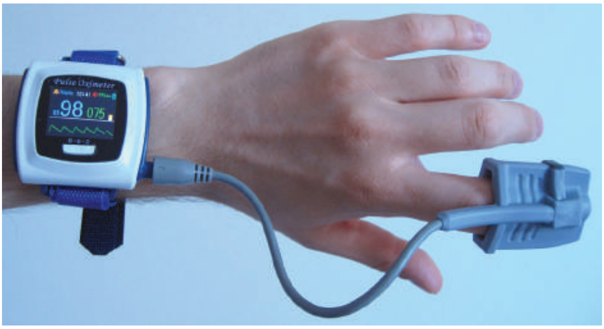

Figure 2. *A pulse oxymeter is an apparatus that measures the amount of oxygen in blood. Oxymeters can be used to determine a person's metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)*

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	125	0.36
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26
Swimming breaststroke	475	1.36
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

Table 5. Energy and Oxygen Consumption Rates[1] (Power).

All bodily functions, from thinking to lifting weights, require energy. (See Figure 3.) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

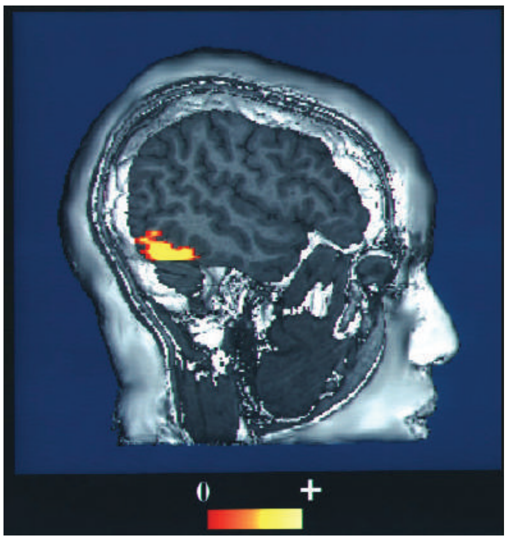

Figure 3. *This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. (credit: NIH via Wikimedia Commons)*

Section Summary

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.

- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Conceptual Questions

1: Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

2: Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

3: Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

4: Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

Problems & Exercises

1: (a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?

2: (a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?

3: Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)

Figure 4. *Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)*

4: (a) What is the efficiency of an out-of-condition professor who does 2.10×10^5 J of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

5: Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from Table 5 for the energy consumption rates of these activities.

6: Using data from Table 5, calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

7: What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See Table 5.)

8: Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?

9: Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.

10: Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.

11: (a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%. (b) What is the average power consumption rate in watts if she does this in 3.00 min?

12: Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus 88*, an aircraft powered by a bicycle-type drive mechanism (see Figure 5). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from Table 2, calculate the food energy in kilojoules he metabolized during the flight.

Figure 5. *The Daedalus 88 in flight. (credit: NASA photo by Beasley)*

13: The swimmer shown in Figure 6 exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.

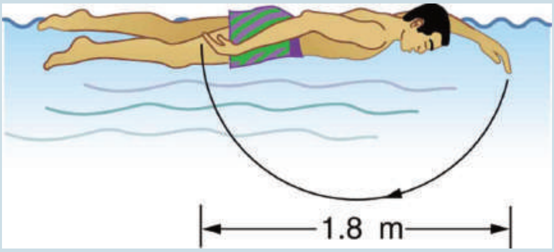

Figure 6.

14: Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

15: The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about 7×10^9 kg. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b)

Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see Figure 7), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

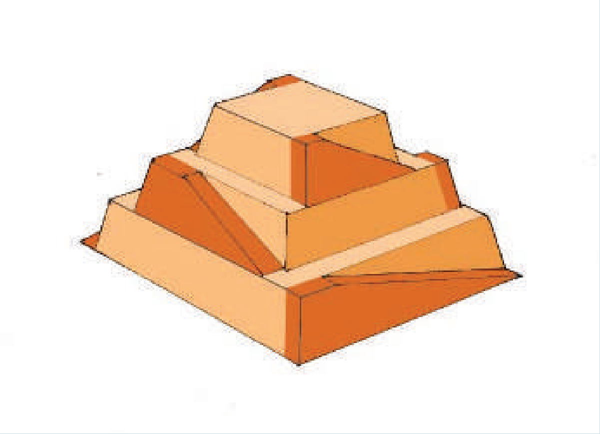

Figure 7. *Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)*

16: (a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

Footnotes

1. 1 for an average 76-kg male

Glossary

metabolic rate
: the rate at which the body uses food energy to sustain life and to do different activities

basal metabolic rate
: the total energy conversion rate of a person at rest

useful work
: work done on an external system

Solutions

Problems & Exercises

1: (a) 9.5 minutes (b) 69 flights of stairs

3: 641 W , 0.860 hp

5: 31 g

7: 14.3%

9: (a) $\mathbf{3.21 \times 10^{4}\ N}$ (b) $\mathbf{2.35 \times 10^{3}\ N}$ (c) Ratio of net force to weight of person is 41.0 in part (a); 3.00 in part (b)

11: (a)108 kJ b) 559 W

13: (a) 144 J b) 288 W

15: (a) $\mathbf{2.50 \times 10^{12}\ J}$ (b) $\boldsymbol{2.52 \%}$ (c) $\mathbf{1.4 \times 10^{4}\ kg}$ (14 metric tons)

PART VIII

CHAPTER 8: ANGULAR KINETICS

Chapter Objectives

After this chapter, you will be able to:

- State the first condition of equilibrium.
- Explain the difference between static equilibrium and dynamic equilibrium.
- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics
- Calculate the mechanical advantage.
- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.
- Calculate angular acceleration of an object.
- Observe the link between linear and angular acceleration.

- Observe the kinematics of rotational motion.
- Understand the relationship between force, mass and acceleration.
- Study the turning effect of force.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.

57. 8.0 Introduction

What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with a constant velocity also has an acceleration of zero. Now, the important part—Newton's second law states that net $\vec{F} = m\vec{a},$ and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

> ### STATICS
>
> Statics is the study of forces in equilibrium, a large group of situations that makes up a special case of Newton's second law. We have already considered a few such situations; in this chapter, we cover the topic more thoroughly, including consideration of such possible effects as the rotation and deformation of an object by the forces acting on it.

How can we guarantee that a body is in equilibrium and what can we learn from systems that are in equilibrium? There are actually two conditions that must be satisfied to achieve equilibrium. These conditions are the topics of the first two sections of this chapter.

58. 8.1 The First Condition for Equilibrium

Summary

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

net force = 0 N

Note that if net ***F*** is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical *x*– and *y*-axes are zero. This is written as

$$\textbf{net } \boldsymbol{F_x} = \mathbf{0} \textbf{ and } \boldsymbol{F_y} = \mathbf{0}$$

Figure 1 and Figure 2 illustrate situations where **net *F* = 0** for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).

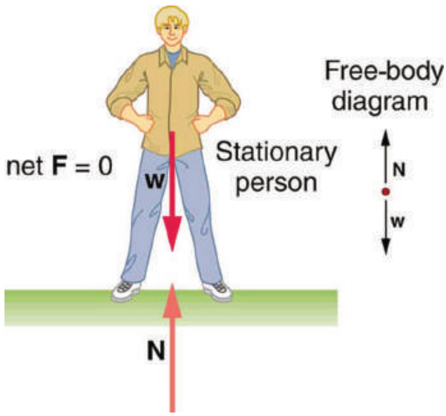

Figure 1. *This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.*

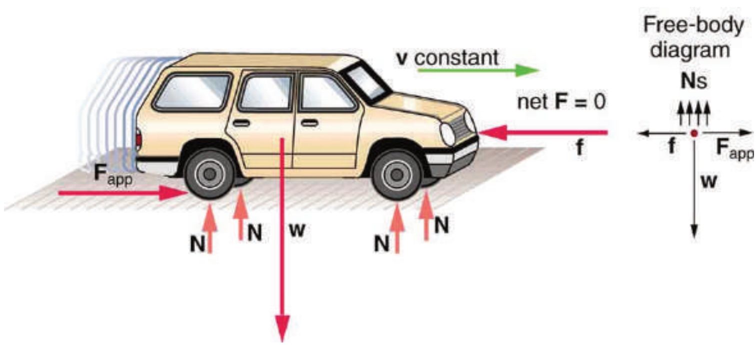

Figure 2. *This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force* $\boldsymbol{F}_{\mathbf{app}}$ *between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.*

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in Figure 3 and Figure 4 where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In Figure 3, the ice hockey stick remains motionless. But in Figure 4, with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

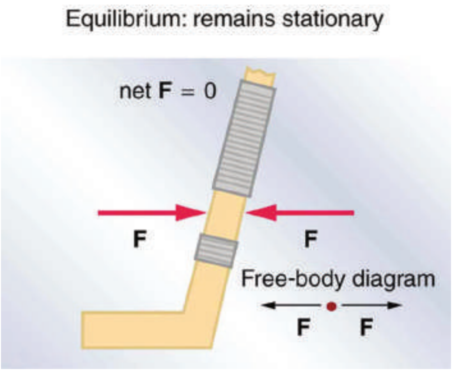

Figure 3. *An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus,* ***net F = 0****. Equilibrium is achieved, which is static equilibrium in this case.*

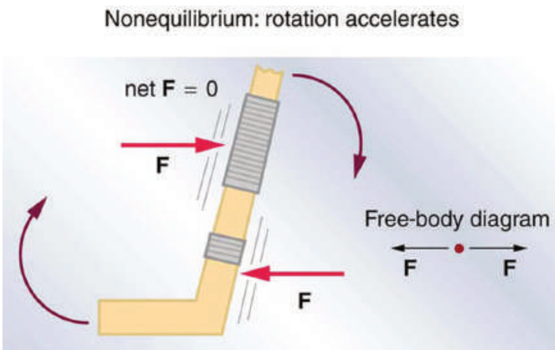

Figure 4. *The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here* ***net F = 0*** *but the system is not at equilibrium. Hence, the* ***net F = 0*** *is a necessary—but not sufficient—condition for achieving equilibrium.*

PHET EXPLORATIONS: TORQUE

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

Figure 5. *Torque*

Section Summary

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that **net *F* = 0**.

Conceptual Questions

1: What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.

2: Under what conditions can a rotating body be in equilibrium? Give an example.

Glossary

static equilibrium
: a state of equilibrium in which the net external force and torque acting on a system is zero

dynamic equilibrium
: a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

59. 8.2 The Second Condition for Equilibrium

Summary

- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics

TORQUE

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity. A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. See Figure 1. First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.

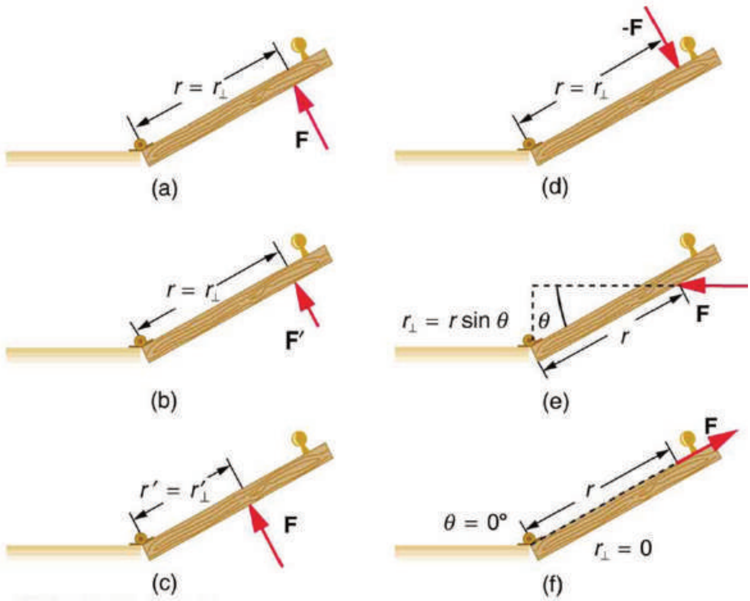

Figure 1. *Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to* ***F****. Note that* $\mathbf{r}_\perp$ *is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force* ***F'*** *acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here,* $\boldsymbol{\theta}$ *is less than* $\mathbf{90°}$*. (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case,* $\boldsymbol{\theta = 0°}$*.*

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

$$\boldsymbol{\tau = rF}\ \textbf{sin}\ \boldsymbol{\theta}$$

where **τ** (the Greek letter tau) is the symbol for torque, ***r*** is the distance from the pivot point to the point where the force is applied, ***F*** is the magnitude of the force, and ***θ*** is the angle between the force and the vector directed from the point of application to the pivot point, as seen in Figure 1 and Figure 2. An alternative expression for torque is given in terms of the **perpendicular lever arm** $\boldsymbol{r_\perp}$ as shown in Figure 1 and Figure 2, which is defined as

$$\boldsymbol{r_\perp = r}\ \textbf{sin}\ \boldsymbol{\theta}$$

so that

$$\boldsymbol{\tau = r_\perp F.}$$

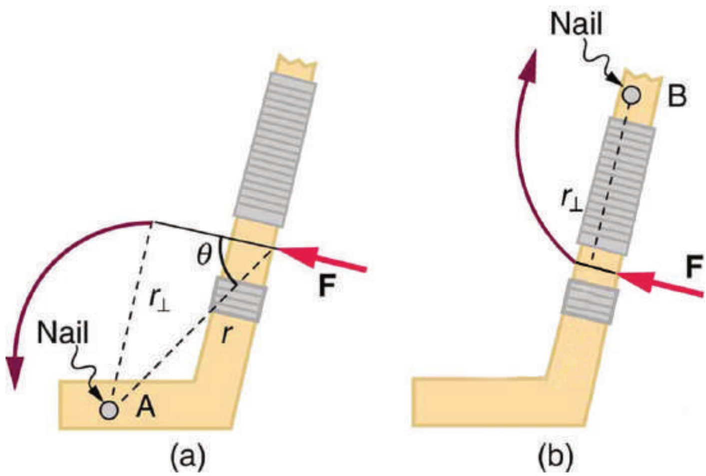

Figure 2. *A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors **r**, **F**, and **θ** for pivot point A on a body are shown here—**r** is the distance from the chosen pivot point to the point where the force **F** is applied, and **θ** is the angle between **F** and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B.*

The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which $\vec{\textbf{F}}$ acts; it is shown as a dashed line in Figure 1 and Figure 2. Note that the line segment that defines the distance $r_\perp$ is perpendicular to $\vec{\textbf{F}}$, as its name implies. It is sometimes easier to find or visualize $r_\perp$ than to find both r and θ. In such cases, it may be more convenient to use $\tau = r_\perp F$ rather than $\tau = rF \sin \theta$ for torque, but both are equally valid.

The **SI unit of torque** is newtons times meters, usually written as **N·m**. For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of **32 N·m (0.800 m × 40 N × sin 90°)** relative to the hinges. If you reduce the force to 20 N, the torque is reduced to **16 N·m**, and so on.

The torque is always calculated with reference to some chosen pivot point. For the same applied force, a different choice for the location of the pivot will give you a different value for the torque, since both ***r*** and **θ** depend on the location of the pivot. Any point in any object can be chosen to calculate the torque about that point. The object may not actually pivot about the chosen "pivot point."

Note that for rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in Figure 2. If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

Now, *the second condition necessary to achieve equilibrium* is that ***the net external torque on a system must be zero***. An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

$$\mathbf{net}\ \boldsymbol{\tau} = \mathbf{0}$$

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

When two children balance a seesaw as shown in Figure 3, they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely.

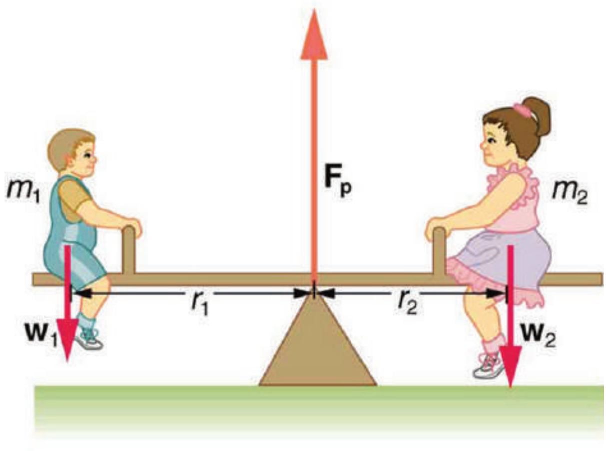

Figure 3. *Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.*

You can explore the PhET simulation called Balancing Act.

https://phet.colorado.edu/en/simulation/balancing-act

Example 1: She Saw Torques On A Seesaw

The two children shown in Figure 3 are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.(a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is $\boldsymbol{F}_{\mathbf{p}}$, the supporting force exerted by the pivot?

Strategy

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

Solution (a)

The three external forces acting on the system are the weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. Torque is defined to be

$$\boldsymbol{\tau = rF \textbf{ sin } \theta.}$$

Here $\boldsymbol{\theta}$ **= 90°**, so that **sin** $\boldsymbol{\theta}$ **= 1** for all three forces. That means $\boldsymbol{r_\perp = r}$ for all three. The torques exerted by the three forces are first,

$$\boldsymbol{\tau_1 = r_1 \, w_1}$$

second,

$$\boldsymbol{\tau_2 = -r_2 \, w_2}$$

and third,

$$\begin{aligned} \boldsymbol{\tau_{\mathrm{p}}} &= \boldsymbol{r_{\mathrm{p}} F_{\mathrm{p}}} \\ &= \boldsymbol{0 \cdot F_{\mathrm{p}}} \\ &= \boldsymbol{0}. \end{aligned}$$

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since $\boldsymbol{F_{\mathrm{p}}}$ acts directly on the pivot point, the distance $\boldsymbol{r_{\mathrm{p}}}$ is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

$$\boldsymbol{\tau_2 = -\tau_1},$$

or

$$\boldsymbol{r_2 \, w_2 = r_1 \, w_1}.$$

Weight is mass times the acceleration due to gravity. Entering $\boldsymbol{mg}$ for $\boldsymbol{w}$, we get

$$\boldsymbol{r_2 m_2 g = r_1 m_1 g}.$$

Solve this for the unknown $\boldsymbol{r_2}$:

$$\boldsymbol{r_2 = r_1 \frac{m_1}{m_2}}.$$

The quantities on the right side of the equation are known; thus, r_2 is

$$r_2 = (1.60 \text{ m}) \frac{26.0 \text{ kg}}{32.0 \text{ kg}} = 1.30 \text{ m}.$$

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

Solution (b)

This part asks for a force F_p. The easiest way to find it is to use the first condition for equilibrium, which is

$$\text{net } \vec{\mathbf{F}} = 0.$$

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

$$\text{net } F_y = 0$$

where we again call the vertical axis the *y*-axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

$$F_\text{p} - w_1 - w_2 = 0.$$

This equation yields what might have been guessed at the beginning:

$$F_\text{p} = w_1 + w_2.$$

So, the pivot supplies a supporting force equal to the total weight of the system:

$$F_\text{p} = m_1 g + m_2 g.$$

Entering known values gives

$$\begin{aligned} F_{\mathrm{p}} &= (26.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) + (32.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) \\ &= 568\ \mathrm{N}. \end{aligned}$$

Discussion

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw's actual pivot!

Several aspects of the preceding example have broad implications. First, the choice of the pivot as the point around which torques are calculated simplified the problem. Since $\boldsymbol{F}_{\mathbf{p}}$ is exerted on the pivot point, its lever arm is zero. Hence, the torque exerted by the supporting force $\boldsymbol{F}_{\mathbf{p}}$ is zero relative to that pivot point. The second condition for equilibrium holds for any choice of pivot point, and so we choose the pivot point to simplify the solution of the problem.

Second, the acceleration due to gravity canceled in this problem, and we were left with a ratio of masses. *This will not always be the case*. Always enter the correct forces—do not jump ahead to enter some ratio of masses.

Third, the weight of each child is distributed over an area of the seesaw, yet we treated the weights as if each force were exerted at a single point. This is not an approximation—the distances $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the distances to points directly below the **center of gravity** of each child. As we shall see in the next section, the mass and weight of a system can act as if they are located at a single point.

Finally, note that the concept of torque has an importance beyond static equilibrium. *Torque plays the same role in rotational motion that force plays in linear motion.* We will examine this in the next chapter.

Section Summary

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is defined to be

  $$\boldsymbol{\tau = rF\ \sin\ \theta}$$

  where **τ** is torque, ***r*** is the distance from the pivot point to the point where the force is applied, ***F*** is the magnitude of the force, and ***θ*** is the angle between ***F*** and the vector directed from the point where the force acts to the pivot point. The perpendicular lever arm $\boldsymbol{r_\perp}$ is defined to be

  $$\boldsymbol{r_\perp = r\ \sin\ \theta}$$

  so that

  $$\boldsymbol{\tau = r_\perp F.}$$

- The perpendicular lever arm $\boldsymbol{r_\perp}$ is the shortest distance from the pivot point to the line along which ***F*** acts. The SI unit for torque is newton-meter (**N·m**). The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:

  $$\mathbf{net}\ \boldsymbol{\tau} = \mathbf{0}$$

  By convention, counterclockwise torques are positive, and clockwise torques are negative.

Conceptual Questions

1: What three factors affect the torque created by a force relative to a specific pivot point?

Problems & Exercises

1: (a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

2: When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton × meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.

3: Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.

4: Use the second condition for equilibrium (net τ =

0) to calculate F_p in Example 1, employing any data given or solved for in part (a) of the example.

5: Repeat the seesaw problem in Example 1 with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.

Glossary

torque
: turning or twisting effectiveness of a force

perpendicular lever arm
: the shortest distance from the pivot point to the line along which FF lies

SI units of torque
: newton times meters, usually written as N·m

center of gravity
: the point where the total weight of the body is assumed to be concentrated

Solutions

Problems & Exercises

1: (a) $\mathbf{46.8\ N\cdot m}$ (b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

3: $\boldsymbol{23.3\textbf{ N}}$

5: Given:

$$\begin{aligned} \boldsymbol{m_1} &= \mathbf{26.0\ kg},\ \boldsymbol{m_2} = \mathbf{32.0\ kg},\ \boldsymbol{m_s} = \mathbf{12.0\ kg}, \\ \boldsymbol{r_1} &= \mathbf{1.60\ m},\ \boldsymbol{r_s} = \mathbf{0.160\ m},\ \textbf{find (a)}\boldsymbol{r_2},\ \textbf{(b)}\boldsymbol{F_\mathrm{p}} \end{aligned}$$

a) Since children are balancing:

$$\begin{gathered} \textbf{net}\ \boldsymbol{\tau_\mathrm{cw}} = -\textbf{net}\ \boldsymbol{\tau_\mathrm{ccw}} \\ \Rightarrow \boldsymbol{w_1 r_1 + m_\mathrm{s} g r_\mathrm{s} = w_2 r_2} \end{gathered}$$

So, solving for $\boldsymbol{r_2}$ gives:

$$\begin{aligned} \boldsymbol{r_2} &= \frac{\boldsymbol{w_1 r_1 + m_\mathrm{s} g r_\mathrm{s}}}{\boldsymbol{w_2}} = \frac{\boldsymbol{m_1 g r_1 + m_\mathrm{s} g r_\mathrm{s}}}{\boldsymbol{m_2 g}} = \frac{\boldsymbol{m_1 r_1 + m_\mathrm{s} r_\mathrm{s}}}{\boldsymbol{m_2}} \\ &= \frac{\mathbf{(26.0\ kg)(1.60\ m)+(12.0\ kg)(0.160\ m)}}{\mathbf{32.0\ kg}} \\ &= \mathbf{1.36\ m} \end{aligned}$$

b) Since the children are not moving:

$$\begin{gathered} \textbf{net F} = \mathbf{0} = \boldsymbol{F_\mathrm{p} - w_1 - w_2 - w_s} \\ \Rightarrow \boldsymbol{F_\mathrm{p} = w_1 + w_2 + w_\mathrm{s}} \end{gathered}$$

So that

$$\begin{aligned} \boldsymbol{F_\mathrm{p}} &= \mathbf{(26.0\ kg + 32.0\ kg + 12.0\ kg)(9.80\ m/s^2)} \\ &= \mathbf{686\ N} \end{aligned}$$

60. 8.3 Stability

Summary

- State the types of equilibrium.
- Describe stable and unstable equilibriums.
- Describe neutral equilibrium.

It is one thing to have a system in equilibrium; it is quite another for it to be stable. The toy doll perched on the man's hand in Figure 1, for example, is not in stable equilibrium. There are *three types of equilibrium*: *stable*, *unstable*, and *neutral*. Figures throughout this module illustrate various examples.

Figure 1 presents a balanced system, such as the toy doll on the man's hand, which has its center of gravity (cg) directly over the pivot, so that the torque of the total weight is zero. This is equivalent to having the torques of the individual parts balanced about the pivot point, in this case the hand. The cgs of the arms, legs, head, and torso are labeled with smaller type.

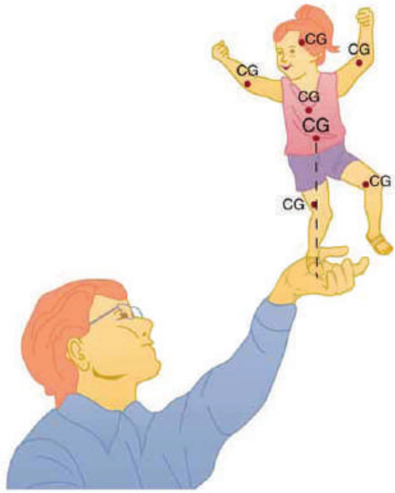

Figure 1. *A man balances a toy doll on one hand.*

A system is said to be in **stable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a marble at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements. For another example of stable equilibrium, see the pencil in Figure 2.

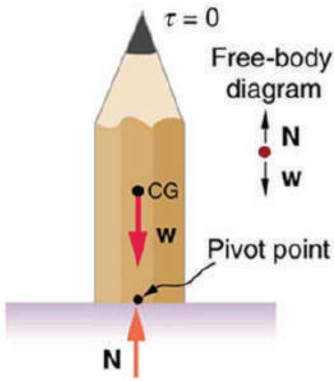

__Figure 2.__ This pencil is in the condition of equilibrium. The net force on the pencil is zero and the total torque about any pivot is zero.

A system is in **unstable equilibrium** if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ball resting on top of a hill. Once displaced, it accelerates away from the crest. See the next several figures for examples of unstable equilibrium.

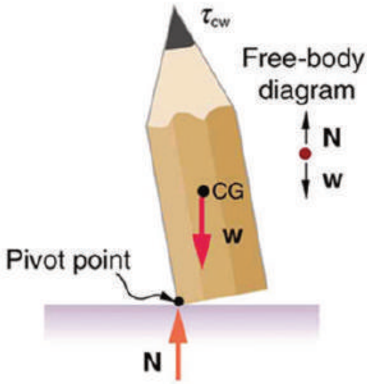

Figure 3. *If the pencil is displaced slightly to the side (counterclockwise), it is no longer in equilibrium. Its weight produces a clockwise torque that returns the pencil to its equilibrium position.*

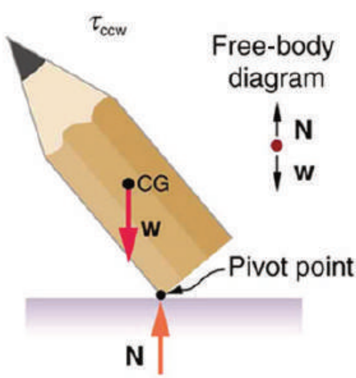

Figure 4. *If the pencil is displaced too far, the torque caused by its weight changes direction to counterclockwise and causes the displacement to increase.*

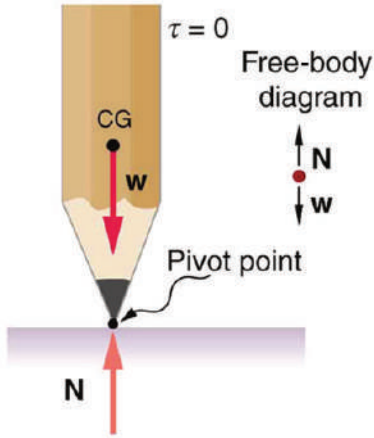

Figure 5. *This figure shows unstable equilibrium, although both conditions for equilibrium are satisfied.*

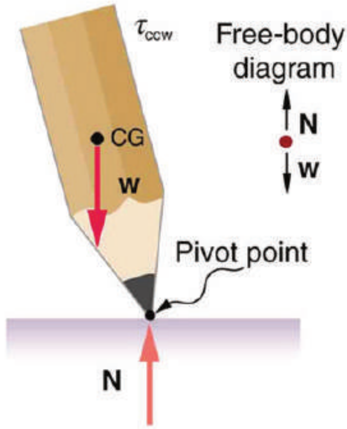

Figure 6. *If the pencil is displaced even slightly, a torque is created by its weight that is in the same direction as the displacement, causing the displacement to increase.*

A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position.

When we consider how far a system in stable equilibrium can be displaced before it becomes unstable, we find that some systems in stable equilibrium are more stable than others. The critical point is reached when the cg is no longer *above* the base of support. Additionally, since the cg of a person's body is above the pivots in the hips, displacements must be quickly controlled. This control is a central nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by lowering one's center of gravity by bending the knees, as when a football player prepares to receive a ball or braces themselves for a tackle. A cane, a crutch, or a walker increases the stability of the user, even more as the base of support widens. Usually, the cg of a female is lower (closer to the ground) than a male. Young children have their center of gravity between their shoulders, which increases the challenge of learning to walk.

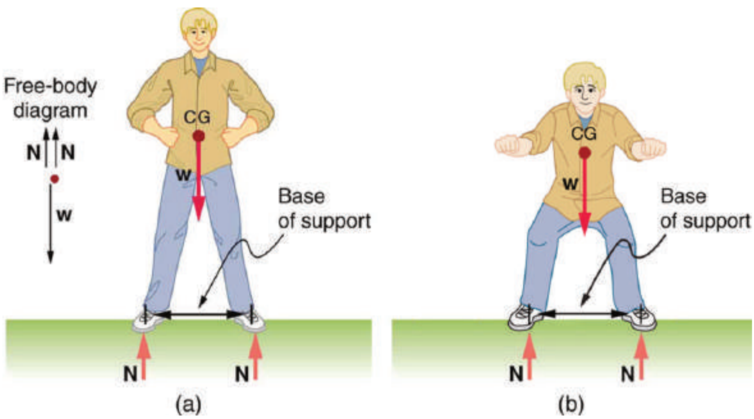

Figure 7. *(a) The center of gravity of an adult is above the hip joints (one of the main pivots in the body) and lies between two narrowly-separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable. Humans are less stable relative to forward and backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart.*

Animals such as chickens have easier systems to control. Figure 8 shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken's cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken's part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

Figure 8 shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is stable for front-to-back displacements as well as for side-to-side displacements.

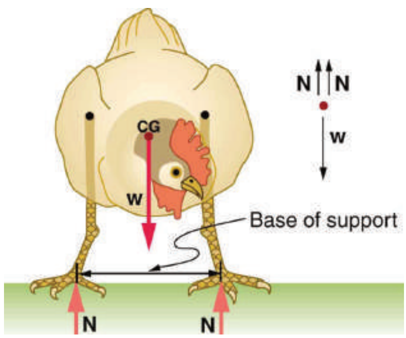

Figure 8. *The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them.*

The basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

TAKE-HOME EXPERIMENT

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

Section Summary

- A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement.
- A system is in unstable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.
- A system is in neutral equilibrium if its equilibrium is independent of displacements from its original position.

Problems & Exercises

1: Suppose a horse leans against a wall as in Figure 9. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal in magnitude and opposite in direction to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.

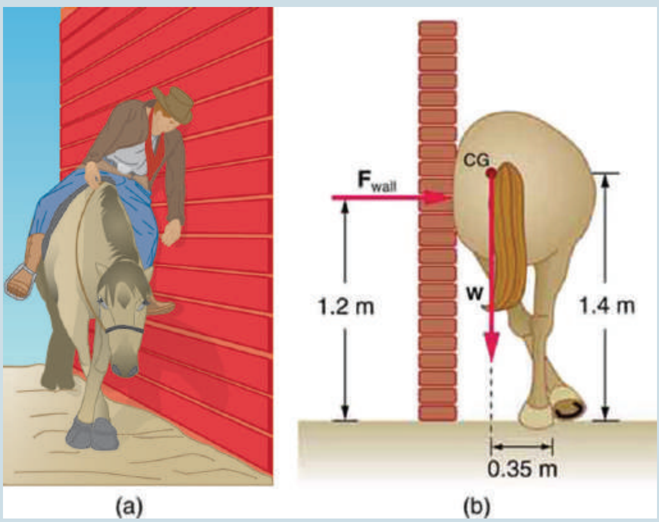

Figure 9.

2: Two children of mass 20.0 kg and 30.0 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3.00 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?

3: A person carries a plank of wood 2.00 m long with one hand pushing down on it at one end with a force F_1 and the other hand holding it up at .500 m from the end of the plank with force $\boldsymbol{F_2}$. If the plank has a mass of 20.0 kg and its center of gravity is at the middle of the plank, what are the magnitudes of the forces F_1 and F_2?

4: A gymnast is attempting to perform splits. From the information given in Figure 10, calculate the magnitude and direction of the force exerted on each foot by the floor.

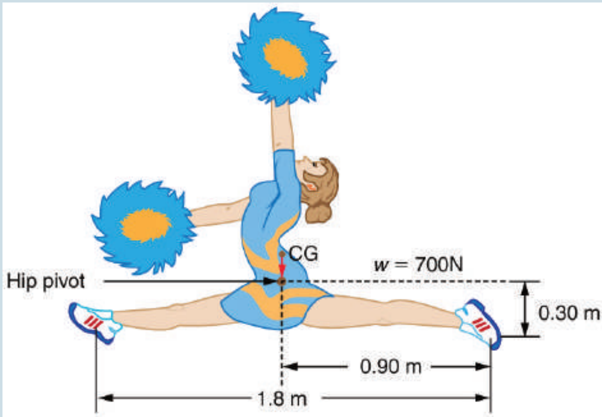

Figure 10. *A gymnast performs full split. The center of gravity and the various distances from it are shown.*

Glossary

neutral equilibrium
: a state of equilibrium that is independent of a system's displacements from its original position

stable equilibrium
: a system, when displaced, experiences a net force or torque in a direction opposite to the direction of the displacement

unstable equilibrium
: a system, when displaced, experiences a net force or torque in the same direction as the displacement from equilibrium

Solutions

Problems & Exercises

1: $\boldsymbol{F_{\text{wall}} = 1.43 \times 10^3 \text{ N}}$

4: $\mathbf{350\ N}$ directly upwards

61. 8.4 Applications of Statics, Including Problem-Solving Strategies

Summary

- Discuss the applications of Statics in real life.
- State and discuss various problem-solving strategies in Statics.

Statics can be applied to a variety of situations, for example, bad posture leading to back strain. We begin with a discussion of problem-solving strategies specifically used for statics.

PROBLEM-SOLVING STRATEGY: STATIC EQUILIBRIUM SITUATIONS

1. The first step is to determine whether or not the system is in **static equilibrium**. This

condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.

2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.
3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations **net *F* = 0** and **net τ =0**, depending on the list of known and unknown factors. If the second condition is involved, *choose the pivot point to simplify the solution*. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then ***r* = 0**), or along a line through the pivot point (then ***θ* = 0**). Always choose a convenient coordinate system for projecting forces.
4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with experience.

Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In Figure 1, the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium (**net F = 0**). The second condition (**net τ =0**) is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In Figure 1, a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole, $F_R = F_L = w/2$. (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See Figure 1. If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

Similar observations can be made using a meter stick held at different locations along its length.

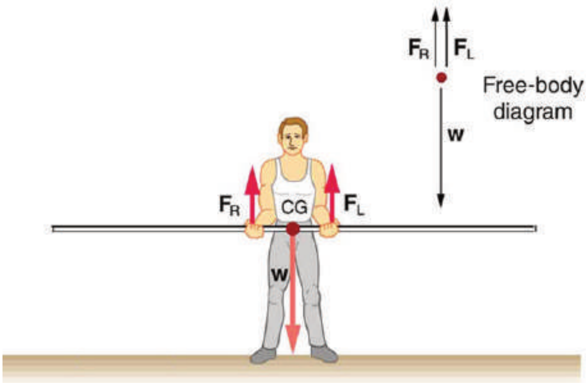

Figure 1. *A pole vaulter holds a pole horizontally with both hands.*

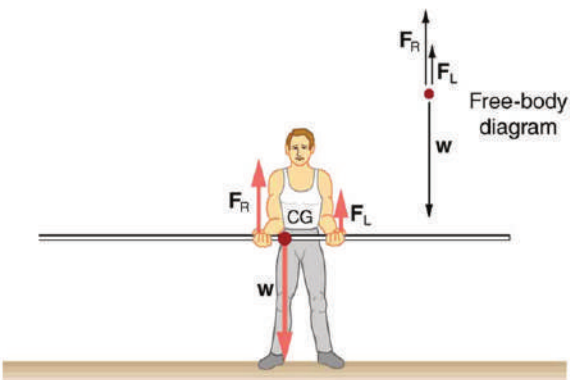

Figure 2. *A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand.*

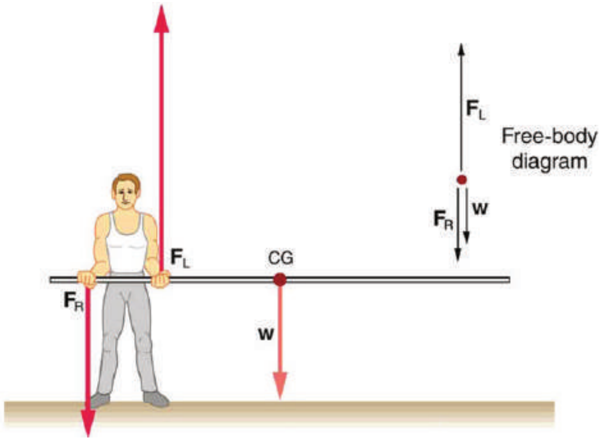

Figure 3. A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.

If the pole vaulter holds the pole as shown in Figure 2, the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If $\boldsymbol{F}_{\mathbf{L}} = \boldsymbol{F}_{\mathbf{R}}$, then the torques about the cg would not be equal since the lever arms are different.) Logically, the right hand should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces $\boldsymbol{F}_{\mathbf{L}}$ and $\boldsymbol{F}_{\mathbf{R}}$ is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole (Figure 3), the direction of the force applied by the right hand of the vaulter reverses its direction.

Example 1: What Force Is Needed to Support a Weight Held Near Its CG?

For the situation shown in Figure 2, calculate: (a) $\boldsymbol{F}_{\mathbf{R}}$, the force exerted by the right hand, and (b) $\boldsymbol{F}_{\mathbf{L}}$, the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

Strategy

Figure 2 includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium (**net *F* = 0**), since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium (**net τ = 0**) if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

Solution for (a)

There are now only two nonzero torques, those from the gravitational force ($\boldsymbol{\tau}_{\mathbf{W}}$) and from the push or pull of the right hand ($\boldsymbol{\tau}_{\mathbf{R}}$). Stating the second condition in terms of clockwise and counterclockwise torques,

$$\mathbf{net}\ \boldsymbol{\tau}_{\mathbf{cw}} = -\ \mathbf{net}\ \boldsymbol{\tau}_{\mathbf{ccw}}.$$

or the algebraic sum of the torques is zero.

Here this is

$$\boldsymbol{\tau}_{\mathbf{R}} = -\boldsymbol{\tau}_{\mathbf{w}}$$

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise torque. Using the definition of torque, **τ = *rF* sin *θ***, noting that ***θ* = 90°**, and substituting known values, we obtain

$$\mathbf{(0.900\ m)}(\boldsymbol{F}_{\mathbf{R}}) = \mathbf{(0.600\ m)}(\boldsymbol{mg})$$

Thus,

$$\begin{aligned} \boldsymbol{F}_{\mathbf{R}} &= \mathbf{(0.667)(5.00\ kg)(9.80\ m/s^2)} \\ &= \mathbf{32.7\ N.} \end{aligned}$$

Solution for (b)

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton's second law:

$$\boldsymbol{F}_{\mathbf{L}} + \boldsymbol{F}_{\mathbf{R}} - \boldsymbol{mg} = \mathbf{0}$$

From this we can conclude:

$$\boldsymbol{F}_{\mathbf{L}} + \boldsymbol{F}_{\mathbf{R}} = \boldsymbol{w} = \boldsymbol{mg}$$

Solving for $\boldsymbol{F}_{\mathbf{L}}$, we obtain

$$\begin{aligned} \boldsymbol{F}_{\mathbf{L}} &= \boldsymbol{mg} - \boldsymbol{F}_{\mathbf{R}} \\ &= \boldsymbol{mg} - \mathbf{32.7\ N} \\ &= \mathbf{(5.00\ kg)(9.80\ m/s^2)} - \mathbf{32.7\ N} \\ &= \mathbf{16.3\ N} \end{aligned}$$

Discussion

F_L is seen to be exactly half of F_R, as we might have guessed, since F_L is applied twice as far from the cg as F_R.

If the pole vaulter holds the pole as he might at the start of a run, shown in Figure 3, the forces change again. Both are considerably greater, and one force reverses direction.

TAKE-HOME EXPERIMENT

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

PHET EXPLORATIONS: BALANCING ACT

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

Figure 4. *Balancing Act*

Summary

- We have discussed the problem-solving strategies specifically useful for statics.

Conceptual Questions

1: When visiting some countries, you may see a person balancing a load on the head. Explain why the center of mass of the load needs to be directly above the person's neck vertebrae.

Problems & Exercises

1: In Figure 3, the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in Figure 1, show that the second condition for equilibrium (net $\tau = 0$) is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.

Glossary

static equilibrium
: equilibrium in which the acceleration of the system is zero and accelerated rotation does not occur

62. 8.5 Mechanical Advantage

Summary

- Calculate the mechanical advantage.

Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we apply the force. The human body can be compared to simple machines depending on muscle attachment and weight of the limb. The word for "machine" comes from the Greek word meaning "to help make things easier." Levers, gears, pulleys, wedges, and screws are some examples of machines. The ratio of output to input force magnitudes for any simple machine is called its **mechanical advantage** (MA).

$$MA = (F_{output}) / (F_{input})$$

One of the simplest machines is the lever, which is a rigid bar pivoted at a fixed place called the fulcrum. Torques are involved in levers, since there is rotation about a pivot point. Distances from the physical pivot of the lever are crucial, and we can obtain a useful expression for the MA in terms of these distances.

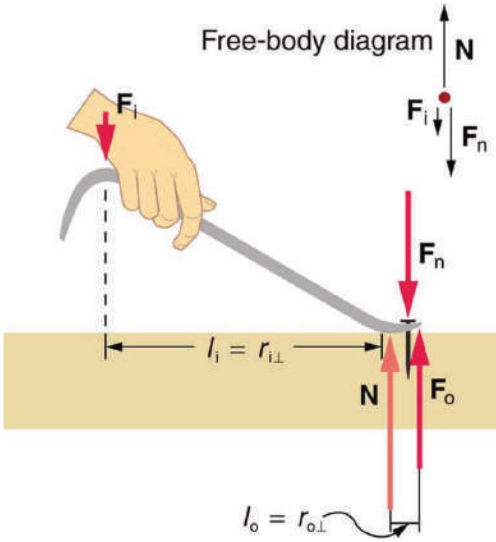

Figure 1. *A nail puller is a lever with a large mechanical advantage. The external forces on the nail puller are represented by solid arrows. The force that the nail puller applies to the nail (*$\mathbf{F_o}$*) is not a force on the nail puller. The reaction force the nail exerts back on the puller (*$\mathbf{F_n}$*) is an external force and is equal and opposite to* $\mathbf{F_o}$*. The perpendicular lever arms of the input and output forces are* $\boldsymbol{l_i}$ *and* $\boldsymbol{l_o}$*.*

Figure 1 shows a lever type that is used as a nail puller. Crowbars, seesaws, and other such levers are all analogous to this one. $\vec{\mathbf{F}}_\mathbf{i}$ is the input force and $\vec{\mathbf{F}}_\mathbf{o}$ is the output force. There are three vertical forces acting on the nail puller (the system of interest) – these are $\vec{\mathbf{F}}_\mathbf{i}, \vec{\mathbf{F}}_\mathbf{o}$ and $\vec{\mathbf{N}}$. $\vec{\mathbf{F}}_\mathbf{n}$ is the reaction force back on the system, equal and opposite to $\vec{\mathbf{F}}_\mathbf{o}$. (Note that $\vec{\mathbf{F}}_\mathbf{o}$ is not a force on the system.) $\vec{\mathbf{N}}$ is the normal force upon the lever, and its torque is zero since it is exerted at the pivot. The torques due to $\vec{\mathbf{F}}_\mathbf{i}$ and $\vec{\mathbf{F}}_\mathbf{n}$ must be equal to each other if the nail is not moving, to satisfy the second condition for equilibrium (**net τ = 0**). (In order for the nail to actually move, the torque due to $\vec{\mathbf{F}}_{\mathbf{i}}$ must be ever-so-slightly greater than torque due to $\vec{\mathbf{F}}_{\mathbf{n}}$.) Hence,

$$l_{\mathbf{i}} F_{\mathbf{i}} = l_{\mathbf{o}} F_{\mathbf{o}}$$

where $l_{\mathbf{i}}$ and $l_{\mathbf{o}}$ are the distances from where the input and output forces are applied to the pivot, as shown in the figure. Rearranging the last equation gives

$$\frac{F_{\mathbf{o}}}{F_{\mathbf{i}}} = \frac{l_{\mathbf{i}}}{l_{\mathbf{o}}}.$$

What interests us most here is that the magnitude of the force exerted by the nail puller, $F_{\mathbf{o}}$, is much greater than the magnitude of the input force applied to the puller at the other end, $F_{\mathbf{i}}$. For the nail puller,

$$\mathbf{MA} = \frac{F_{\mathbf{o}}}{F_{\mathbf{i}}} = \frac{l_{\mathbf{i}}}{l_{\mathbf{o}}}.$$

This equation is true for levers in general. For the nail puller, the MA is certainly greater than one. The longer the handle on the nail puller, the greater the force you can exert with it.

Two other types of levers that differ slightly from the nail puller are a wheelbarrow and a shovel, shown in Figure 2. All these lever types are similar in that only three forces are involved – the input force, the output force, and the force on the pivot – and thus their MAs are given by $\mathbf{MA} = \frac{F_{\mathbf{o}}}{F_{\mathbf{i}}}$ and $\mathbf{MA} = \frac{d_1}{d_2}$, with distances being measured relative to the physical pivot. The wheelbarrow and shovel differ from the nail puller because both the input and output forces are on the same side of the pivot.

In the case of the wheelbarrow, the output force or load is between the pivot (the wheel's axle) and the input or applied force. In the case of the shovel, the input force is between the pivot (at the end of the handle) and the load, but the input lever arm is shorter than the output lever arm. In this case, the MA is less than one.

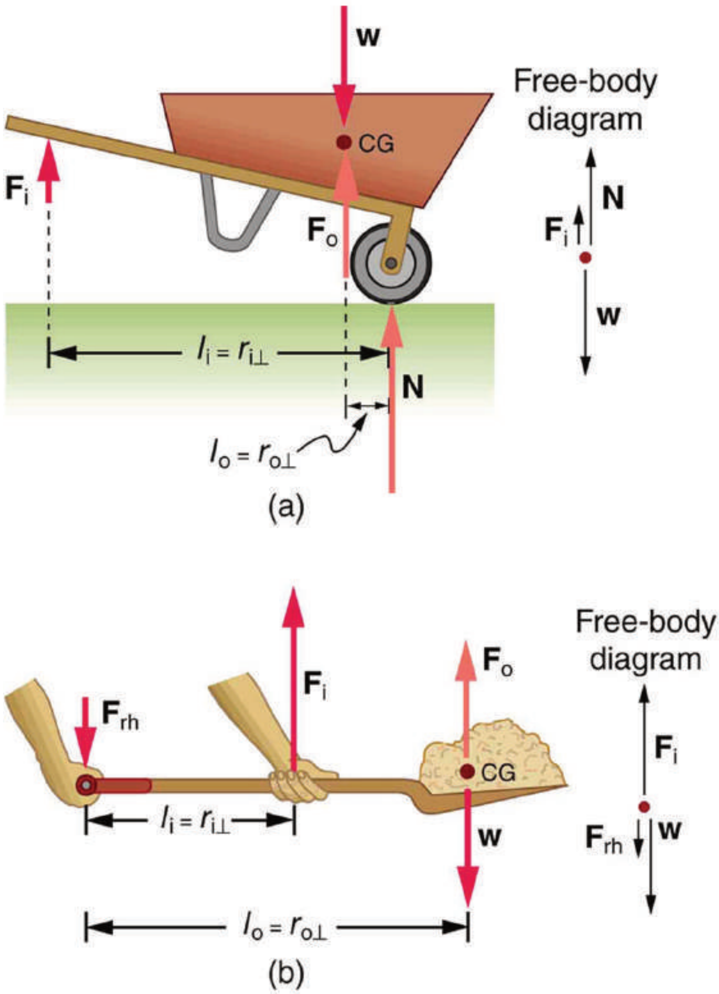

Figure 2. *(a) In the case of the wheelbarrow, the output force or load is between the pivot and the input force. The pivot is the wheel's axle. Here, the output force is greater than the input force. Thus, a wheelbarrow enables you to lift much heavier loads than you could with your body alone. (b) In the case of the shovel, the input force is between the pivot and the load, but the input lever arm is shorter than the output lever arm. The pivot is at the handle held by the right hand. Here, the output force (supporting the shovel's load) is less than the input force (from the hand nearest the load), because the input is exerted closer to the pivot than is the output.*

Example 1: What is the Advantage for the Wheelbarrow?

In the wheelbarrow of Figure 2, the load has a perpendicular lever arm of 7.50 cm, while the hands have a perpendicular lever arm of 1.02 m. (a) What upward force must you exert to support the wheelbarrow and its load if their combined mass is 45.0 kg? (b) What force does the wheelbarrow exert on the ground?

Strategy

Here, we use the concept of mechanical advantage.

Solution

(a) In this case, $\frac{\boldsymbol{F}_{\mathbf{o}}}{\boldsymbol{F}_{\mathbf{i}}} = \frac{\boldsymbol{l}_{\mathbf{i}}}{\boldsymbol{l}_{\mathbf{o}}}$ becomes

$$\boldsymbol{F}_{\mathbf{i}} = \boldsymbol{F}_{\mathbf{o}} \frac{\boldsymbol{l}_{\mathbf{o}}}{\boldsymbol{l}_{\mathbf{i}}}.$$

Adding values into this equation yields

$$\boldsymbol{F}_{\mathbf{i}} = \mathbf{(45.0\,kg)(9.80\,m/s^2)}\frac{\mathbf{0.075\,m}}{\mathbf{1.02\,m}} = \mathbf{32.4\,N}.$$

The free-body diagram (see Figure 2) gives the following normal force: $\boldsymbol{F}_{\mathbf{i}}$ **+** ***N*** **=** ***W***. Therefore, ***N*** **= (45.0 kg)(9.80 m/s^2) -32.4 N = 409 N**. ***N*** is the normal force acting on the wheel; by Newton's third law, the force the wheel exerts on the ground is **409 N**.

Discussion

An even longer handle would reduce the force needed to lift the load. The MA here is $\mathbf{MA=1.02/0.0750=13.6}$.

Another very simple machine is the inclined plane. Pushing a cart up a plane is easier than lifting the same cart straight up to the top using a ladder, because the applied force is less. However, the work done in both cases (assuming the work done by friction is negligible) is the same.

A crank is a lever that can be rotated $\mathbf{360°}$ about its pivot, as shown in Figure 3. Such a machine may not look like a lever, but the physics of its actions remain the same. The MA for a crank is simply the ratio of the radii r_i/r_0. Wheels and gears have this simple expression for their MAs too. The MA can be greater than 1, as it is for the crank, or less than 1, as it is for the simplified car axle driving the wheels, as shown. If the axle's radius is **2.0 cm** and the wheel's radius is **24.0 cm**, then $\mathbf{MA=2.0/24.0=0.083}$ and the axle would have to exert a force of **12,000 N** on the wheel to enable it to exert a force of **1000 N** on the ground.

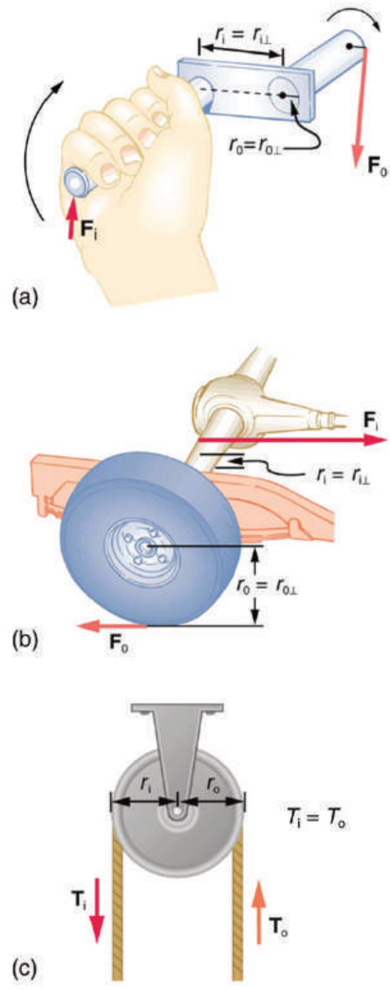

Figure 3. *(a) A crank is a type of lever that can be rotated* ***360°*** *about its pivot. Cranks are usually designed to have a large MA. (b) A simplified automobile axle drives a wheel, which has a much larger diameter than the axle. The MA is less than 1. (c) An ordinary pulley is used to lift a heavy load. The pulley changes the direction of the force* ***T*** *exerted by the cord without changing its magnitude. Hence, this machine has*

an MA of 1.

Section Summary

- Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we have to apply the force.
- The ratio of output to input forces for any simple machine is called its mechanical advantage
- A few simple machines are the lever, nail puller, wheelbarrow, crank, etc.

Conceptual Questions

1: Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

2: Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces (see previous Question)?

Glossary

mechanical advantage
: the ratio of output to input forces for any simple machine

63. 8.6 Forces and Torques in Muscles and Joints

Summary

- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. Figure 1 shows a forearm holding a book and a schematic diagram of an analogous lever system. The schematic is a good approximation for the forearm, which looks more complicated than it is, and we can get some insight into the way typical muscle systems function by analyzing it.

Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor—that is, it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs apply to the outside world. The reason is clear once we realize that most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in Figure 1.

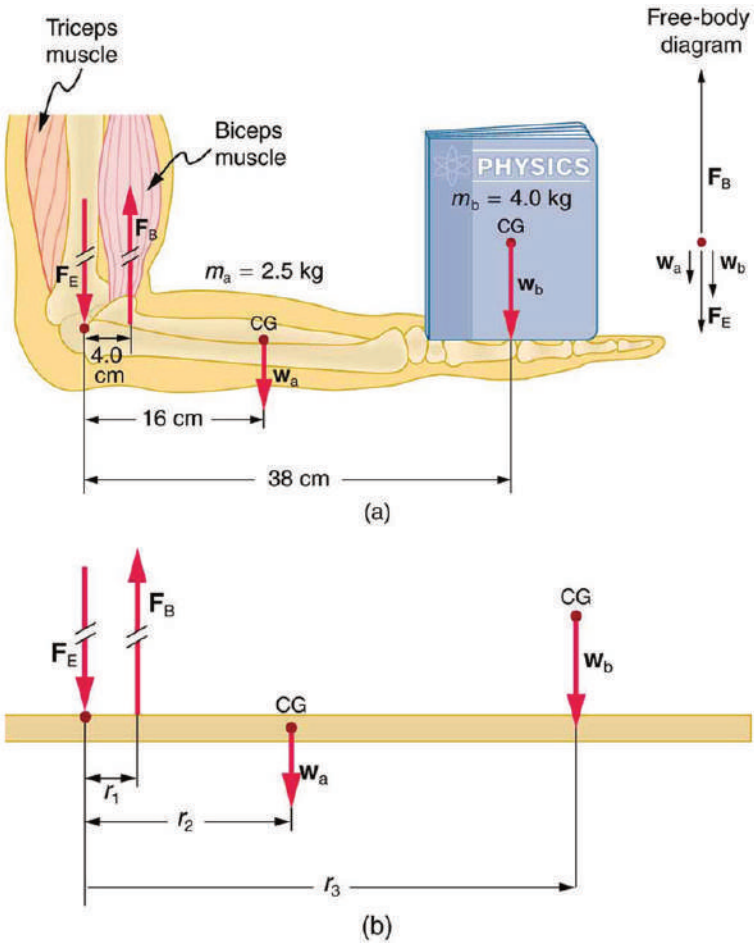

Figure 1. *(a) The figure shows the forearm of a person holding a book. The biceps exert a force* $\mathbf{F_B}$ *to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in Example 1.*

Example 1: Muscles Exert Bigger Forces Than You Might Think

Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in Figure 1, and compare this force with the weight of the forearm plus its load. You may take the data in the figure to be accurate to three significant figures.

Strategy

There are four forces acting on the forearm and its load (the system of interest). The magnitude of the force of the biceps is $\boldsymbol{F_B}$; that of the elbow joint is $\boldsymbol{F_E}$; that of the weights of the forearm is $\boldsymbol{w_a}$, and its load is $\boldsymbol{w_b}$.Two of these are unknown ($\boldsymbol{F_B}$ and $\boldsymbol{F_E}$), so that the first condition for equilibrium cannot by itself yield $\boldsymbol{F_B}$. But if we use the second condition and choose the pivot to be at the elbow, then the torque due to $\boldsymbol{F_E}$ is zero, and the only unknown becomes $\boldsymbol{F_B}$.

Solution

The torques created by the weights are clockwise relative to the pivot, while the torque created by the biceps is counterclockwise; thus, the second condition for equilibrium (**net τ = 0**) becomes

$$\boldsymbol{r_2 w_a + r_3 w_b = r_1 F_B}.$$

Note that $\boldsymbol{\sin\theta=1}$ for all forces, since $\boldsymbol{\theta=90°}$ for all forces. This equation can easily be solved for $\boldsymbol{F_B}$ in terms of known quantities, yielding

$$\boldsymbol{F_B} = \frac{r_2 w_a + r_3 w_b}{r_1}.$$

Entering the known values gives

$$F_B = \frac{(0.160\ \mathrm{m})(2.50\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) + (0.380\ \mathrm{m})(4.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{0.0400\ \mathrm{m}}$$

which yields

$$F_B = 470\ \mathrm{N}.$$

Now, the combined weight of the arm and its load is **(6.50 kg)(9.80 m/s^2)=63.7 N**, so that the ratio of the force exerted by the biceps to the total weight is

$$\frac{F_B}{w_a + w_b} = \frac{470}{63.7} = 7.38.$$

Discussion

This means that the biceps muscle is exerting a force 7.38 times the weight supported.

In the above example of the biceps muscle, the angle between the forearm and upper arm is 90°. If this angle changes, the force exerted by the biceps muscle also changes. In addition, the length of the biceps muscle changes. The force the biceps muscle can exert depends upon its length; it is smaller when it is shorter than when it is stretched.

Very large forces are also created in the joints. In the previous example, the downward force $\boldsymbol{F_E}$ exerted by the humerus at the elbow joint equals 407 N, or 6.38 times the total weight supported. (The calculation of $\boldsymbol{F_E}$ is straightforward and is left as an end-of-chapter problem.) Because muscles can contract, but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions and thus subtract. (In the above example, the upward force of the muscle minus the downward force of the joint equals the weight supported—that is, **470 N-407 N=63 N**, approximately equal to the weight supported.) Forces in muscles and joints are largest when their load is a long distance from the joint, as the book is in the previous example.

In racquet sports such as tennis the constant extension of the arm during game play creates large forces in this way. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as "tennis elbow," can result from repetitive motion, undue torques, and possibly poor racquet selection in such sports. Various tried techniques for holding and using a racquet or bat or stick not only increases sporting prowess but can minimize fatigue and long-term damage to the body. For example, tennis balls correctly hit at the "sweet spot" on the racquet will result in little vibration or impact force being felt in the racquet and the body—less torque. Twisting the hand to provide top spin on the ball or using an extended rigid elbow in a backhand stroke can also aggravate the tendons in the elbow.

Training coaches and physical therapists use the knowledge of relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque which can, over a period of time, revive muscles and joints. Some exercises are designed to be carried out under water, because this requires greater forces to be exerted, further strengthening muscles. However, connecting tissues in the limbs, such as tendons and cartilage as well as joints are sometimes damaged by the large forces they carry. Often, this is due to accidents, but heavily muscled athletes, such as weightlifters, can tear muscles and connecting tissue through effort alone.

The back is considerably more complicated than the arm or leg, with various muscles and joints between vertebrae, all having mechanical advantages less than 1. Back muscles must, therefore, exert very large forces, which are borne by the spinal column. Discs crushed by mere exertion are very common. The jaw is somewhat exceptional—the masseter muscles that close the jaw have a mechanical advantage greater than 1 for the back teeth, allowing us to exert very large forces with them. A cause of stress headaches is persistent clenching of teeth where the sustained large force translates into fatigue in muscles around the skull.

Figure 2 shows how bad posture causes back strain. In part (a), we see a person with good posture. Note that her upper body's cg is directly above the pivot point in the hips, which in turn is directly above the base of support at her feet. Because of this, her upper body's weight exerts no torque about the hips. The only force needed is a vertical force at the hips equal to the weight supported. No muscle action is required, since the bones are rigid and transmit this force from the floor. This is a position of unstable equilibrium, but only small forces are needed to bring the upper body back to vertical if it is slightly displaced. Bad posture is shown in part (b); we see that the upper body's cg is in front of the pivot in the hips. This creates a clockwise torque around the hips that is counteracted by muscles in the lower back. These muscles must exert large forces, since they have typically small mechanical advantages. (In other words, the perpendicular lever arm for the muscles is much smaller than for the cg.) Poor posture can also cause muscle strain for people sitting at their desks using computers. Special chairs are available that allow the body's CG to be more easily situated above the seat, to reduce back pain. Prolonged muscle action produces muscle strain. Note that the cg of the entire body is still directly above the base of support in part (b) of Figure 2. This is compulsory; otherwise the person would not be in equilibrium. We lean forward for the same reason when carrying a load on our backs, to the side when carrying a load in one arm, and backward when carrying a load in front of us, as seen in Figure 3.

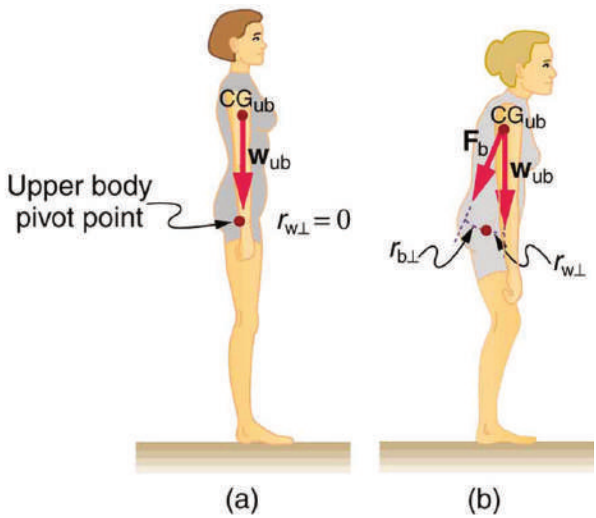

Figure 2. *(a) Good posture places the upper body's cg over the pivots in the hips, eliminating the need for muscle action to balance the body. (b) Poor posture requires exertion by the back muscles to counteract the clockwise torque produced around the pivot by the upper body's weight. The back muscles have a small effective perpendicular lever arm,* $\mathbf{r_{b\perp}}$*, and must therefore exert a large force* $\mathbf{F_b}$*. Note that the legs lean backward to keep the cg of the entire body above the base of support in the feet.*

You have probably been warned against lifting objects with your back. This action, even more than bad posture, can cause muscle strain and damage discs and vertebrae, since abnormally large forces are created in the back muscles and spine.

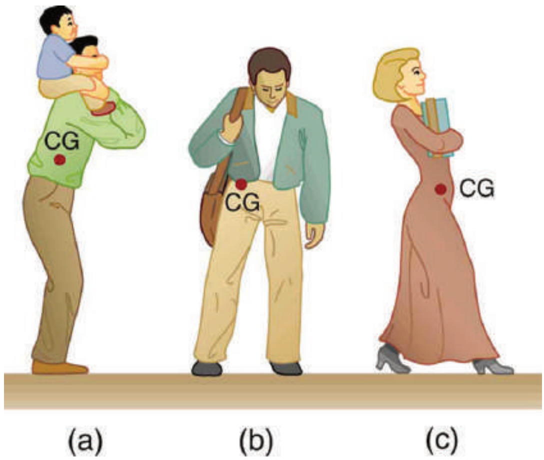

Figure 3. *People adjust their stance to maintain balance. (a) A father carrying his son piggyback leans forward to position their overall cg above the base of support at his feet. (b) A student carrying a shoulder bag leans to the side to keep the overall cg over his feet. (c) Another student carrying a load of books in her arms leans backward for the same reason.*

Example 2: Do Not Lift with Your Back

Consider the person lifting a heavy box with his back, shown in Figure 4. (a) Calculate the magnitude of the force $\boldsymbol{F}_{\mathbf{B}}$– in the back muscles that is needed to support the upper body plus the box and compare this with his weight. The mass of the upper body is 55.0 kg and the mass of the box is 30.0 kg. (b) Calculate the magnitude and direction of the force $\boldsymbol{F}_{\mathbf{V}}$– exerted by the vertebrae on the spine at the indicated pivot point. Again, data in the figure may be taken to be accurate to three significant figures.

Strategy

By now, we sense that the second condition for equilibrium is a good place to start, and inspection of the known values confirms that it can be used to solve for $\boldsymbol{F}_{\mathbf{B}}$– if the pivot is chosen to be at the hips. The torques created by $\boldsymbol{w}_{\mathbf{ub}}$ and $\boldsymbol{w}_{\mathbf{box}}$– are clockwise, while that created by $\vec{\mathbf{F}}_{\mathbf{B}}$ – is counterclockwise.

Solution for (a)

Using the perpendicular lever arms given in the figure, the second condition for equilibrium (**net τ=0**) becomes

$$(0.350\text{ m})(55.0\text{ kg})(9.80\text{ m/s}^2) + (0.500\text{ m})(30.0\text{ kg})(9.80\text{ m/s}^2) = (0.0800\text{ m})F_{\text{B}}.$$

Solving for $\boldsymbol{F}_{\mathbf{B}}$ yields

$$F_{\text{B}} = 4.20 \times 10^3\text{ N}.$$

The ratio of the force the back muscles exert to the weight of the upper body plus its load is

$$\frac{F_{\text{B}}}{w_{\text{ub}} + w_{\text{box}}} = \frac{4200\text{ N}}{833\text{ N}} = 5.04.$$

This force is considerably larger than it would be if the load were not present.

Solution for (b)

More important in terms of its damage potential is the force on the vertebrae F_V. The first condition for equilibrium (**net** $\boldsymbol{F}$**=0**) can be used to find its magnitude and direction. Using ***y*** for vertical and ***x*** for horizontal, the condition for the net external forces along those axes to be zero

$$\textbf{net } \boldsymbol{F_y} = \mathbf{0} \textbf{ and net } \boldsymbol{F_x} = \mathbf{0}.$$

Starting with the vertical (***y***) components, this yields

$$\boldsymbol{F}_{\mathbf{Vy}} - \boldsymbol{w}_{\mathbf{ub}} - \boldsymbol{w}_{\mathbf{box}} - \boldsymbol{F}_{\mathbf{B}} \textbf{ sin } \mathbf{29.0^0} = \mathbf{0}.$$

Thus,

$$\begin{aligned} \boldsymbol{F}_{\mathbf{Vy}} &= \boldsymbol{w}_{\mathbf{ub}} + \boldsymbol{w}_{\mathbf{box}} + \boldsymbol{F}_{\mathbf{B}} \textbf{ sin } \mathbf{29.0^0} \\ &= \mathbf{833\ N} + (\mathbf{4200\ N}) \textbf{ sin } \mathbf{29.0^0} \end{aligned}$$

yielding

$$\boldsymbol{F}_{\mathbf{Vy}} = \mathbf{2.87 \times 10^3\ N}.$$

Similarly, for the horizontal (***x***) components,

$$\boldsymbol{F}_{\mathbf{Vx}} - \boldsymbol{F}_{\mathbf{B}} \textbf{ cos } \mathbf{29.0^0} = \mathbf{0}$$

yielding

$$\boldsymbol{F}_{\mathbf{Vx}} = \mathbf{3.67 \times 10^3\ N.}$$

The magnitude of $\vec{\mathbf{F}}_{\mathbf{V}}$ is given by the Pythagorean theorem:

$$\boldsymbol{F}_{\mathbf{V}} = \sqrt{\boldsymbol{F}_{\mathbf{Vx}}^{\mathbf{2}} + \boldsymbol{F}_{\mathbf{Vy}}^{\mathbf{2}}} = \mathbf{4.66 \times 10^3\ N.}$$

The direction of $\vec{\mathbf{F}}_{\mathbf{V}}$ is

$$\boldsymbol{\theta} = \mathbf{tan}^{\mathbf{-1}}\left(\frac{\boldsymbol{F}_{\mathbf{Vy}}}{\boldsymbol{F}_{\mathbf{Vx}}}\right) = \mathbf{38.0^0}.$$

Note that the ratio of F_V to the weight supported is

$$\frac{F_V}{w_{ub} + w_{box}} = \frac{4660 \text{ N}}{833 \text{ N}} = 5.59.$$

Discussion

This force is about 5.6 times greater than it would be if the person were standing erect. The trouble with the back is not so much that the forces are large—because similar forces are created in our hips, knees, and ankles—but that our spines are relatively weak. Proper lifting, performed with the back erect and using the legs to raise the body and load, creates much smaller forces in the back—in this case, about 5.6 times smaller.

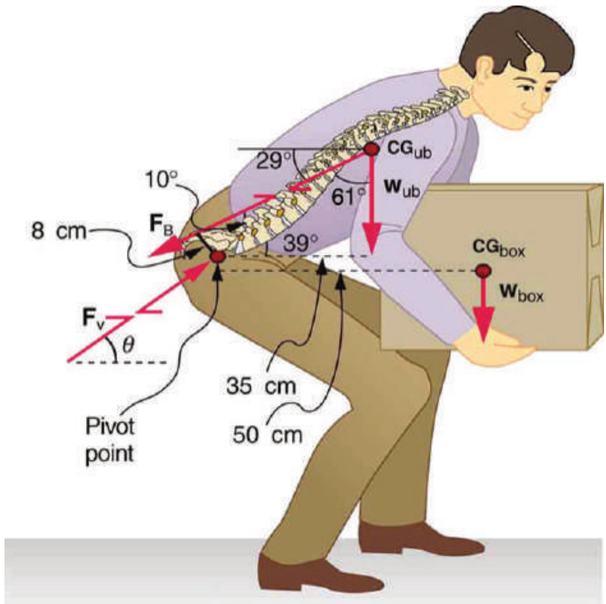

Figure 4. *This figure shows that large forces are exerted by the back muscles and experienced in the vertebrae when a person lifts with their back, since these muscles have small effective perpendicular lever arms. The data shown here are analyzed in the preceding example, Example 2.*

What are the benefits of having most skeletal muscles attached so close to joints? One advantage is speed because small muscle contractions can produce large movements of limbs in a short period of time. Other advantages are flexibility and agility, made possible by the large numbers of joints and the ranges over which they function. For example, it is difficult to imagine a system with biceps muscles attached at the wrist that would be capable of the broad range of movement we vertebrates possess.

There are some interesting complexities in real systems of muscles, bones, and joints. For instance, the pivot point in many joints changes location as the joint is flexed, so that the perpendicular lever arms and the mechanical advantage of the system change, too. Thus the force the biceps muscle must exert to hold up a book varies as the forearm is flexed. Similar mechanisms operate in the legs, which explain, for example, why there is less leg strain when a bicycle seat is set at the proper height. The methods employed in this section give a reasonable description of real systems provided enough is known about the dimensions of the system. There are many other interesting examples of force and torque in the body—a few of these are the subject of end-of-chapter problems.

Section Summary

- Statics plays an important part in understanding everyday strains in our muscles and bones.
- Many lever systems in the body have a mechanical advantage of significantly less than one, as many of our muscles are attached close to joints.
- Someone with good posture stands or sits in such as way that their center of gravity lies directly above the pivot point in their hips, thereby avoiding back strain and damage to disks.

Conceptual Questions

1: Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

2: Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces?

3: Certain types of dinosaurs were bipedal (walked on two legs). What is a good reason that these creatures invariably had long tails if they had long necks?

4: Swimmers and athletes during competition need to go through certain postures at the beginning of the race. Consider the balance of the person and why start-offs are so important for races.

5: If the maximum force the biceps muscle can exert is 1000 N, can we pick up an object that weighs 1000 N? Explain your answer.

6: Suppose the biceps muscle was attached through tendons to the upper arm close to the elbow and the forearm near the wrist. What would be the advantages and disadvantages of this type of construction for the motion of the arm?

7: Explain one of the reasons why pregnant women often suffer from back strain late in their pregnancy.

Problems & Exercises

1: Verify that the force in the elbow joint in Example 1 is 407 N, as stated in the text.

2: Two muscles in the back of the leg pull on the Achilles tendon as shown in Figure 5. What total force do they exert?

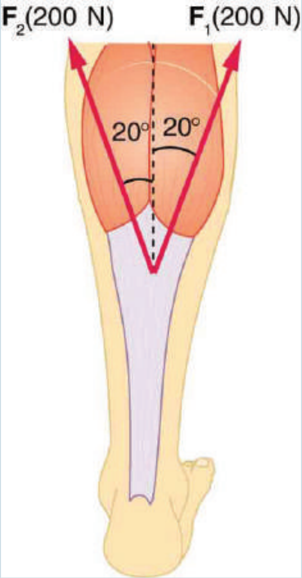

Figure 5. *The Achilles tendon of the posterior leg serves to attach plantaris, gastrocnemius, and soleus muscles to calcaneus bone.*

3: The upper leg muscle (quadriceps) exerts a force of

1250 N, which is carried by a tendon over the kneecap (the patella) at the angles shown in Figure 6. Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).

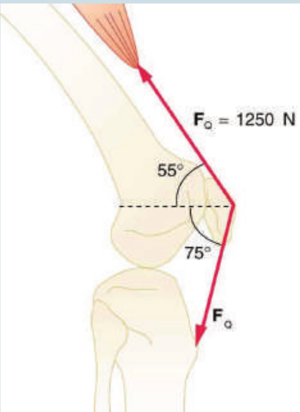

Figure 6. The knee joint works like a hinge to bend and straighten the lower leg. It permits a person to sit, stand, and pivot.

4: A device for exercising the upper leg muscle is shown in Figure 7, together with a schematic representation of an equivalent lever system. Calculate the force exerted by the upper leg muscle to lift the mass at a constant speed. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium in Chapter 9.4 Applications of Statistics, Including Problem-Solving Strategies.

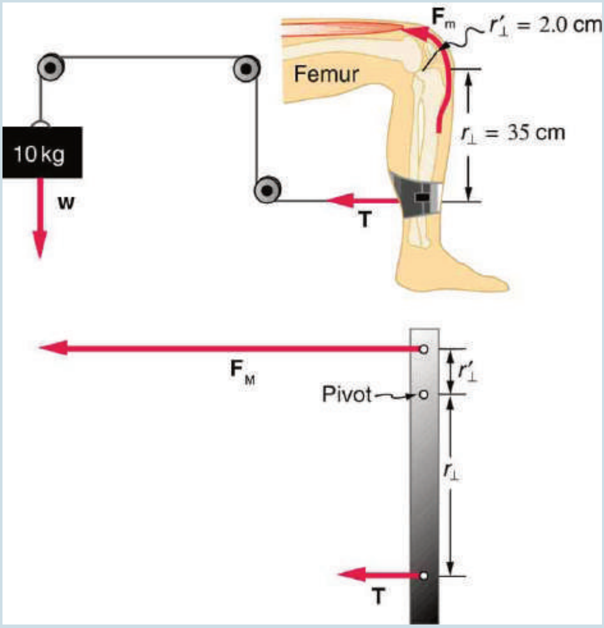

Figure 7. *A mass is connected by pulleys and wires to the ankle in this exercise device.*

5: A person working at a drafting board may hold her head as shown in Figure 8, requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae F_V to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.

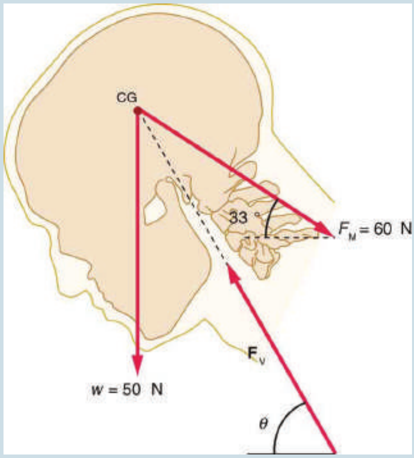

Figure 8.

6: We analyzed the biceps muscle example with the angle between forearm and upper arm set at 90°. Using the same numbers as in Example 1, find the force exerted by the biceps muscle when the angle is 120° and the forearm is in a downward position.

7: Even when the head is held erect, as in Figure 9, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?

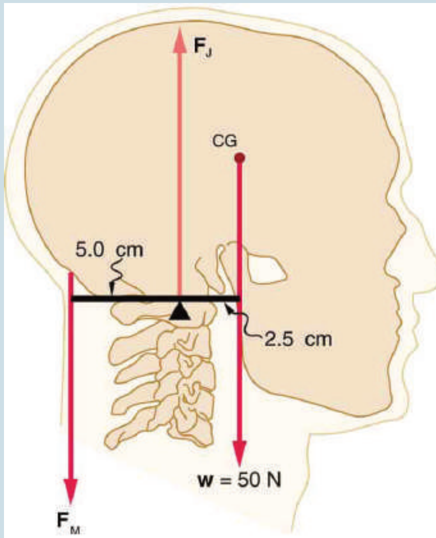

Figure 9. The center of mass of the head lies in front of its major point of support, requiring muscle action to hold the head erect. A simplified lever system is shown.

8: A 75-kg man stands on his toes by exerting an upward force through the Achilles tendon, as in Figure 10. (a) What is the force in the Achilles tendon if he stands on one foot? (b) Calculate the force at the pivot of the simplified lever system shown—that force is representative of forces in the ankle joint.

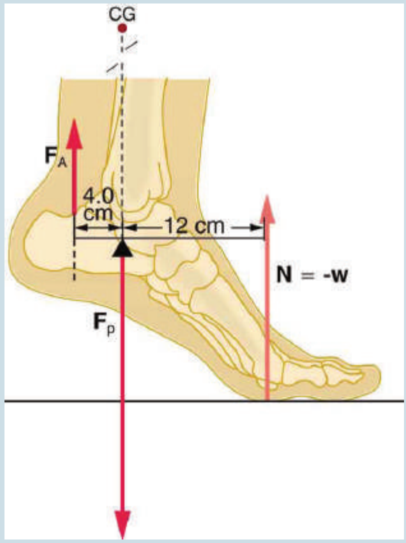

Figure 10. *The muscles in the back of the leg pull the Achilles tendon when one stands on one's toes. A simplified lever system is shown.*

9: A father lifts his child as shown in Figure 11. What force should the upper leg muscle exert to lift the child at a constant speed?

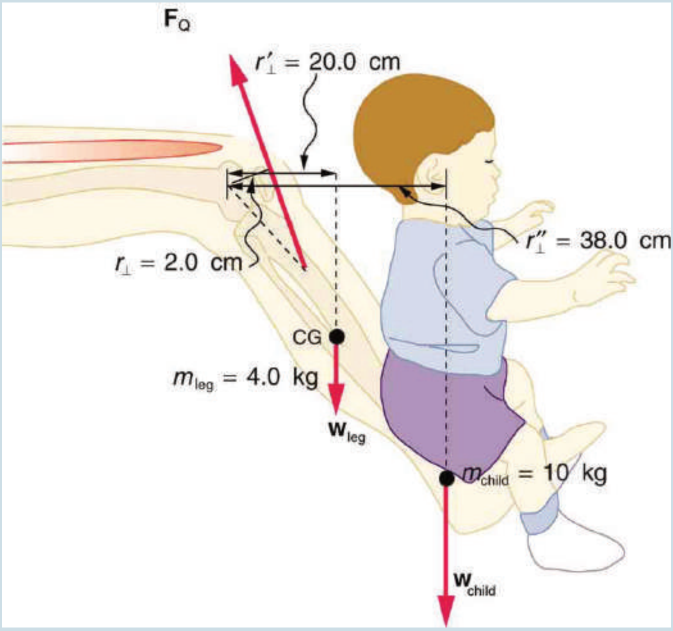

Figure 11. *A child being lifted by a father's lower leg.*

10: Unlike most of the other muscles in our bodies, the masseter muscle in the jaw, as illustrated in Figure 12, is attached relatively far from the joint, enabling large forces to be exerted by the back teeth. (a) Using the information in the figure, calculate the force exerted by the lower teeth on the bullet. (b) Calculate the force on the joint.

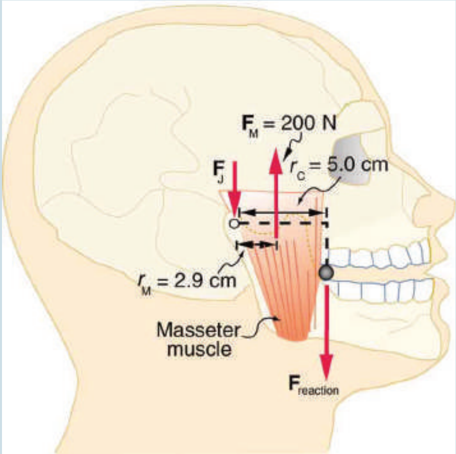

Figure 12. *A person clenching a bullet between his teeth.*

11: Integrated Concepts

Suppose we replace the 4.0-kg book in Exercise 6 of the biceps muscle with an elastic exercise rope that obeys Hooke's Law. Assume its force constant k=600 N/m. (a) How much is the rope stretched (past equilibrium) to provide the same force F_B as in this example? Assume the rope is held in the hand at the same location as the book. (b) What force is on the biceps muscle if the exercise rope is pulled straight up so that the forearm makes an angle of 25° with the horizontal? Assume the biceps muscle is still perpendicular to the forearm.

12: (a) What force should the woman in Figure 13 exert on the floor with each hand to do a push-up?

Assume that she moves up at a constant speed. (b) The triceps muscle at the back of her upper arm has an effective lever arm of 1.75 cm, and she exerts force on the floor at a horizontal distance of 20.0 cm from the elbow joint. Calculate the magnitude of the force in each triceps muscle, and compare it to her weight. (c) How much work does she do if her center of mass rises 0.240 m? (d) What is her useful power output if she does 25 pushups in one minute?

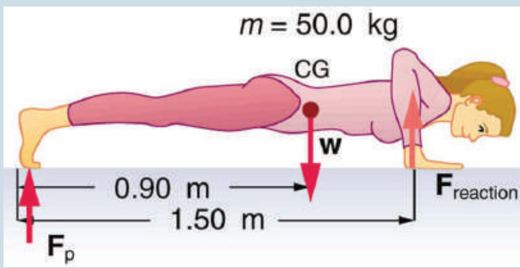

Figure 13. *A woman doing pushups.*

13: You have just planted a sturdy 2-m-tall palm tree in your front lawn for your mother's birthday. Your brother kicks a 500 g ball, which hits the top of the tree at a speed of 5 m/s and stays in contact with it for 10 ms. The ball falls to the ground near the base of the tree and the recoil of the tree is minimal. (a) What is the force on the tree? (b) The length of the sturdy section of the root is only 20 cm. Furthermore, the soil around the roots is loose and we can assume that an effective force is applied at the tip of the 20 cm length. What is the effective force exerted by the end of the tip of the root to keep the tree from toppling? Assume the tree will be uprooted rather than bend. (c) What could you have done to ensure that the tree does not uproot easily?

14: Unreasonable Results

Suppose two children are using a uniform seesaw that is 3.00 m long and has its center of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. (a) Calculate where the second 18.0 kg child must sit to balance the seesaw. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

15: Construct Your Own Problem

Consider a method for measuring the mass of a person's arm in anatomical studies. The subject lies on her back, extends her relaxed arm to the side and two scales are placed below the arm. One is placed under the elbow and the other under the back of her hand. Construct a problem in which you calculate the mass of the arm and find its center of mass based on the scale readings and the distances of the scales from the shoulder joint. You must include a free body diagram of the arm to direct the analysis. Consider changing the position of the scale under the hand to provide more information, if needed. You may wish to consult references to obtain reasonable mass values.

Problems & Exercises

1:

$$\begin{aligned} F_{\mathrm{B}} &= 470\ \mathrm{N}; r_1 = 4.00\ \mathrm{cm}; w_{\mathrm{a}} = 2.50\ \mathrm{kg}; r_2 = 16.0\ \mathrm{cm}; w_{\mathrm{b}} = 4.00\ \mathrm{kg}; r_3 = 38.0\ \mathrm{cm} \\ F_{\mathrm{E}} &= w_{\mathrm{a}}\left(\frac{r_2}{r_1} - 1\right) + w_{\mathrm{b}}\left(\frac{r_3}{r_1} - 1\right) \\ &= (2.50\ \mathrm{kg})(9.80\ \mathrm{m/s}^2)\left(\frac{16.0\ \mathrm{cm}}{4.0\ \mathrm{cm}} - 1\right) \\ &\quad + (4.00\ \mathrm{kg})(9.80\ \mathrm{m/s}^2)\left(\frac{38.0\ \mathrm{cm}}{4.00\ \mathrm{cm}} - 1\right) \\ &= 407\ \mathrm{N} \end{aligned}$$

3: $1.1 \times 10^3\ \mathrm{N}$, $\theta = 190^0$ ccw from positive x axis

5: $F_{\mathrm{V}} = 97\ \mathrm{N}, \theta = 59^0$

7: (a) $25\ \mathrm{N}$ downward (b) $75\ \mathrm{N}$ upward

8: (a) $F_{\mathrm{A}} = 2.21 \times 10^3\ \mathrm{N}$ upward (b) $F_{\mathrm{B}} = 2.94 \times 10^3\ \mathrm{N}$ downward

10: (a) $F_{\text{teeth on bullet}} = 1.2 \times 10^2\ \mathrm{N}$ upward (b) $F_{\mathrm{J}} = 84\ \mathrm{N}$ downward

12: (a) $147\ \mathrm{N}$ downward (b) $1680\ \mathrm{N}, 3.4$ times her weight (c) $118\ \mathrm{J}$ (d) $49.0\ \mathrm{W}$

14: (a) $\bar{x}_2 = 2.33\ \mathrm{m}$ (b) The seesaw is 3.0 m long, and hence, there is only 1.50 m of board on the other side of the pivot. The second child is off the board. (c) The position of the first child must be shortened, i.e. brought closer to the pivot.

PART IX

CHAPTER 9: MECHANICS OF HUMAN TISSUES

Chapter Objectives

After this chapter, you will be able to:

- Describe the anatomy of muscle-tendon complex.
- Describe the function and structure of muscle
- Explain how muscles work with tendons to move the body
- Describe how muscles contract and relax
- Explain how the nervous system controls muscle tension
- Define concentric action, eccentric action, electromechanical delay, fatigue, hyperplasia, hypertrophy, atrophy, isometric action, motor unit.
- Describe how the muscle-tendon complex acts together to create movement.
- List the factors that affect how much force the muscle-tendon complex can produce.
- Describe how the body moves as a series of lever systems.

64. 9.1 Overview of Muscle Tissues

Muscle is one of the four primary tissue types of the body, and the body contains three types of muscle tissue: skeletal muscle, cardiac muscle, and smooth muscle (Figure 1). All three muscle tissues have some properties in common; they all exhibit a quality called **excitability** as their plasma membranes can change their electrical states (from polarized to depolarized) and send an electrical wave called an action potential along the entire length of the membrane. Skeletal muscle completely depends on signaling from the nervous system to work properly. On the other hand, both cardiac muscle and smooth muscle can respond to other stimuli, such as hormones and local stimuli.

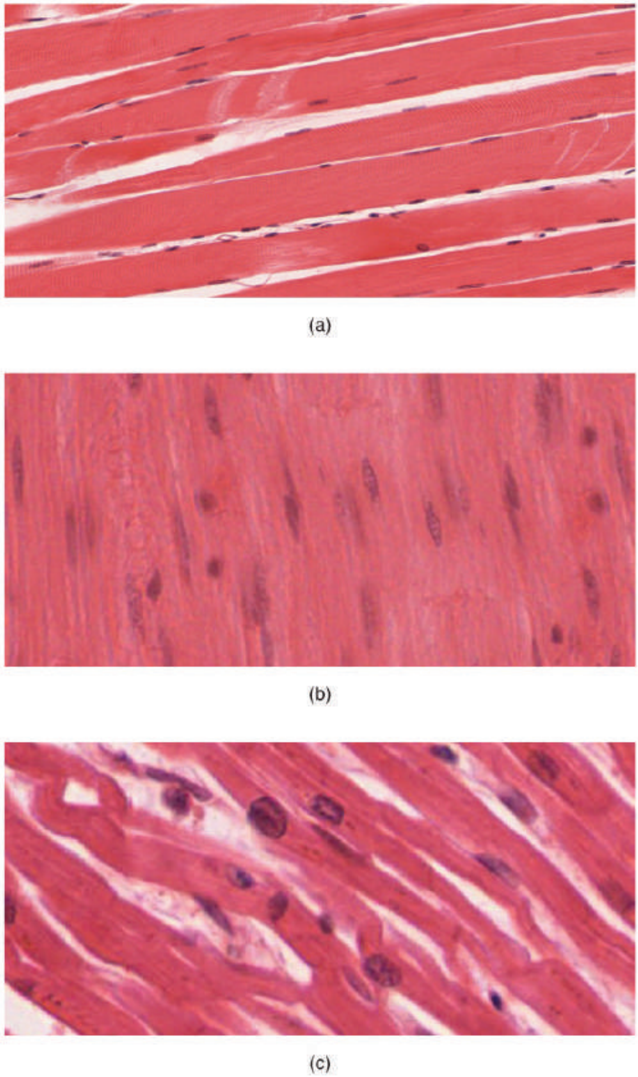

Figure 1. The Three Types of Muscle Tissue. The body contains three types of muscle tissue: (a) skeletal muscle, (b) smooth muscle, and (c) cardiac muscle. From top, LM × 1600, LM × 1600, LM × 1600. (Micrographs provided by the Regents of University of Michigan Medical School © 2012)

The muscles all begin the actual process of contracting (shortening) when a protein called actin is pulled by a protein called myosin. This occurs in striated muscle (skeletal and cardiac) after specific binding sites on the actin have been exposed in response to the interaction between calcium ions (Ca^{++}) and proteins (troponin and tropomyosin) that "shield" the actin-binding sites. Ca^{++} also is required for the contraction of smooth muscle, although its role is different: here Ca^{++} activates enzymes, which in turn activate myosin heads. All muscles require adenosine triphosphate (ATP) to continue the process of contracting, and they all relax when the Ca^{++} is removed and the actin-binding sites are re-shielded.

A muscle can return to its original length when relaxed due to a quality of muscle tissue called **elasticity**. It can recoil back to its original length due to elastic fibers. Muscle tissue also has the quality of **extensibility**; it can stretch or extend. **Contractility** allows muscle tissue to pull on its attachment points and shorten with force.

Skeletal muscles are made up of contractile proteins—actin and myosin. The actin and myosin proteins are arranged very regularly in the cytoplasm of individual muscle cells (referred to as fibers), which creates a pattern, or stripes, called striations. The striations are visible with a light microscope under high magnification (see Figure 1). **Skeletal muscle fibers are multinucleated structures that compose the skeletal muscle.**

Review Questions

1. Muscle that has a striped appearance is described as being ________.

A. elastic
B. nonstriated
C. excitable
D. striated

2. Which element is important in directly triggering contraction?

A. sodium (Na^+)
B. calcium (Ca^{++})
C. potassium (K^+)
D. chloride (Cl^-)

3. Which of the following properties is *not* common to all three muscle tissues?

A. excitability
B. the need for ATP
C. at rest, uses shielding proteins to cover actin-binding sites
D. elasticity

Critical Thinking Questions

1. Why is elasticity an important quality of muscle tissue?

Glossary

cardiac muscle
: striated muscle found in the heart; joined to one another at intercalated discs and under the regulation of pacemaker cells, which contract as one unit to pump blood through the circulatory system. Cardiac muscle is under involuntary control.

contractility
: ability to shorten (contract) forcibly

elasticity
: ability to stretch and rebound

excitability
: ability to undergo neural stimulation

extensibility
: ability to lengthen (extend)

skeletal muscle
: striated, multinucleated muscle that requires signaling from the nervous system to trigger contraction; most skeletal muscles are referred to as voluntary muscles that move bones and produce movement

smooth muscle
: nonstriated, mononucleated muscle in the skin that is associated with hair follicles; assists in moving materials in the walls of internal organs, blood vessels, and internal passageways

Solutions

Answers for Review Questions

1. D

2. B
3. C

Answers for Critical Thinking Questions

1. It allows muscle to return to its original length during relaxation after contraction.

65. 9.2 Skeletal Muscle

The best-known feature of skeletal muscle is its ability to contract and cause movement. Skeletal muscles act not only to produce movement but also to stop movement, such as resisting gravity to maintain posture. Small, constant adjustments of the skeletal muscles are needed to hold a body upright or balanced in any position. Muscles also prevent excess movement of the bones and joints, maintaining skeletal stability and preventing skeletal structure damage or deformation. Joints can become misaligned or dislocated entirely by pulling on the associated bones; muscles work to keep joints stable. Skeletal muscles are located throughout the body at the openings of internal tracts to control the movement of various substances. These muscles allow functions, such as swallowing, urination, and defecation, to be under voluntary control. Skeletal muscles also protect internal organs (particularly abdominal and pelvic organs) by acting as an external barrier or shield to external trauma and by supporting the weight of the organs.

Skeletal muscles contribute to the maintenance of homeostasis in the body by generating heat. Muscle contraction requires energy, and when ATP is broken down, heat is produced. This heat is very noticeable during exercise, when sustained muscle movement causes body temperature to rise, and in cases of extreme cold, when shivering produces random skeletal muscle contractions to generate heat.

Each skeletal muscle is an organ that consists of various integrated tissues. These tissues include the skeletal muscle fibers, blood vessels, nerve fibers, and connective tissue. Each skeletal muscle has three layers of connective tissue (called “mysia”) that enclose it and provide structure to the muscle as a whole, and also compartmentalize the muscle fibers within the muscle (Figure 1). Each muscle is wrapped in a sheath of dense, irregular connective tissue called the **epimysium**, which allows a muscle to contract and move powerfully while maintaining its structural integrity. The epimysium also separates muscle from other tissues and organs in the area, allowing the muscle to move independently.

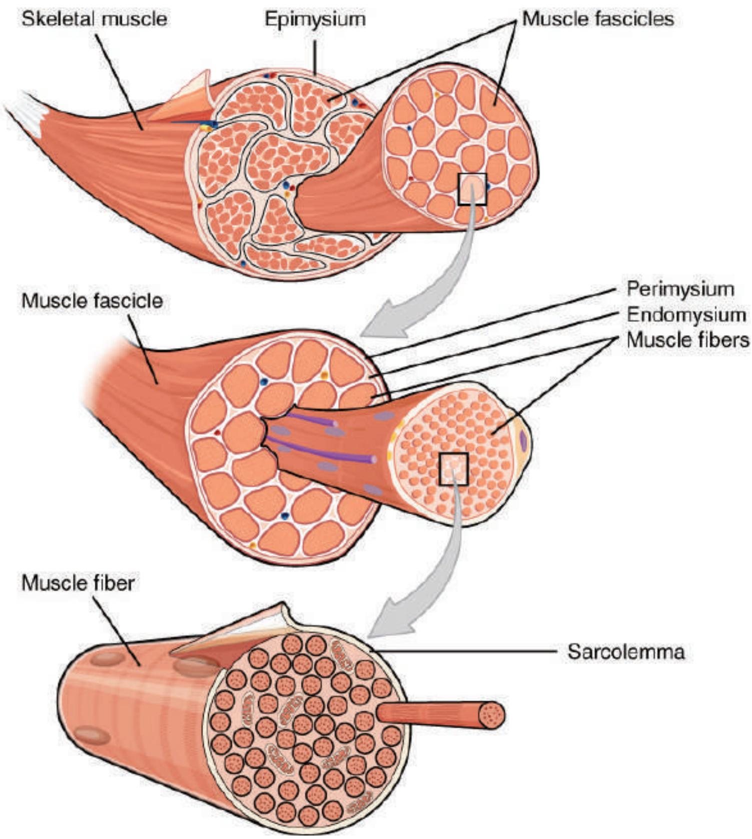

Figure 1. The Three Connective Tissue Layers. Bundles of muscle fibers, called fascicles, are covered by the perimysium. Muscle fibers are covered by the endomysium.

Inside each skeletal muscle, muscle fibers are organized into individual bundles, each called a **fascicle**, by a middle layer of connective tissue called the **perimysium**. This fascicular organization is common in muscles of the limbs; it allows the nervous system to trigger a specific movement of a muscle by activating a subset of muscle fibers within a bundle, or fascicle of the muscle. Inside each fascicle, each muscle fiber is encased in a thin connective tissue layer of collagen and reticular fibers called the **endomysium**. The endomysium contains the extracellular fluid and nutrients to support the muscle fiber. These nutrients are supplied via blood to the muscle tissue.

In skeletal muscles that work with tendons to pull on bones, the collagen in the three tissue layers (the mysia) intertwines with the collagen of a tendon. At the other end of the tendon, it fuses with the periosteum coating the bone. The tension created by contraction of the muscle fibers is then transferred though the mysia, to the tendon, and then to the periosteum to pull on the bone for movement of the skeleton. In other places, the mysia may fuse with a broad, tendon-like sheet called an **aponeurosis**, or to fascia, the connective tissue between skin and bones. The broad sheet of connective tissue in the lower back that the latissimus dorsi muscles (the "lats") fuse into is an example of an aponeurosis.

Every skeletal muscle is also richly supplied by blood vessels for nourishment, oxygen delivery, and waste removal. In addition, every muscle fiber in a skeletal muscle is supplied by the axon branch of a somatic motor neuron, which signals the fiber to contract. Unlike cardiac and smooth muscle, the only way to functionally contract a skeletal muscle is through signaling from the nervous system.

Skeletal Muscle Fibers

Because skeletal muscle cells are long and cylindrical, they are commonly referred to as muscle fibers. Skeletal muscle fibers can be quite large for human cells, with diameters up to 100 μm and lengths up to 30 cm (11.8 in) in the Sartorius of the upper leg. During early development, embryonic myoblasts, each with its own nucleus, fuse with up to hundreds of other myoblasts to form the multinucleated skeletal muscle fibers. Multiple nuclei mean multiple copies of genes, permitting the production of the large amounts of proteins and enzymes needed for muscle contraction.

Some other terminology associated with muscle fibers is rooted in the Greek *sarco*, which means “flesh.” The plasma membrane of muscle fibers is called the **sarcolemma**, the cytoplasm is referred to as **sarcoplasm**, and the specialized smooth endoplasmic reticulum, which stores, releases, and retrieves calcium ions (Ca^{++}) is called the **sarcoplasmic reticulum (SR)** (Figure 2). As will soon be described, the functional unit of a skeletal muscle fiber is the sarcomere, a highly organized arrangement of the contractile myofilaments **actin** (thin filament) and **myosin** (thick filament), along with other support proteins.

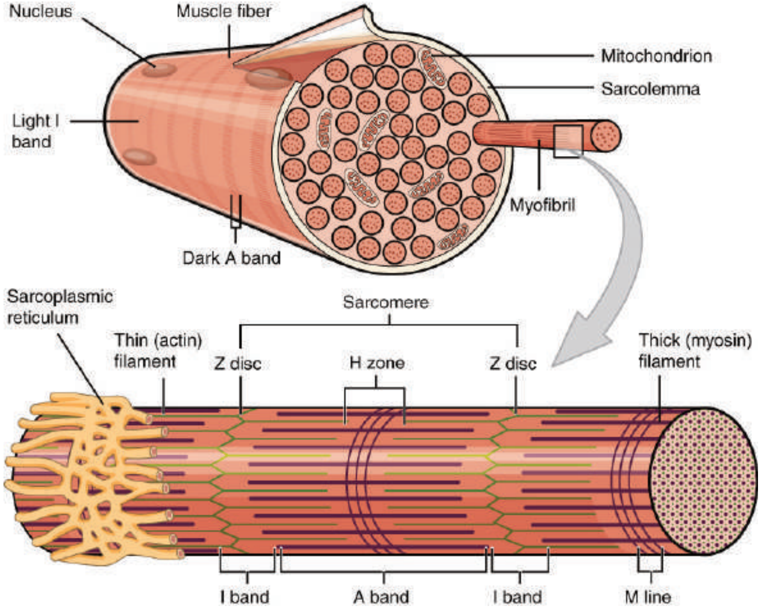

Figure 2. Muscle Fiber. A skeletal muscle fiber is surrounded by a plasma membrane called the sarcolemma, which contains sarcoplasm, the cytoplasm of muscle cells. A muscle fiber is composed of many fibrils, which give the cell its striated appearance.

The Sarcomere

The striated appearance of skeletal muscle fibers is due to the arrangement of the myofilaments of actin and myosin in sequential order from one end of the muscle fiber to the other. Each packet of these microfilaments and their regulatory proteins, **troponin** and **tropomyosin** (along with other proteins) is called a **sarcomere**.

Watch this video to learn more about macro- and microstructures of skeletal muscles.

Watch this video to learn more about macro- and microstructures of skeletal muscles. (a) What are the names of the "junction points" between sarcomeres? (b) What are the names of the "subunits" within the myofibrils that run the length of skeletal muscle fibers? (c) What is the "double strand of pearls" described in the video? (d) What gives a skeletal muscle fiber its striated appearance?

The sarcomere is the functional unit of the muscle fiber. The sarcomere itself is bundled within the myofibril that runs the entire length of the muscle fiber and attaches to the sarcolemma at its end. As myofibrils contract, the entire muscle cell contracts. Because myofibrils are only approximately 1.2 μm in diameter, hundreds to thousands (each with thousands of sarcomeres) can be found inside one muscle fiber. Each sarcomere is approximately 2 μm in length with a three-dimensional cylinder-like arrangement and is bordered by structures called Z-discs (also called Z-lines, because pictures are two-dimensional), to which the actin myofilaments are anchored (Figure 3). Because the actin and its troponin-tropomyosin complex (projecting from the Z-discs toward the center of the sarcomere) form strands that are thinner than the myosin, it is called the **thin filament** of the sarcomere. Likewise, because the myosin strands and their multiple heads (projecting from the center of the sarcomere, toward but not all to way to, the Z-discs) have more mass and are thicker, they are called the **thick filament** of the sarcomere.

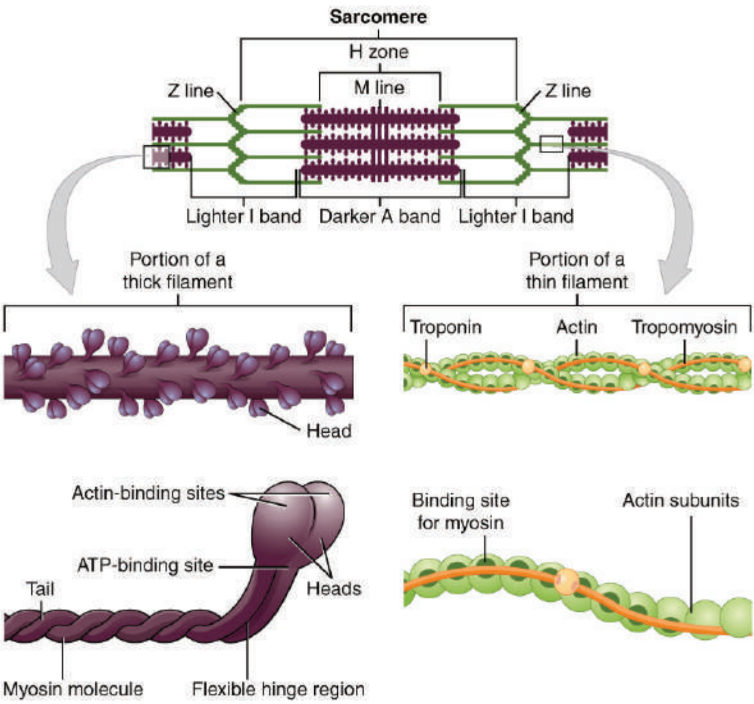

Figure 3. The Sarcomere. The sarcomere, the region from one Z-line to the next Z-line, is the functional unit of a skeletal muscle fiber.

The Neuromuscular Junction

Another specialization of the skeletal muscle is the site where a motor neuron's terminal meets the muscle fiber—called the **neuromuscular junction (NMJ)**. This is where the muscle fiber first responds to signaling by the motor neuron. Every skeletal muscle fiber in every skeletal muscle is innervated by a motor neuron at the NMJ. Excitation signals from the neuron are the only way to functionally activate the fiber to contract.

Watch this video to learn more about what happens at the NMJ.

Every skeletal muscle fiber is supplied by a motor neuron at the NMJ. Watch this video to learn more about what happens at the NMJ. (a) What is the definition of a motor unit? (b) What is the structural and functional difference between a large motor unit and a small motor unit? (c) Can you give an example of each? (d) Why is the neurotransmitter acetylcholine degraded after binding to its receptor?

Excitation-Contraction Coupling

All living cells have membrane potentials, or electrical gradients across their membranes. The inside of the membrane is usually around -60 to -90 mV, relative to the outside. This is referred to as a cell's membrane potential. Neurons and muscle cells can use their membrane potentials to generate electrical signals. They do this by controlling the movement of charged particles, called ions, across their membranes to create electrical currents. This is achieved by opening and closing specialized proteins in the membrane called ion channels. Although the currents generated by ions moving through these channel proteins are very small, they form the basis of both neural signaling and muscle contraction.

Both neurons and skeletal muscle cells are electrically excitable, meaning that they are able to generate action potentials. An action potential is a special type of electrical signal that can travel along a cell membrane as a wave. This allows a signal to be transmitted quickly and faithfully over long distances.

Although the term **excitation-contraction coupling** confuses or scares some students, it comes down to this: for a skeletal muscle fiber to contract, its membrane must first be "excited"—in other words, it must be stimulated to fire an action potential. The muscle fiber action potential, which sweeps along the sarcolemma as a wave, is "coupled" to the actual contraction through the release of calcium ions (Ca^{++}) from the SR. Once released, the Ca^{++} interacts with the shielding proteins, forcing them to move aside so that the actin-binding sites are available for attachment by myosin heads. The myosin then pulls the actin filaments toward the center, shortening the muscle fiber.

In skeletal muscle, this sequence begins with signals from the somatic motor division of the nervous system. In other words, the "excitation" step in skeletal muscles is always triggered by signaling from the nervous system (Figure 4).

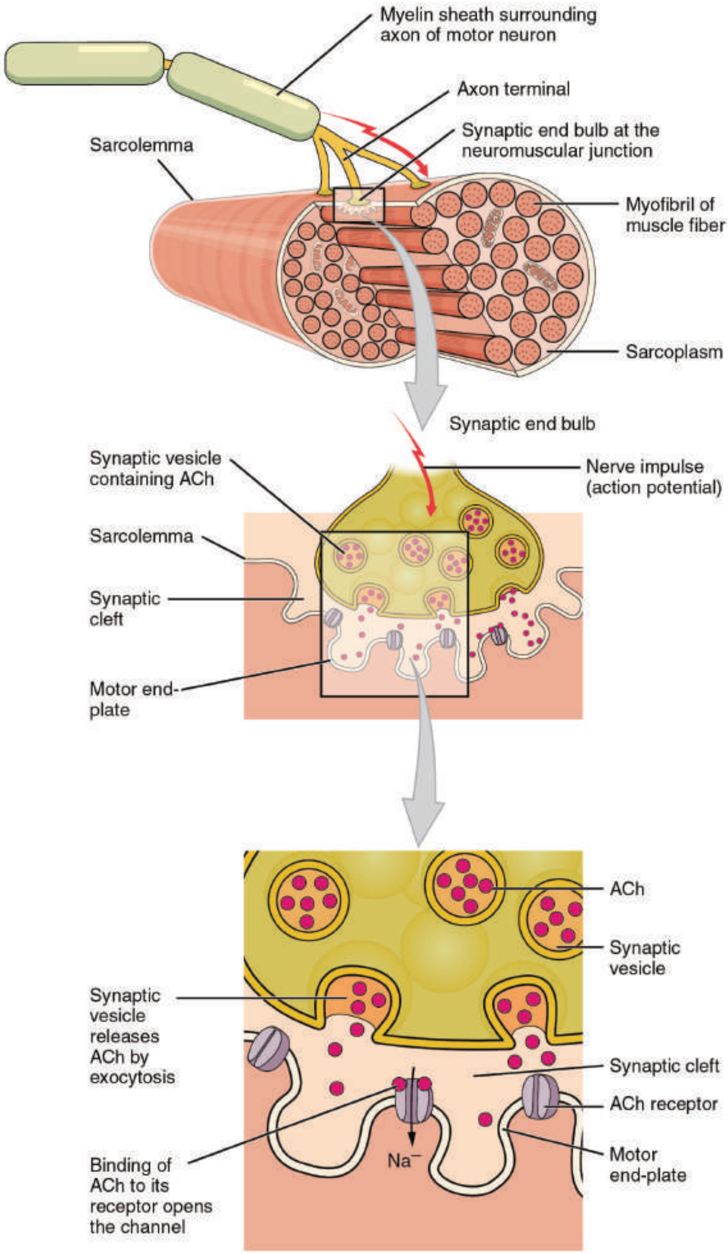

Figure 4. Motor End-Plate and Innervation. At the NMJ, the axon

terminal releases ACh. The motor end-plate is the location of the ACh-receptors in the muscle fiber sarcolemma. When ACh molecules are released, they diffuse across a minute space called the synaptic cleft and bind to the receptors.

The motor neurons that tell the skeletal muscle fibers to contract originate in the spinal cord, with a smaller number located in the brainstem for activation of skeletal muscles of the face, head, and neck. These neurons have long processes, called axons, which are specialized to transmit action potentials long distances— in this case, all the way from the spinal cord to the muscle itself (which may be up to three feet away). The axons of multiple neurons bundle together to form nerves, like wires bundled together in a cable.

Signaling begins when a neuronal **action potential** travels along the axon of a motor neuron, and then along the individual branches to terminate at the NMJ. At the NMJ, the axon terminal releases a chemical messenger, or **neurotransmitter**, called **acetylcholine (ACh)**. The ACh molecules diffuse across a minute space called the **synaptic cleft** and bind to ACh receptors located within the **motor end-plate** of the sarcolemma on the other side of the synapse. Once ACh binds, a channel in the ACh receptor opens and positively charged ions can pass through into the muscle fiber, causing it to **depolarize**, meaning that the membrane potential of the muscle fiber becomes less negative (closer to zero.)

As the membrane depolarizes, another set of ion channels called **voltage-gated sodium channels** are triggered to open. Sodium ions enter the muscle fiber, and an action potential rapidly spreads (or “fires”) along the entire membrane to initiate excitation-contraction coupling.

Things happen very quickly in the world of excitable membranes (just think about how quickly you can snap your fingers as soon as you decide to do it). Immediately following depolarization of the membrane, it repolarizes, re-establishing the negative membrane potential. Meanwhile, the ACh in the synaptic cleft is degraded by the enzyme acetylcholinesterase (AChE) so that the ACh cannot rebind to a receptor and reopen its channel, which would cause unwanted extended muscle excitation and contraction.

Propagation of an action potential along the sarcolemma is the excitation portion of excitation-contraction coupling. Recall that this excitation actually triggers the release of calcium ions (Ca^{++}) from its storage in the cell's SR. For the action potential to reach the membrane of the SR, there are periodic invaginations in the sarcolemma, called **T-tubules** ("T" stands for "transverse"). You will recall that the diameter of a muscle fiber can be up to 100 μm, so these T-tubules ensure that the membrane can get close to the SR in the sarcoplasm. The arrangement of a T-tubule with the membranes of SR on either side is called a **triad** (Figure 5). The triad surrounds the cylindrical structure called a **myofibril**, which contains actin and myosin.

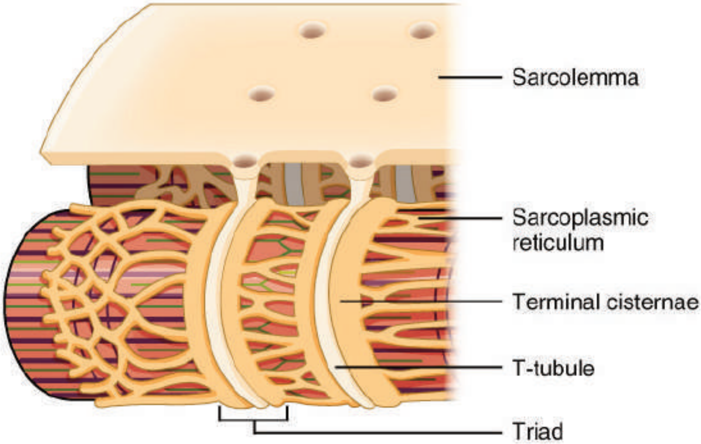

Figure 5. The T-tubule. Narrow T-tubules permit the conduction of electrical impulses. The SR functions to regulate intracellular levels of calcium. Two terminal cisternae (where enlarged SR connects to the T-tubule) and one T-tubule comprise a triad—a "threesome" of membranes, with those of SR on two sides and the T-tubule sandwiched between them.

The T-tubules carry the action potential into the interior of the cell, which triggers the opening of calcium channels in the membrane of the adjacent SR, causing Ca^{++} to diffuse out of the SR and into the sarcoplasm. It is the arrival of Ca^{++} in the sarcoplasm that initiates contraction of the muscle fiber by its contractile units, or sarcomeres.

Chapter Review

Skeletal muscles contain connective tissue, blood vessels, and nerves. There are three layers of connective tissue: epimysium, perimysium, and endomysium. Skeletal muscle fibers are organized into groups called fascicles. Blood vessels and nerves enter the connective tissue and branch in the cell. Muscles attach to bones directly or through tendons or aponeuroses. Skeletal muscles maintain posture, stabilize bones and joints, control internal movement, and generate heat.

Skeletal muscle fibers are long, multinucleated cells. The membrane of the cell is the sarcolemma; the cytoplasm of the cell is the sarcoplasm. The sarcoplasmic reticulum (SR) is a form of endoplasmic reticulum. Muscle fibers are composed of myofibrils. The striations are created by the organization of actin and myosin resulting in the banding pattern of myofibrils.

Interactive Link Questions

Watch this video to learn more about macro- and microstructures of skeletal muscles. (a) What are the names of the “junction points” between sarcomeres? (b) What are the names of the “subunits” within the myofibrils that run the length of skeletal muscle fibers? (c) What is the “double strand of pearls” described in the video? (d) What gives a skeletal muscle fiber its striated appearance?

(a) Z-lines. (b) Sarcomeres. (c) This is the arrangement of the actin and myosin filaments in a sarcomere. (d) The alternating strands of actin and myosin filaments.

Every skeletal muscle fiber is supplied by a motor neuron at the NMJ. Watch this video to learn more about what happens at the neuromuscular junction. (a) What is the definition of a motor unit? (b) What is the structural and functional difference between a large motor unit and a small motor unit? Can you give an example of each? (c) Why is the neurotransmitter acetylcholine degraded after binding to its receptor?

(a) It is the number of skeletal muscle fibers supplied by a single motor neuron. (b) A large motor unit has one neuron supplying many skeletal muscle fibers for gross movements, like the Temporalis muscle, where 1000 fibers are supplied by one neuron. A small motor has one neuron supplying few skeletal muscle fibers for very fine movements, like the extraocular eye muscles, where six fibers are supplied by one neuron. (c) To avoid prolongation of muscle contraction.

Review Questions

1. The correct order for the smallest to the largest unit of organization in muscle tissue is ________.

A. fascicle, filament, muscle fiber, myofibril
B. filament, myofibril, muscle fiber, fascicle
C. muscle fiber, fascicle, filament, myofibril
D. myofibril, muscle fiber, filament, fascicle

2. Depolarization of the sarcolemma means ________.

A. the inside of the membrane has become less negative as sodium ions accumulate
B. the outside of the membrane has become less negative as sodium ions accumulate
C. the inside of the membrane has become more negative as sodium ions accumulate
D. the sarcolemma has completely lost any electrical charge

Critical Thinking Questions

1. What would happen to skeletal muscle if the epimysium were destroyed?

2. Describe how tendons facilitate body movement.

3. What are the five primary functions of skeletal muscle?

4. What are the opposite roles of voltage-gated sodium channels and voltage-gated potassium channels?

Glossary

acetylcholine (ACh)
: neurotransmitter that binds at a motor end-plate to trigger depolarization

actin
: protein that makes up most of the thin myofilaments in a sarcomere muscle fiber

action potential
: change in voltage of a cell membrane in response to a stimulus that results in transmission of an electrical signal; unique to neurons and muscle fibers

aponeurosis
: broad, tendon-like sheet of connective tissue that attaches a skeletal muscle to another skeletal muscle or to a bone

depolarize
: to reduce the voltage difference between the inside and outside of a cell's plasma membrane (the sarcolemma for

a muscle fiber), making the inside less negative than at rest

endomysium
loose, and well-hydrated connective tissue covering each muscle fiber in a skeletal muscle

epimysium
outer layer of connective tissue around a skeletal muscle

excitation-contraction coupling
sequence of events from motor neuron signaling to a skeletal muscle fiber to contraction of the fiber's sarcomeres

fascicle
bundle of muscle fibers within a skeletal muscle

motor end-plate
sarcolemma of muscle fiber at the neuromuscular junction, with receptors for the neurotransmitter acetylcholine

myofibril
long, cylindrical organelle that runs parallel within the muscle fiber and contains the sarcomeres

myosin
protein that makes up most of the thick cylindrical myofilament within a sarcomere muscle fiber

neuromuscular junction (NMJ)
synapse between the axon terminal of a motor neuron and the section of the membrane of a muscle fiber with receptors for the acetylcholine released by the terminal

neurotransmitter
signaling chemical released by nerve terminals that bind to and activate receptors on target cells

perimysium
connective tissue that bundles skeletal muscle fibers into fascicles within a skeletal muscle

sarcomere
longitudinally, repeating functional unit of skeletal muscle,

with all of the contractile and associated proteins involved in contraction

sarcolemma
plasma membrane of a skeletal muscle fiber

sarcoplasm
cytoplasm of a muscle cell

sarcoplasmic reticulum (SR)
specialized smooth endoplasmic reticulum, which stores, releases, and retrieves Ca^{++}

synaptic cleft
space between a nerve (axon) terminal and a motor end-plate

T-tubule
projection of the sarcolemma into the interior of the cell

thick filament
the thick myosin strands and their multiple heads projecting from the center of the sarcomere toward, but not all to way to, the Z-discs

thin filament
thin strands of actin and its troponin-tropomyosin complex projecting from the Z-discs toward the center of the sarcomere

triad
the grouping of one T-tubule and two terminal cisternae

troponin
regulatory protein that binds to actin, tropomyosin, and calcium

tropomyosin
regulatory protein that covers myosin-binding sites to prevent actin from binding to myosin

voltage-gated sodium channels
membrane proteins that open sodium channels in response to a sufficient voltage change, and initiate and transmit the action potential as Na^{+} enters through the channel

Solutions

Answers for Review Questions

1. B
2. A

Answers for Critical Thinking Questions

1. Muscles would lose their integrity during powerful movements, resulting in muscle damage.
2. When a muscle contracts, the force of movement is transmitted through the tendon, which pulls on the bone to produce skeletal movement.
3. Produce movement of the skeleton, maintain posture and body position, support soft tissues, encircle openings of the digestive, urinary, and other tracts, and maintain body temperature.
4. The opening of voltage-gated sodium channels, followed by the influx of Na^+, transmits an Action Potential after the membrane has sufficiently depolarized. The delayed opening of potassium channels allows K^+ to exit the cell, to repolarize the membrane.

66. 9.3 Muscle Fiber Contraction and Relaxation

The sequence of events that result in the contraction of an individual muscle fiber begins with a signal—the neurotransmitter, ACh—from the motor neuron innervating that fiber. The local membrane of the fiber will depolarize as positively charged sodium ions (Na^+) enter, triggering an action potential that spreads to the rest of the membrane will depolarize, including the T-tubules. This triggers the release of calcium ions (Ca^{++}) from storage in the sarcoplasmic reticulum (SR). The Ca^{++} then initiates contraction, which is sustained by ATP (Figure 1). As long as Ca^{++} ions remain in the sarcoplasm to bind to troponin, which keeps the actin-binding sites "unshielded," and as long as ATP is available to drive the cross-bridge cycling and the pulling of actin strands by myosin, the muscle fiber will continue to shorten to an anatomical limit.

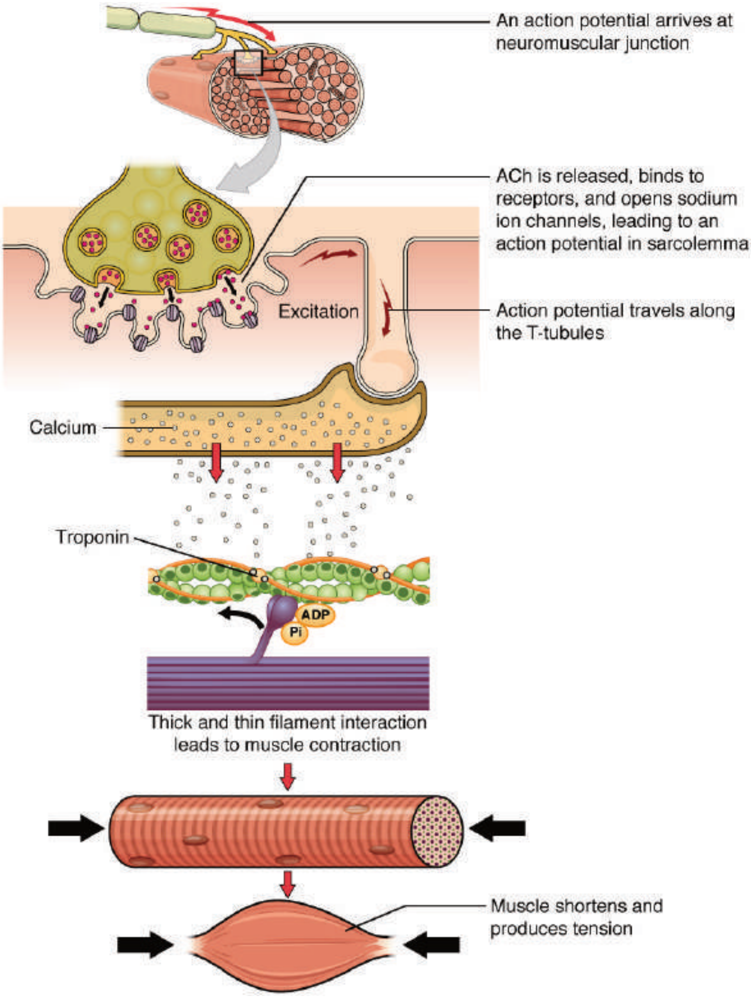

Figure 1. Contraction of a Muscle Fiber. A cross-bridge forms between actin and the myosin heads triggering contraction. As long as Ca++ ions remain in the sarcoplasm to bind to troponin, and as long as ATP is available, the muscle fiber will continue to shorten.

Muscle contraction usually stops when signaling from the motor neuron ends, which repolarizes the sarcolemma and T-tubules, and closes the voltage-gated calcium channels in the SR. Ca^{++} ions are then pumped back into the SR, which causes the tropomyosin to reshield (or re-cover) the binding sites on the actin strands. A muscle also can stop contracting when it runs out of ATP and becomes fatigued (Figure 2).

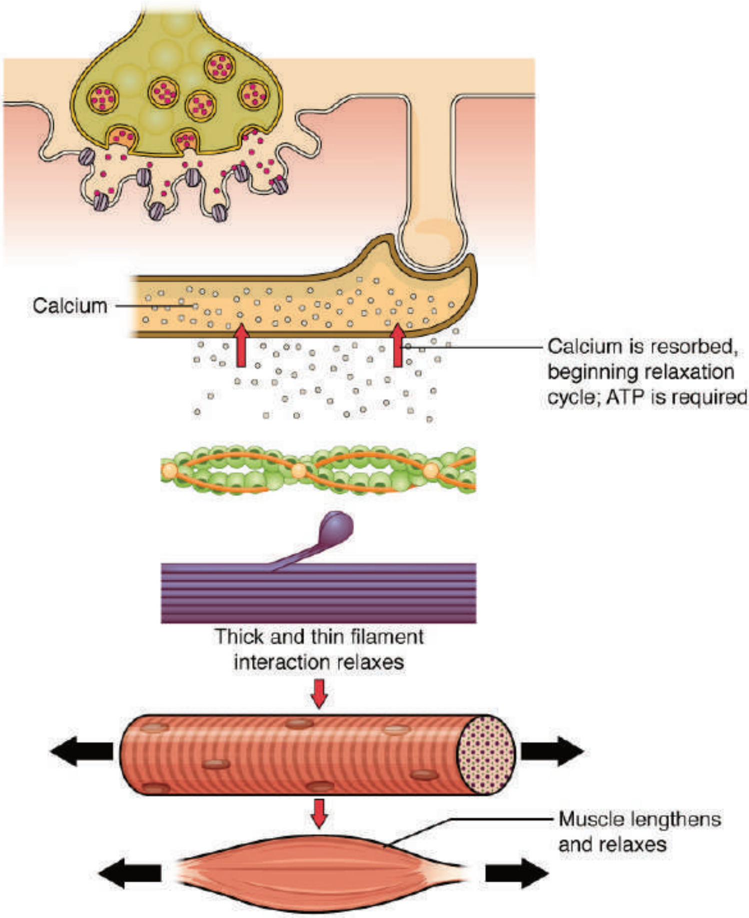

Figure 2. Relaxation of a Muscle Fiber. Ca++ ions are pumped back into the SR, which causes the tropomyosin to reshield the binding sites on the actin strands. A muscle may also stop contracting when it runs out of ATP and becomes fatigued.

Watch this video to learn more about the role of calcium.

The release of calcium ions initiates muscle contractions. Watch this video to learn more about the role of calcium. (a) What are "T-tubules" and what is their role? (b) Please describe how actin-binding sites are made available for cross-bridging with myosin heads during contraction.

The molecular events of muscle fiber shortening occur within the fiber's sarcomeres (see Figure 3). The contraction of a striated muscle fiber occurs as the sarcomeres, linearly arranged within myofibrils, shorten as myosin heads pull on the actin filaments.

The region where thick and thin filaments overlap has a dense appearance, as there is little space between the filaments. This zone where thin and thick filaments overlap is very important to muscle contraction, as it is the site where filament movement starts. Thin filaments, anchored at their ends by the Z-discs, do not extend completely into the central region that only contains thick filaments, anchored at their bases at a spot called the M-line. A myofibril is composed of many sarcomeres running along its length; thus, myofibrils and muscle cells contract as the sarcomeres contract.

The Sliding Filament Model of Contraction

When signaled by a motor neuron, a skeletal muscle fiber contracts as the thin filaments are pulled and then slide past the thick filaments within the fiber's sarcomeres. This process is known as the sliding filament model of muscle contraction (Figure 3). The sliding can only occur when myosin-binding sites on the actin filaments are exposed by a series of steps that begins with Ca^{++} entry into the sarcoplasm.

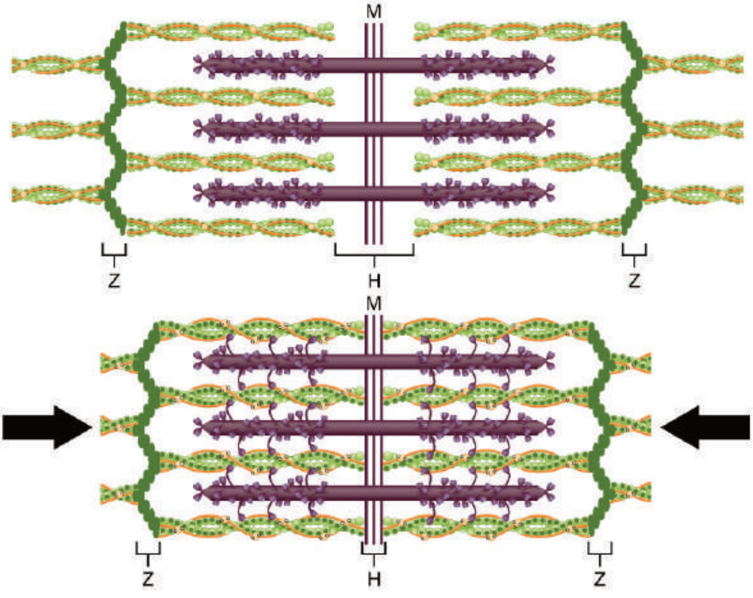

Figure 3. The Sliding Filament Model of Muscle Contraction. When a sarcomere contracts, the Z lines move closer together, and the I band becomes smaller. The A band stays the same width. At full contraction, the thin and thick filaments overlap.

Tropomyosin is a protein that winds around the chains of the actin filament and covers the myosin-binding sites to prevent actin from binding to myosin. Tropomyosin binds to troponin to form a troponin-tropomyosin complex. The troponin-tropomyosin complex prevents the myosin "heads" from binding to the active sites on the actin microfilaments. Troponin also has a binding site for Ca^{++} ions.

To initiate muscle contraction, tropomyosin has to expose the myosin-binding site on an actin filament to allow cross-bridge formation between the actin and myosin microfilaments. The first step in the process of contraction is for Ca^{++} to bind to troponin so that tropomyosin can slide away from the binding sites on the actin strands. This allows the myosin heads to bind to these exposed binding sites and form cross-bridges. The thin filaments are then pulled by the myosin heads to slide past the thick filaments toward the center of the sarcomere. But each head can only pull a very short distance before it has reached its limit and must be "re-cocked" before it can pull again, a step that requires ATP.

ATP and Muscle Contraction

For thin filaments to continue to slide past thick filaments during muscle contraction, myosin heads must pull the actin at the binding sites, detach, re-cock, attach to more binding sites, pull, detach, re-cock, etc. This repeated movement is known as the cross-bridge cycle. This motion of the myosin heads is similar to the oars when an individual rows a boat: The paddle of the oars (the myosin heads) pull, are lifted from the water (detach), repositioned (re-cocked) and then immersed again to pull (Figure 4). Each cycle requires energy, and the action of the myosin heads in the sarcomeres repetitively pulling on the thin filaments also requires energy, which is provided by ATP.

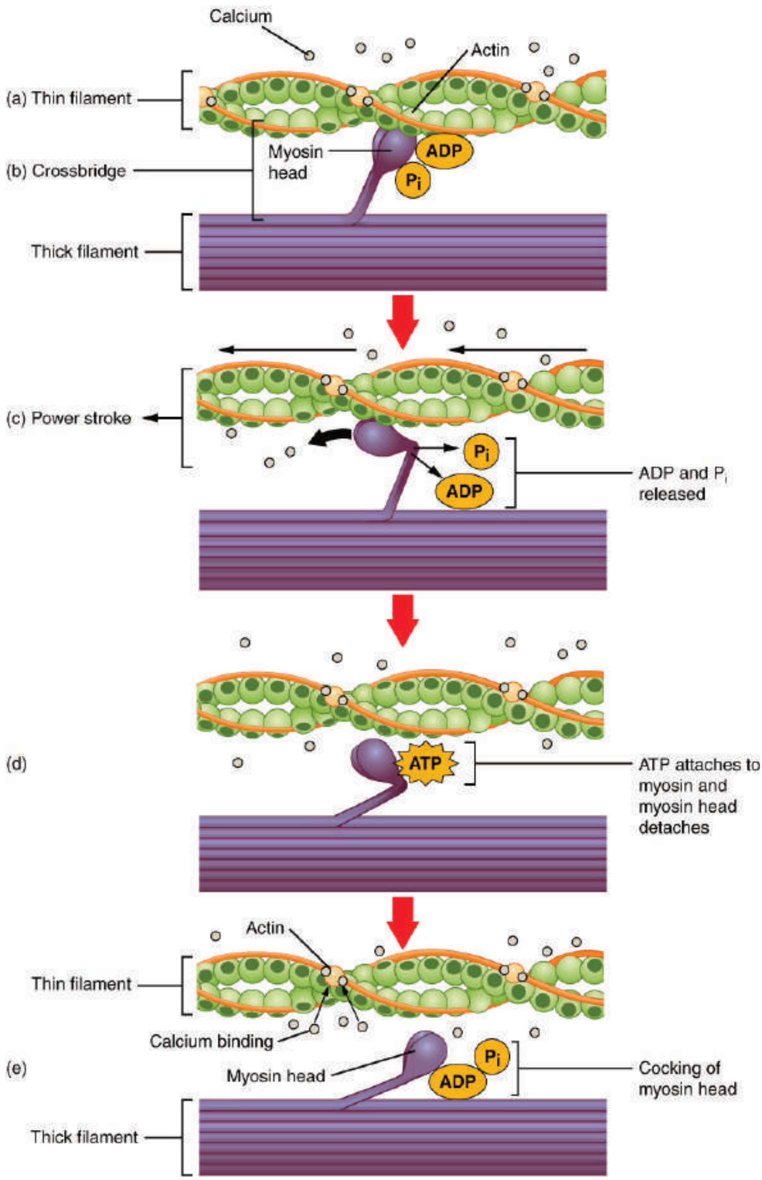

Figure 4. Skeletal Muscle Contraction. (a) The active site on actin is exposed as calcium binds to troponin. (b) The myosin head is attracted to actin, and myosin binds actin at its actin-binding site, forming the cross-bridge. (c) During the power stroke, the phosphate generated in the previous contraction cycle is released. This results in

the myosin head pivoting toward the center of the sarcomere, after which the attached ADP and phosphate group are released. (d) A new molecule of ATP attaches to the myosin head, causing the cross-bridge to detach. (e) The myosin head hydrolyzes ATP to ADP and phosphate, which returns the myosin to the cocked position.

Cross-bridge formation occurs when the myosin head attaches to the actin while adenosine diphosphate (ADP) and inorganic phosphate (P_i) are still bound to myosin (Figure 4**a,b**). P_i is then released, causing myosin to form a stronger attachment to the actin, after which the myosin head moves toward the M-line, pulling the actin along with it. As actin is pulled, the filaments move approximately 10 nm toward the M-line. This movement is called the **power stroke**, as movement of the thin filament occurs at this step (Figure 4**c**). In the absence of ATP, the myosin head will not detach from actin.

One part of the myosin head attaches to the binding site on the actin, but the head has another binding site for ATP. ATP binding causes the myosin head to detach from the actin (Figure 4**d**). After this occurs, ATP is converted to ADP and P_i by the intrinsic **ATPase** activity of myosin. The energy released during ATP hydrolysis changes the angle of the myosin head into a cocked position (Figure 4**e**). The myosin head is now in position for further movement.

When the myosin head is cocked, myosin is in a high-energy configuration. This energy is expended as the myosin head moves through the power stroke, and at the end of the power stroke, the myosin head is in a low-energy position. After the power stroke, ADP is released; however, the formed cross-bridge is still in place, and actin and myosin are bound together. As long as ATP is available, it readily attaches to myosin, the cross-bridge cycle can recur, and muscle contraction can continue.

Note that each thick filament of roughly 300 myosin molecules has multiple myosin heads, and many cross-bridges form and break continuously during muscle contraction. Multiply this by all of the sarcomeres in one myofibril, all the myofibrils in one muscle fiber, and all of the muscle fibers in one skeletal muscle, and you can understand why so much energy (ATP) is needed to keep skeletal muscles working. In fact, it is the loss of ATP that results in the rigor mortis observed soon after someone dies. With no further ATP production possible, there is no ATP available for myosin heads to detach from the actin-binding sites, so the cross-bridges stay in place, causing the rigidity in the skeletal muscles.

Sources of ATP

ATP supplies the energy for muscle contraction to take place. In addition to its direct role in the cross-bridge cycle, ATP also provides the energy for the active-transport Ca^{++} pumps in the SR. Muscle contraction does not occur without sufficient amounts of ATP. The amount of ATP stored in muscle is very low, only sufficient to power a few seconds worth of contractions. As it is broken down, ATP must therefore be regenerated and replaced quickly to allow for sustained contraction. There are three mechanisms by which ATP can be regenerated: creatine phosphate metabolism, anaerobic glycolysis, fermentation and aerobic respiration.

Creatine phosphate is a molecule that can store energy in its phosphate bonds. In a resting muscle, excess ATP transfers its energy to creatine, producing ADP and creatine phosphate. This acts as an energy reserve that can be used to quickly create more ATP. When the muscle starts to contract and needs energy, creatine phosphate transfers its phosphate back to ADP to form ATP and creatine. This reaction is catalyzed by the enzyme creatine kinase and occurs very quickly; thus, creatine phosphate-derived ATP powers the first few seconds of muscle contraction. However, creatine phosphate can only provide approximately 15 seconds worth of energy, at which point another energy source has to be used (Figure 5).

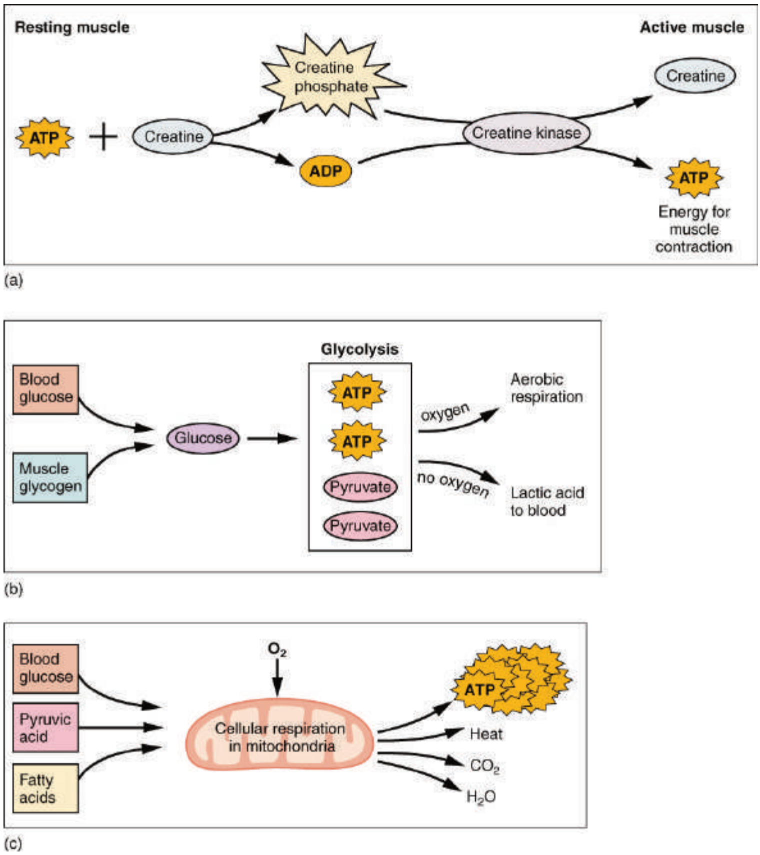

Figure 5. Muscle Metabolism. (a) Some ATP is stored in a resting muscle. As contraction starts, it is used up in seconds. More ATP is generated from creatine phosphate for about 15 seconds. (b) Each glucose molecule produces two ATP and two molecules of pyruvic acid, which can be used in aerobic respiration or converted to lactic acid. If oxygen is not available, pyruvic acid is converted to lactic acid, which may contribute to muscle fatigue. This occurs during strenuous exercise when high amounts of energy are needed but oxygen cannot be sufficiently delivered to muscle. (c) Aerobic respiration is the breakdown of glucose in the presence of oxygen (O2) to produce carbon dioxide, water, and ATP. Approximately 95 percent of the ATP required for resting or moderately active muscles is provided by aerobic respiration, which takes place in mitochondria.

As the ATP produced by creatine phosphate is depleted, muscles turn to glycolysis as an ATP source. **Glycolysis** is an anaerobic (non-oxygen-dependent) process that breaks down glucose (sugar) to produce ATP; however, glycolysis cannot generate ATP as quickly as creatine phosphate. Thus, the switch to glycolysis results in a slower rate of ATP availability to the muscle. The sugar used in glycolysis can be provided by blood glucose or by metabolizing glycogen that is stored in the muscle. The breakdown of one glucose molecule produces two ATP and two molecules of **pyruvic acid**, which can be used in aerobic respiration or when oxygen levels are low, converted to lactic acid (Figure 5**b**).

If oxygen is available, pyruvic acid is used in aerobic respiration. However, if oxygen is not available, pyruvic acid is converted to **lactic acid**, which may contribute to muscle fatigue. This conversion allows the recycling of the enzyme NAD^+ from NADH, which is needed for glycolysis to continue. This occurs during strenuous exercise when high amounts of energy are needed but oxygen cannot be sufficiently delivered to muscle. Glycolysis itself cannot be sustained for very long (approximately 1 minute of muscle activity), but it is useful in facilitating short bursts of high-intensity output. This is because glycolysis does not utilize glucose very efficiently, producing a net gain of two ATPs per molecule of glucose, and the end product of lactic acid, which may contribute to muscle fatigue as it accumulates.

Aerobic respiration is the breakdown of glucose or other nutrients in the presence of oxygen (O_2) to produce carbon dioxide, water, and ATP. Approximately 95 percent of the ATP required for resting or moderately active muscles is provided by aerobic respiration, which takes place in mitochondria. The inputs for aerobic respiration include glucose circulating in the bloodstream, pyruvic acid, and fatty acids. Aerobic respiration is much more efficient than anaerobic glycolysis, producing approximately 36 ATPs per molecule of glucose versus four from glycolysis. However, aerobic respiration cannot be sustained without a steady supply of O_2 to the skeletal muscle and is much slower (Figure 5**c**). To compensate, muscles store small amount of excess oxygen in proteins call myoglobin, allowing for more efficient muscle contractions and less fatigue. Aerobic training also increases the efficiency of the circulatory system so that O_2 can be supplied to the muscles for longer periods of time.

Muscle fatigue occurs when a muscle can no longer contract in response to signals from the nervous system. The exact causes of muscle fatigue are not fully known, although certain factors have been correlated with the decreased muscle contraction that occurs during fatigue. ATP is needed for normal muscle contraction, and as ATP reserves are reduced, muscle function may decline. This may be more of a factor in brief, intense muscle output rather than sustained, lower intensity efforts. Lactic acid buildup may lower intracellular pH, affecting enzyme and protein activity. Imbalances in Na^+ and K^+ levels as a result of membrane depolarization may disrupt Ca^{++} flow out of the SR. Long periods of sustained exercise may damage the SR and the sarcolemma, resulting in impaired Ca^{++} regulation.

Intense muscle activity results in an **oxygen debt**, which is the amount of oxygen needed to compensate for ATP produced without oxygen during muscle contraction. Oxygen is required to restore ATP and creatine phosphate levels, convert lactic acid to pyruvic acid, and, in the liver, to convert lactic acid into glucose or glycogen. Other systems used during exercise also require oxygen, and all of these combined processes result in the increased breathing rate that occurs after exercise. Until the oxygen debt has been met, oxygen intake is elevated, even after exercise has stopped.

Relaxation of a Skeletal Muscle

Relaxing skeletal muscle fibers, and ultimately, the skeletal muscle, begins with the motor neuron, which stops releasing its chemical signal, ACh, into the synapse at the NMJ. The muscle fiber will repolarize, which closes the gates in the SR where Ca^{++} was being released. ATP-driven pumps will move Ca^{++} out of the sarcoplasm back into the SR. This results in the "reshielding" of the actin-binding sites on the thin filaments. Without the ability to form cross-bridges between the thin and thick filaments, the muscle fiber loses its tension and relaxes.

Muscle Strength

The number of skeletal muscle fibers in a given muscle is genetically determined and does not change. Muscle strength is directly related to the amount of myofibrils and sarcomeres within each fiber. Factors, such as hormones and stress (and artificial anabolic steroids), acting on the muscle can increase the production of sarcomeres and myofibrils within the muscle fibers, a change called hypertrophy, which results in the increased mass and bulk in a skeletal muscle. Likewise, decreased use of a skeletal muscle results in atrophy, where the number of sarcomeres and myofibrils disappear (but not the number of muscle fibers). It is common for a limb in a cast to show atrophied muscles when the cast is removed, and certain diseases, such as polio, show atrophied muscles.
Disorders of the ...

Muscular System

Duchenne muscular dystrophy (DMD) is a progressive weakening of the skeletal muscles. It is one of several diseases collectively referred to as "muscular dystrophy." DMD is caused by a lack of the protein dystrophin, which helps the thin filaments of myofibrils bind to the sarcolemma. Without sufficient dystrophin, muscle contractions cause the sarcolemma to tear, causing an influx of Ca^{++}, leading to cellular damage and muscle fiber degradation. Over time, as muscle damage accumulates, muscle mass is lost, and greater functional impairments develop.

DMD is an inherited disorder caused by an abnormal X chromosome. It primarily affects males, and it is usually diagnosed in early childhood. DMD usually first appears as difficulty with balance and motion, and then progresses to an inability to walk. It continues progressing upward in the body from the lower extremities to the upper body, where it affects the muscles responsible for breathing and circulation. It ultimately causes death due to respiratory failure, and those afflicted do not usually live past their 20s.

Because DMD is caused by a mutation in the gene that codes for dystrophin, it was thought that introducing healthy myoblasts into patients might be an effective treatment. Myoblasts are the embryonic cells responsible for muscle development, and ideally, they would carry healthy genes that could produce the dystrophin needed for normal muscle contraction. This approach has been largely unsuccessful in humans. A recent approach has involved attempting to boost the muscle's production of utrophin, a protein similar to dystrophin that may be able to assume the role of dystrophin and prevent cellular damage from occurring.

Chapter Review

A sarcomere is the smallest contractile portion of a muscle. Myofibrils are composed of thick and thin filaments. Thick filaments are composed of the protein myosin; thin filaments are composed of the protein actin. Troponin and tropomyosin are regulatory proteins.

Muscle contraction is described by the sliding filament model of contraction. ACh is the neurotransmitter that binds at the neuromuscular junction (NMJ) to trigger depolarization, and an action potential travels along the sarcolemma to trigger calcium release from SR. The actin sites are exposed after Ca^{++} enters the sarcoplasm from its SR storage to activate the troponin-tropomyosin complex so that the tropomyosin shifts away from the sites. The cross-bridging of myposin heads docking into actin-binding sites is followed by the "power stroke"—the sliding of the thin filaments by thick filaments. The power strokes are powered by ATP. Ultimately, the sarcomeres, myofibrils, and muscle fibers shorten to produce movement.

Interactive Link Questions

The release of calcium ions initiates muscle contractions. Watch this video to learn more about the role of calcium. (a) What are "T-tubules" and what is their role? (b) Please also describe how actin-binding sites are made available for cross-bridging with myosin heads during contraction.

(a) The T-tubules are inward extensions of the sarcolemma that trigger the release of Ca^{++} from SR during an Action Potential. (b) Ca^{++} binds to tropomyosin, and this slides the tropomyosin rods away from the binding sites.

Review Questions

1. In relaxed muscle, the myosin-binding site on actin is blocked by ________.

A. titin
B. troponin
C. myoglobin
D. tropomyosin

2. According to the sliding filament model, binding sites on actin open when ________.

A. creatine phosphate levels rise
B. ATP levels rise
C. acetylcholine levels rise
D. calcium ion levels rise

3. The cell membrane of a muscle fiber is called ________.

A. myofibril
B. sarcolemma
C. sarcoplasm
D. myofilament

4. Muscle relaxation occurs when ________.

A. calcium ions are actively transported out of the sarcoplasmic reticulum
B. calcium ions diffuse out of the sarcoplasmic reticulum
C. calcium ions are actively transported into the sarcoplasmic reticulum

D. calcium ions diffuse into the sarcoplasmic reticulum

5. During muscle contraction, the cross-bridge detaches when ________.

A. the myosin head binds to an ADP molecule
B. the myosin head binds to an ATP molecule
C. calcium ions bind to troponin
D. calcium ions bind to actin

6. Thin and thick filaments are organized into functional units called ________.

A. myofibrils
B. myofilaments
C. T-tubules
D. sarcomeres

Review Questions

Critical Thinking Questions

1. How would muscle contractions be affected if skeletal muscle fibers did not have T-tubules?

2. What causes the striated appearance of skeletal muscle tissue?

3. How would muscle contractions be affected if ATP was completely depleted in a muscle fiber?

Glossary

aerobic respiration
: production of ATP in the presence of oxygen

ATPase
: enzyme that hydrolyzes ATP to ADP

creatine phosphate
: phosphagen used to store energy from ATP and transfer it to muscle

glycolysis
: anaerobic breakdown of glucose to ATP

lactic acid
: product of anaerobic glycolysis

oxygen debt
: amount of oxygen needed to compensate for ATP produced without oxygen during muscle contraction

power stroke
: action of myosin pulling actin inward (toward the M line)

pyruvic acid
: product of glycolysis that can be used in aerobic respiration or converted to lactic acid

Solutions

Answers for Review Questions

1. D
2. D
3. B
4. C
5. C
6. D

Answers for Critical Thinking Questions

1. Without T-tubules, action potential conduction into the interior of the cell would happen much more slowly, causing delays between neural stimulation and muscle contraction, resulting in slower, weaker contractions.
2. Dark A bands and light I bands repeat along myofibrils, and the alignment of myofibrils in the cell cause the entire cell to appear striated.
3. Without ATP, the myosin heads cannot detach from the actin-binding sites. All of the "stuck" cross-bridges result in muscle stiffness. In a live person, this can cause a condition like "writer's cramps." In a recently dead person, it results in rigor mortis.

67. 9.4 Nervous System Control of Muscle Tension

To move an object, referred to as load, the sarcomeres in the muscle fibers of the skeletal muscle must shorten. The force generated by the contraction of the muscle (or shortening of the sarcomeres) is called **muscle tension**. However, muscle tension also is generated when the muscle is contracting against a load that does not move, resulting in two main types of skeletal muscle contractions: isotonic contractions and isometric contractions.

In **isotonic contractions**, where the tension in the muscle stays constant, a load is moved as the length of the muscle changes (shortens). There are two types of isotonic contractions: concentric and eccentric. A **concentric contraction** involves the muscle shortening to move a load. An example of this is the biceps brachii muscle contracting when a hand weight is brought upward with increasing muscle tension. As the biceps brachii contract, the angle of the elbow joint decreases as the forearm is brought toward the body. Here, the biceps brachii contracts as sarcomeres in its muscle fibers are shortening and cross-bridges form; the myosin heads pull the actin. An **eccentric contraction** occurs as the muscle tension diminishes and the muscle lengthens. In this case, the hand weight is lowered in a slow and controlled manner as the amount of cross-bridges being activated by nervous system stimulation decreases. In this case, as tension is released from the biceps brachii, the angle of the elbow joint increases. Eccentric contractions are also used for movement and balance of the body.

An **isometric contraction** occurs as the muscle produces tension without changing the angle of a skeletal joint. Isometric contractions involve sarcomere shortening and increasing muscle tension, but do not move a load, as the force produced cannot overcome the resistance provided by the load. For example, if one attempts to lift a hand weight that is too heavy, there will be sarcomere activation and shortening to a point, and ever-increasing muscle tension, but no change in the angle of the elbow joint. In everyday living, isometric contractions are active in maintaining posture and maintaining bone and joint stability. However, holding your head in an upright position occurs not because the muscles cannot move the head, but because the goal is to remain stationary and not produce movement. Most actions of the body are the result of a combination of isotonic and isometric contractions working together to produce a wide range of outcomes (Figure 1).

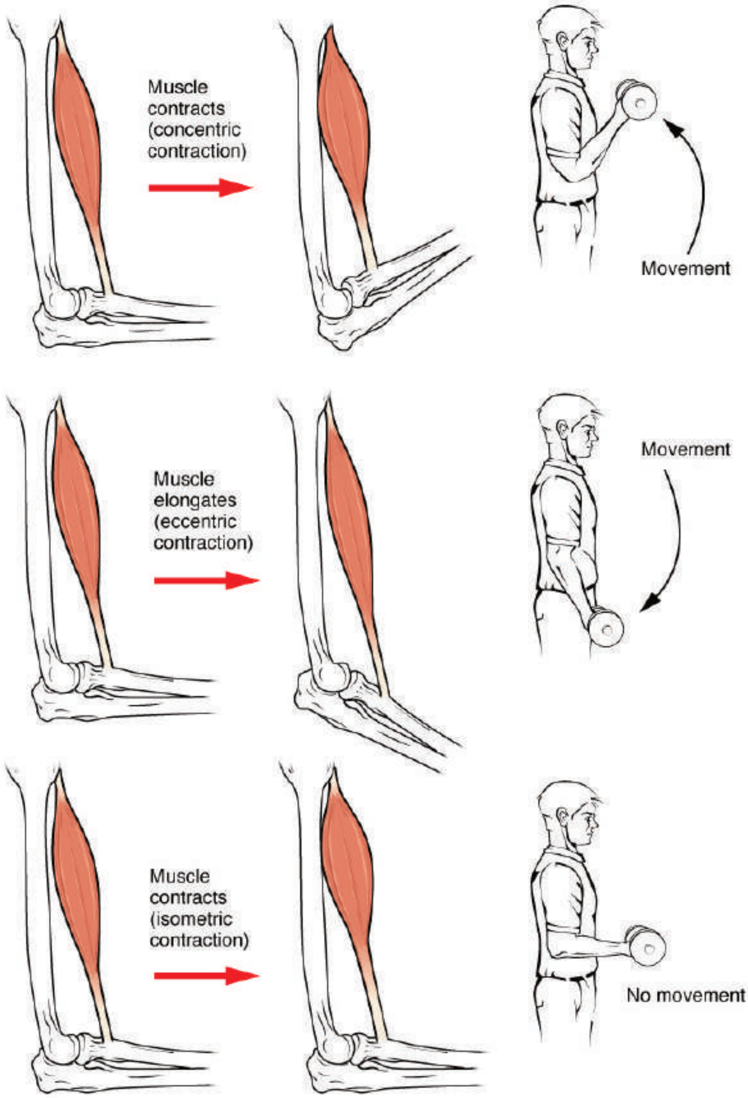

Figure 1. Types of Muscle Contractions. During isotonic contractions, muscle length changes to move a load. During isometric contractions, muscle length does not change because the load exceeds the tension the muscle can generate.

All of these muscle activities are under the exquisite control of the nervous system. Neural control regulates concentric, eccentric and isometric contractions, muscle fiber recruitment, and muscle tone. A crucial aspect of nervous system control of skeletal muscles is the role of motor units.

Motor Units

As you have learned, every skeletal muscle fiber must be innervated by the axon terminal of a motor neuron in order to contract. Each muscle fiber is innervated by only one motor neuron. The actual group of muscle fibers in a muscle innervated by a single motor neuron is called a **motor unit**. The size of a motor unit is variable depending on the nature of the muscle.

A small motor unit is an arrangement where a single motor neuron supplies a small number of muscle fibers in a muscle. Small motor units permit very fine motor control of the muscle. The best example in humans is the small motor units of the extraocular eye muscles that move the eyeballs. There are thousands of muscle fibers in each muscle, but every six or so fibers are supplied by a single motor neuron, as the axons branch to form synaptic connections at their individual NMJs. This allows for exquisite control of eye movements so that both eyes can quickly focus on the same object. Small motor units are also involved in the many fine movements of the fingers and thumb of the hand for grasping, texting, etc.

A large motor unit is an arrangement where a single motor neuron supplies a large number of muscle fibers in a muscle. Large motor units are concerned with simple, or “gross,” movements, such as powerfully extending the knee joint. The best example is the large motor units of the thigh muscles or back muscles, where a single motor neuron will supply thousands of muscle fibers in a muscle, as its axon splits into thousands of branches.

There is a wide range of motor units within many skeletal muscles, which gives the nervous system a wide range of control over the muscle. The small motor units in the muscle will have smaller, lower-threshold motor neurons that are more excitable, firing first to their skeletal muscle fibers, which also tend to be the smallest. Activation of these smaller motor units, results in a relatively small degree of contractile strength (tension) generated in the muscle. As more strength is needed, larger motor units, with bigger, higher-threshold motor neurons are enlisted to activate larger muscle fibers. This increasing activation of motor units produces an increase in muscle contraction known as **recruitment**. As more motor units are recruited, the muscle contraction grows progressively stronger. In some muscles, the largest motor units may generate a contractile force of 50 times more than the smallest motor units in the muscle. This allows a feather to be picked up using the biceps brachii arm muscle with minimal force, and a heavy weight to be lifted by the same muscle by recruiting the largest motor units.

When necessary, the maximal number of motor units in a muscle can be recruited simultaneously, producing the maximum force of contraction for that muscle, but this cannot last for very long because of the energy requirements to sustain the contraction. To prevent complete muscle fatigue, motor units are generally not all simultaneously active, but instead some motor units rest while others are active, which allows for longer muscle contractions. The nervous system uses recruitment as a mechanism to efficiently utilize a skeletal muscle.

The Length-Tension Range of a

Sarcomere

When a skeletal muscle fiber contracts, myosin heads attach to actin to form cross-bridges followed by the thin filaments sliding over the thick filaments as the heads pull the actin, and this results in sarcomere shortening, creating the tension of the muscle contraction. The cross-bridges can only form where thin and thick filaments already overlap, so that the length of the sarcomere has a direct influence on the force generated when the sarcomere shortens. This is called the length-tension relationship.

The ideal length of a sarcomere to produce maximal tension occurs at 80 percent to 120 percent of its resting length, with 100 percent being the state where the medial edges of the thin filaments are just at the most-medial myosin heads of the thick filaments (Figure 2). This length maximizes the overlap of actin-binding sites and myosin heads. If a sarcomere is stretched past this ideal length (beyond 120 percent), thick and thin filaments do not overlap sufficiently, which results in less tension produced. If a sarcomere is shortened beyond 80 percent, the zone of overlap is reduced with the thin filaments jutting beyond the last of the myosin heads and shrinks the H zone, which is normally composed of myosin tails. Eventually, there is nowhere else for the thin filaments to go and the amount of tension is diminished. If the muscle is stretched to the point where thick and thin filaments do not overlap at all, no cross-bridges can be formed, and no tension is produced in that sarcomere. This amount of stretching does not usually occur, as accessory proteins and connective tissue oppose extreme stretching.

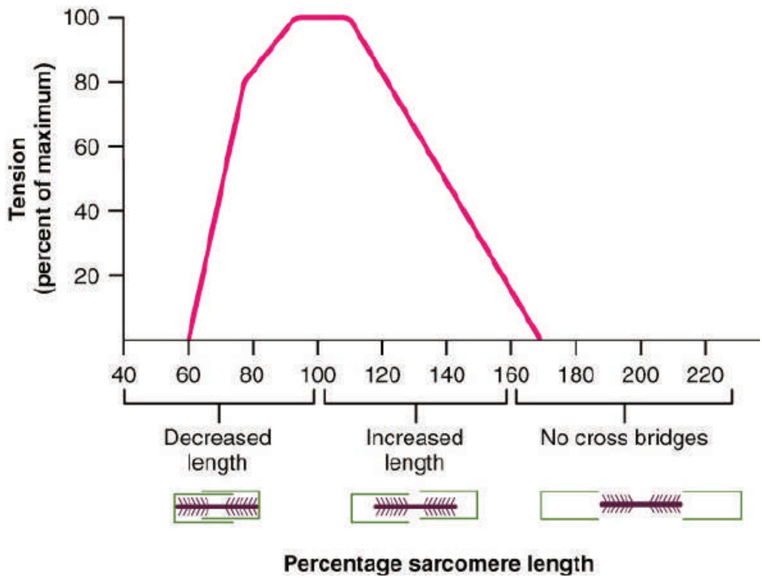

Figure 2. The Ideal Length of a Sarcomere. Sarcomeres produce maximal tension when thick and thin filaments overlap between about 80 percent to 120 percent.

The Frequency of Motor Neuron Stimulation

A single action potential from a motor neuron will produce a single contraction in the muscle fibers of its motor unit. This isolated contraction is called a **twitch**. A twitch can last for a few milliseconds or 100 milliseconds, depending on the muscle type. The tension produced by a single twitch can be measured by a **myogram**, an instrument that measures the amount of tension produced over time (Figure 3). Each twitch undergoes three phases. The first phase is the **latent period**, during which the action potential is being propagated along the sarcolemma and Ca^{++} ions are released from the SR. This is the phase during which excitation and contraction are being coupled but contraction has yet to occur. The **contraction phase** occurs next. The Ca^{++} ions in the sarcoplasm have bound to troponin, tropomyosin has shifted away from actin-binding sites, cross-bridges formed, and sarcomeres are actively shortening to the point of peak tension. The last phase is the **relaxation phase**, when tension decreases as contraction stops. Ca^{++} ions are pumped out of the sarcoplasm into the SR, and cross-bridge cycling stops, returning the muscle fibers to their resting state.

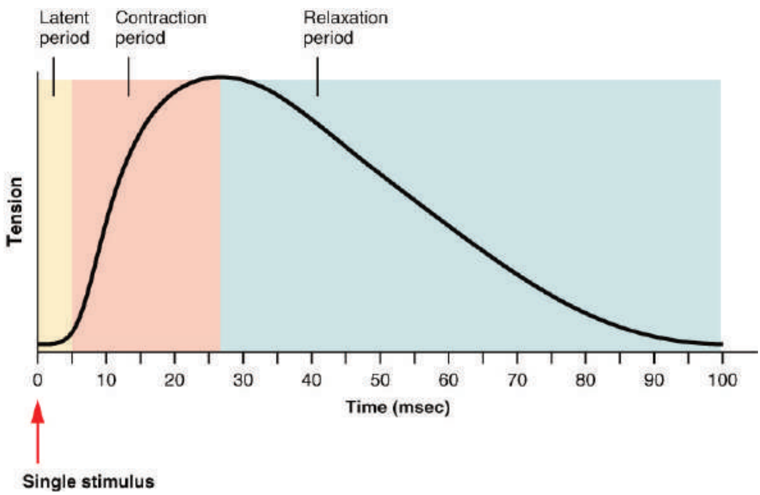

Figure 3. A Myogram of a Muscle Twitch. A single muscle twitch has a latent period, a contraction phase when tension increases, and a relaxation phase when tension decreases. During the latent period, the action potential is being propagated along the sarcolemma. During the contraction phase, Ca++ ions in the sarcoplasm bind to troponin, tropomyosin moves from actin-binding sites, cross-bridges form, and sarcomeres shorten. During the relaxation phase, tension decreases as Ca++ ions are pumped out of the sarcoplasm and cross-bridge cycling stops.

Although a person can experience a muscle "twitch," a single twitch does not produce any significant muscle activity in a living body. A series of action potentials to the muscle fibers is necessary to produce a muscle contraction that can produce work. Normal muscle contraction is more sustained, and it can be modified by input from the nervous system to produce varying amounts of force; this is called a **graded muscle response**. The frequency of action potentials (nerve impulses) from a motor neuron and the number of motor neurons transmitting action potentials both affect the tension produced in skeletal muscle.

The rate at which a motor neuron fires action potentials affects the tension produced in the skeletal muscle. If the fibers are stimulated while a previous twitch is still occurring, the second twitch will be stronger. This response is called **wave summation**, because the excitation-contraction coupling effects of successive motor neuron signaling is summed, or added together (Figure 4**a**). At the molecular level, summation occurs because the second stimulus triggers the release of more Ca^{++} ions, which become available to activate additional sarcomeres while the muscle is still contracting from the first stimulus. Summation results in greater contraction of the motor unit.

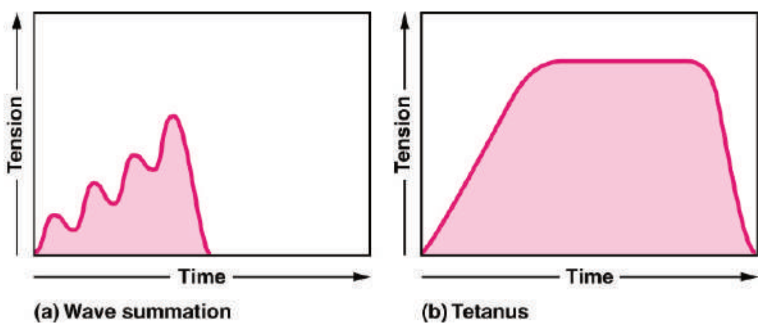

Figure 4. Wave Summation and Tetanus. (a) The excitation-contraction coupling effects of successive motor neuron signaling is added together which is referred to as wave summation. The bottom of each wave, the end of the relaxation phase, represents the point of stimulus. (b) When the stimulus frequency is so high that the relaxation phase disappears completely, the contractions become continuous; this is called tetanus.

If the frequency of motor neuron signaling increases, summation and subsequent muscle tension in the motor unit continues to rise until it reaches a peak point. The tension at this point is about three to four times greater than the tension of a single twitch, a state referred to as incomplete tetanus. During incomplete tetanus, the muscle goes through quick cycles of contraction with a short relaxation phase for each. If the stimulus frequency is so high that the relaxation phase disappears completely, contractions become continuous in a process called complete **tetanus** (Figure 4**b**).

During tetanus, the concentration of Ca^{++} ions in the sarcoplasm allows virtually all of the sarcomeres to form cross-bridges and shorten, so that a contraction can continue uninterrupted (until the muscle fatigues and can no longer produce tension).

Treppe

When a skeletal muscle has been dormant for an extended period and then activated to contract, with all other things being equal, the initial contractions generate about one-half the force of later contractions. The muscle tension increases in a graded manner that to some looks like a set of stairs. This tension increase is called **treppe**, a condition where muscle contractions become more efficient. It’s also known as the “staircase effect” (Figure 5).

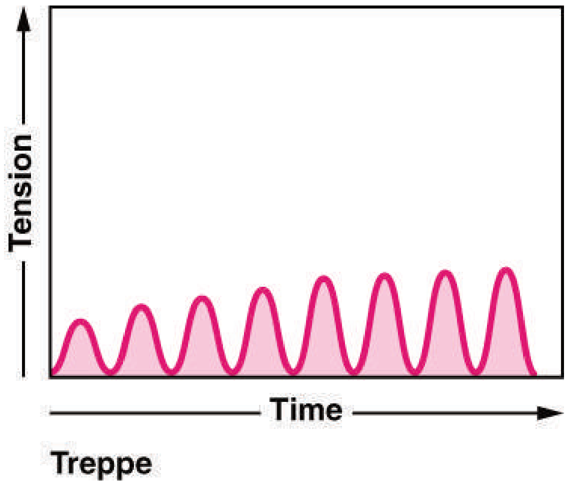

Figure 5. Treppe. When muscle tension increases in a graded manner that looks like a set of stairs, it is called treppe. The bottom of each wave represents the point of stimulus.

It is believed that treppe results from a higher concentration of Ca^{++} in the sarcoplasm resulting from the steady stream of signals from the motor neuron. It can only be maintained with adequate ATP.

Muscle Tone

Skeletal muscles are rarely completely relaxed, or flaccid. Even if a muscle is not producing movement, it is contracted a small amount to maintain its contractile proteins and produce **muscle tone**. The tension produced by muscle tone allows muscles to continually stabilize joints and maintain posture.

Muscle tone is accomplished by a complex interaction between the nervous system and skeletal muscles that results in the activation of a few motor units at a time, most likely in a cyclical manner. In this manner, muscles never fatigue completely, as some motor units can recover while others are active.

The absence of the low-level contractions that lead to muscle tone is referred to as **hypotonia** or atrophy, and can result from damage to parts of the central nervous system (CNS), such as the cerebellum, or from loss of innervations to a skeletal muscle, as in poliomyelitis. Hypotonic muscles have a flaccid appearance and display functional impairments, such as weak reflexes. Conversely, excessive muscle tone is referred to as **hypertonia**, accompanied by hyperreflexia (excessive reflex responses), often the result of damage to upper motor neurons in the CNS. Hypertonia can present with muscle rigidity (as seen in Parkinson's disease) or spasticity, a phasic change in muscle tone, where a limb will "snap" back from passive stretching (as seen in some strokes).

Chapter Review

The number of cross-bridges formed between actin and myosin determines the amount of tension produced by a muscle. The length of a sarcomere is optimal when the zone of overlap between thin and thick filaments is greatest. Muscles that are stretched or compressed too greatly do not produce maximal amounts of power. A motor unit is formed by a motor neuron and all of the muscle fibers that are innervated by that same motor neuron. A single contraction is called a twitch. A muscle twitch has a latent period, a contraction phase, and a relaxation phase. A graded muscle response allows variation in muscle tension. Summation occurs as successive stimuli are added together to produce a stronger muscle contraction. Tetanus is the fusion of contractions to produce a continuous contraction. Increasing the number of motor neurons involved increases the amount of motor units activated in a muscle, which is called recruitment. Muscle tone is the constant low-level contractions that allow for posture and stability.

Review Questions

1. During which phase of a twitch in a muscle fiber is tension the greatest?

A. resting phase
B. repolarization phase
C. contraction phase
D. relaxation phase

Exercises

1. Why does a motor unit of the eye have few muscle fibers compared to a motor unit of the leg?

2. What factors contribute to the amount of tension produced in an individual muscle fiber?

Glossary

concentric contraction
: muscle contraction that shortens the muscle to move a load

contraction phase
: twitch contraction phase when tension increases

eccentric contraction
: muscle contraction that lengthens the muscle as the tension is diminished

graded muscle response
: modification of contraction strength

hypertonia
: abnormally high muscle tone

hypotonia
: abnormally low muscle tone caused by the absence of low-level contractions

isometric contraction
: muscle contraction that occurs with no change in muscle length

isotonic contraction
: muscle contraction that involves changes in muscle length

latent period
: the time when a twitch does not produce contraction

motor unit
: motor neuron and the group of muscle fibers it innervates

muscle tension
: force generated by the contraction of the muscle; tension generated during isotonic contractions and isometric contractions

muscle tone
: low levels of muscle contraction that occur when a muscle is not producing movement

myogram
: instrument used to measure twitch tension

recruitment
: increase in the number of motor units involved in contraction

relaxation phase
: period after twitch contraction when tension decreases

tetanus
: a continuous fused contraction

treppe
: stepwise increase in contraction tension

twitch
: single contraction produced by one action potential

wave summation
: addition of successive neural stimuli to produce greater contraction

Solutions

Answers for Review Questions

1. C

Answers for Critical Thinking Questions

1. Eyes require fine movements and a high degree of control, which is permitted by having fewer muscle fibers associated with a neuron.
2. The length, size and types of muscle fiber and the frequency of neural stimulation contribute to the amount of tension produced in an individual muscle fiber.

68. 9.5 Types of Muscle Fibers

Two criteria to consider when classifying the types of muscle fibers are how fast some fibers contract relative to others, and how fibers produce ATP. Using these criteria, there are three main types of skeletal muscle fibers. **Slow oxidative (SO)** fibers contract relatively slowly and use aerobic respiration (oxygen and glucose) to produce ATP. **Fast oxidative (FO)** fibers have fast contractions and primarily use aerobic respiration, but because they may switch to anaerobic respiration (glycolysis), can fatigue more quickly than SO fibers. Lastly, **fast glycolytic (FG)** fibers have fast contractions and primarily use anaerobic glycolysis. The FG fibers fatigue more quickly than the others. Most skeletal muscles in a human contain(s) all three types, although in varying proportions.

The speed of contraction is dependent on how quickly myosin's ATPase hydrolyzes ATP to produce cross-bridge action. Fast fibers hydrolyze ATP approximately twice as quickly as slow fibers, resulting in much quicker cross-bridge cycling (which pulls the thin filaments toward the center of the sarcomeres at a faster rate). The primary metabolic pathway used by a muscle fiber determines whether the fiber is classified as oxidative or glycolytic. If a fiber primarily produces ATP through aerobic pathways it is oxidative. More ATP can be produced during each metabolic cycle, making the fiber more resistant to fatigue. Glycolytic fibers primarily create ATP through anaerobic glycolysis, which produces less ATP per cycle. As a result, glycolytic fibers fatigue at a quicker rate.

The oxidative fibers contain many more mitochondria than the glycolytic fibers, because aerobic metabolism, which uses oxygen (O_2) in the metabolic pathway, occurs in the mitochondria. The SO fibers possess a large number of mitochondria and are capable of contracting for longer periods because of the large amount of ATP they can produce, but they have a relatively small diameter and do not produce a large amount of tension. SO fibers are extensively supplied with blood capillaries to supply O_2 from the red blood cells in the bloodstream. The SO fibers also possess myoglobin, an O_2-carrying molecule similar to O_2-carrying hemoglobin in the red blood cells. The myoglobin stores some of the needed O_2 within the fibers themselves (and gives SO fibers their red color). All of these features allow SO fibers to produce large quantities of ATP, which can sustain muscle activity without fatiguing for long periods of time.

The fact that SO fibers can function for long periods without fatiguing makes them useful in maintaining posture, producing isometric contractions, stabilizing bones and joints, and making small movements that happen often but do not require large amounts of energy. They do not produce high tension, and thus they are not used for powerful, fast movements that require high amounts of energy and rapid cross-bridge cycling.

FO fibers are sometimes called intermediate fibers because they possess characteristics that are intermediate between fast fibers and slow fibers. They produce ATP relatively quickly, more quickly than SO fibers, and thus can produce relatively high amounts of tension. They are oxidative because they produce ATP aerobically, possess high amounts of mitochondria, and do not fatigue quickly. However, FO fibers do not possess significant myoglobin, giving them a lighter color than the red SO fibers. FO fibers are used primarily for movements, such as walking, that require more energy than postural control but less energy than an explosive movement, such as sprinting. FO fibers are useful for this type of movement because they produce more tension than SO fibers but they are more fatigue-resistant than FG fibers.

FG fibers primarily use anaerobic glycolysis as their ATP source. They have a large diameter and possess high amounts of glycogen, which is used in glycolysis to generate ATP quickly to produce high levels of tension. Because they do not primarily use aerobic metabolism, they do not possess substantial numbers of mitochondria or significant amounts of myoglobin and therefore have a white color. FG fibers are used to produce rapid, forceful contractions to make quick, powerful movements. These fibers fatigue quickly, permitting them to only be used for short periods. Most muscles possess a mixture of each fiber type. The predominant fiber type in a muscle is determined by the primary function of the muscle.

Chapter Review

ATP provides the energy for muscle contraction. The three mechanisms for ATP regeneration are creatine phosphate, anaerobic glycolysis, and aerobic metabolism. Creatine phosphate provides about the first 15 seconds of ATP at the beginning of muscle contraction. Anaerobic glycolysis produces small amounts of ATP in the absence of oxygen for a short period. Aerobic metabolism utilizes oxygen to produce much more ATP, allowing a muscle to work for longer periods. Muscle fatigue, which has many contributing factors, occurs when muscle can no longer contract. An oxygen debt is created as a result of muscle use. The three types of muscle fiber are slow oxidative (SO), fast oxidative (FO) and fast glycolytic (FG). SO fibers use aerobic metabolism to produce low power contractions over long periods and are slow to fatigue. FO fibers use aerobic metabolism to produce ATP but produce higher tension contractions than SO fibers. FG fibers use anaerobic metabolism to produce powerful, high-tension contractions but fatigue quickly.

Review Questions

1. Muscle fatigue is caused by ________.

A. buildup of ATP and lactic acid levels
B. exhaustion of energy reserves and buildup of lactic acid levels
C. buildup of ATP and pyruvic acid levels
D. exhaustion of energy reserves and buildup of pyruvic acid levels

2. A sprinter would experience muscle fatigue sooner than a marathon runner due to ________.

A. anaerobic metabolism in the muscles of the sprinter
B. anaerobic metabolism in the muscles of the marathon runner
C. aerobic metabolism in the muscles of the sprinter
D. glycolysis in the muscles of the marathon runner

3. What aspect of creatine phosphate allows it to supply energy to muscles?

A. ATPase activity
B. phosphate bonds
C. carbon bonds
D. hydrogen bonds

4. Drug X blocks ATP regeneration from ADP and phosphate. How will muscle cells respond to this drug?

A. by absorbing ATP from the bloodstream
B. by using ADP as an energy source
C. by using glycogen as an energy source
D. none of the above

Critical Thinking Questions

1. Why do muscle cells use creatine phosphate instead of glycolysis to supply ATP for the first few seconds of muscle contraction?

2. Is aerobic respiration more or less efficient than glycolysis? Explain your answer.

Glossary

fast glycolytic (FG)
: muscle fiber that primarily uses anaerobic glycolysis

fast oxidative (FO)
: intermediate muscle fiber that is between slow oxidative and fast glycolytic fibers

slow oxidative (SO)
: muscle fiber that primarily uses aerobic respiration

Solutions

Answers for Review Questions

1. B
2. A
3. B
4. D

Answers for Critical Thinking Questions

1. Creatine phosphate is used because creatine phosphate and ADP are converted very quickly into ATP by creatine kinase. Glycolysis cannot generate ATP as quickly as creatine phosphate.
2. Aerobic respiration is much more efficient than anaerobic glycolysis, yielding 36 ATP per molecule of glucose, as opposed to two ATP produced by glycolysis.

69. 9.6 Exercise and Muscle Performance

Physical training alters the appearance of skeletal muscles and can produce changes in muscle performance. Conversely, a lack of use can result in decreased performance and muscle appearance. Although muscle cells can change in size, new cells are not formed when muscles grow. Instead, structural proteins are added to muscle fibers in a process called **hypertrophy**, so cell diameter increases. The reverse, when structural proteins are lost and muscle mass decreases, is called **atrophy**. Age-related muscle atrophy is called **sarcopenia**. Cellular components of muscles can also undergo changes in response to changes in muscle use.

Endurance Exercise

Slow fibers are predominantly used in endurance exercises that require little force but involve numerous repetitions. The aerobic metabolism used by slow-twitch fibers allows them to maintain contractions over long periods. Endurance training modifies these slow fibers to make them even more efficient by producing more mitochondria to enable more aerobic metabolism and more ATP production. Endurance exercise can also increase the amount of myoglobin in a cell, as increased aerobic respiration increases the need for oxygen. Myoglobin is found in the sarcoplasm and acts as an oxygen storage supply for the mitochondria.

The training can trigger the formation of more extensive capillary networks around the fiber, a process called

angiogenesis, to supply oxygen and remove metabolic waste. To allow these capillary networks to supply the deep portions of the muscle, muscle mass does not greatly increase in order to maintain a smaller area for the diffusion of nutrients and gases. All of these cellular changes result in the ability to sustain low levels of muscle contractions for greater periods without fatiguing.

The proportion of SO muscle fibers in muscle determines the suitability of that muscle for endurance, and may benefit those participating in endurance activities. Postural muscles have a large number of SO fibers and relatively few FO and FG fibers, to keep the back straight (Figure 1). Endurance athletes, like marathon-runners also would benefit from a larger proportion of SO fibers, but it is unclear if the most-successful marathoners are those with naturally high numbers of SO fibers, or whether the most successful marathon runners develop high numbers of SO fibers with repetitive training. Endurance training can result in overuse injuries such as stress fractures and joint and tendon inflammation.

Figure 1. Marathoners. Long-distance runners have a large number of SO fibers and relatively few FO and FG fibers. (credit: "Tseo2"/Wikimedia Commons)

Resistance Exercise

Resistance exercises, as opposed to endurance exercise, require large amounts of FG fibers to produce short, powerful movements that are not repeated over long periods. The high rates of ATP hydrolysis and cross-bridge formation in FG fibers result in powerful muscle contractions. Muscles used for power have a higher ratio of FG to SO/FO fibers, and trained athletes possess even higher levels of FG fibers in their muscles. Resistance exercise affects muscles by increasing the formation of myofibrils, thereby increasing the thickness of muscle fibers. This added structure causes hypertrophy, or the enlargement of muscles, exemplified by the large skeletal muscles seen in body builders and other athletes (Figure 2). Because this muscular enlargement is achieved by the addition of structural proteins, athletes trying to build muscle mass often ingest large amounts of protein.

Figure 2. Hypertrophy. Body builders have a large number of FG fibers and relatively few FO and SO fibers. (credit: Lin Mei/flickr)

Except for the hypertrophy that follows an increase in the number of sarcomeres and myofibrils in a skeletal muscle, the cellular changes observed during endurance training do not usually occur with resistance training. There is usually no significant increase in mitochondria or capillary density. However, resistance training does increase the development of connective tissue, which adds to the overall mass of the muscle and helps to contain muscles as they produce increasingly powerful contractions. Tendons also become stronger to prevent tendon damage, as the force produced by muscles is transferred to tendons that attach the muscle to bone.

For effective strength training, the intensity of the exercise must continually be increased. For instance, continued weight lifting without increasing the weight of the load does not increase muscle size. To produce ever-greater results, the weights lifted must become increasingly heavier, making it more difficult for muscles to move the load. The muscle then adapts to this heavier load, and an even heavier load must be used if even greater muscle mass is desired.

If done improperly, resistance training can lead to overuse injuries of the muscle, tendon, or bone. These injuries can occur if the load is too heavy or if the muscles are not given sufficient time between workouts to recover or if joints are not aligned properly during the exercises. Cellular damage to muscle fibers that occurs after intense exercise includes damage to the sarcolemma and myofibrils. This muscle damage contributes to the feeling of soreness after strenuous exercise, but muscles gain mass as this damage is repaired, and additional structural proteins are added to replace the damaged ones. Overworking skeletal muscles can also lead to tendon damage and even skeletal damage if the load is too great for the muscles to bear.

Performance-Enhancing Substances

Some athletes attempt to boost their performance by using various agents that may enhance muscle performance. Anabolic steroids are one of the more widely known agents used to boost muscle mass and increase power output. Anabolic steroids are a form of testosterone, a male sex hormone that stimulates muscle formation, leading to increased muscle mass.

Endurance athletes may also try to boost the availability of oxygen to muscles to increase aerobic respiration by using substances such as erythropoietin (EPO), a hormone normally produced in the kidneys, which triggers the production of red blood cells. The extra oxygen carried by these blood cells can then be used by muscles for aerobic respiration. Human growth hormone (hGH) is another supplement, and although it can facilitate building muscle mass, its main role is to promote the healing of muscle and other tissues after strenuous exercise. Increased hGH may allow for faster recovery after muscle damage, reducing the rest required after exercise, and allowing for more sustained high-level performance.

Although performance-enhancing substances often do improve performance, most are banned by governing bodies in sports and are illegal for nonmedical purposes. Their use to enhance performance raises ethical issues of cheating because they give users an unfair advantage over nonusers. A greater concern, however, is that their use carries serious health risks. The side effects of these substances are often significant, nonreversible, and in some cases fatal. The physiological strain caused by these substances is often greater than what the body can handle, leading to effects that are unpredictable and dangerous. Anabolic steroid use has been linked to infertility, aggressive behavior, cardiovascular disease, and brain cancer.

Similarly, some athletes have used creatine to increase power output. Creatine phosphate provides quick bursts of ATP to muscles in the initial stages of contraction. Increasing the amount of creatine available to cells is thought to produce more ATP and therefore increase explosive power output, although its effectiveness as a supplement has been questioned.

Everyday Connection

Aging and Muscle Tissue

Although atrophy due to disuse can often be reversed with exercise, muscle atrophy with age, referred to as sarcopenia, is irreversible. This is a primary reason why even highly trained athletes succumb to declining performance with age. This decline is noticeable in athletes whose sports require strength and powerful movements, such as sprinting, whereas the effects of age are less noticeable in endurance athletes such as marathon runners or long-distance cyclists. As muscles age, muscle fibers die, and they are replaced by connective tissue and adipose tissue (Figure 3). Because those tissues cannot contract and generate force as muscle can, muscles lose the ability to produce powerful contractions. The decline in muscle mass causes a loss of strength, including the strength required for posture and mobility. This may be caused by a reduction in FG fibers that hydrolyze ATP quickly to produce short, powerful contractions. Muscles in older people sometimes possess greater numbers of SO fibers, which are responsible for longer contractions and do not produce powerful movements. There may also be a reduction in the size of motor units, resulting in fewer fibers being stimulated and less muscle tension being produced.

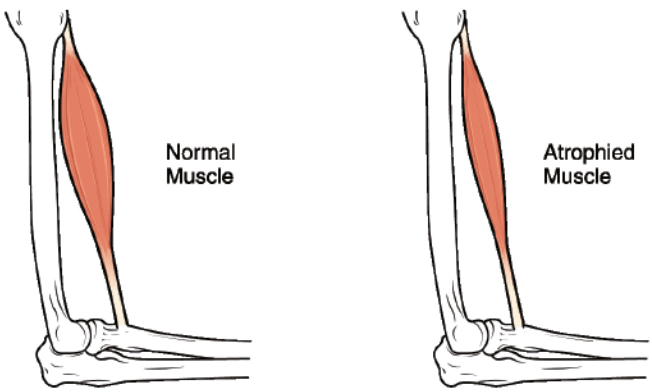

Figure 3. Atrophy. Muscle mass is reduced as muscles atrophy with disuse.

Sarcopenia can be delayed to some extent by exercise, as training adds structural proteins and causes cellular changes that can offset the effects of atrophy. Increased exercise can produce greater numbers of cellular mitochondria, increase capillary density, and increase the mass and strength of connective tissue. The effects of age-related atrophy are especially pronounced in people who are sedentary, as the loss of muscle cells is displayed as functional impairments such as trouble with locomotion, balance, and posture. This can lead to a decrease in quality of life and medical problems, such as joint problems because the muscles that stabilize bones and joints are weakened. Problems with locomotion and balance can also cause various injuries due to falls.

Chapter Review

Hypertrophy is an increase in muscle mass due to the addition of structural proteins. The opposite of hypertrophy is atrophy, the loss of muscle mass due to the breakdown of structural proteins. Endurance exercise causes an increase in cellular mitochondria, myoglobin, and capillary networks in SO fibers. Endurance athletes have a high level of SO fibers relative to the other fiber types. Resistance exercise causes hypertrophy. Power-producing muscles have a higher number of FG fibers than of slow fibers. Strenuous exercise causes muscle cell damage that requires time to heal. Some athletes use performance-enhancing substances to enhance muscle performance. Muscle atrophy due to age is called sarcopenia and occurs as muscle fibers die and are replaced by connective and adipose tissue.

Review Questions

1. The muscles of a professional sprinter are most likely to have ________.

A. 80 percent fast-twitch muscle fibers and 20 percent slow-twitch muscle fibers

B. 20 percent fast-twitch muscle fibers and 80 percent slow-twitch muscle fibers

C. 50 percent fast-twitch muscle fibers and 50 percent slow-twitch muscle fibers

D. 40 percent fast-twitch muscle fibers and 60 percent slow-twitch muscle fibers

2. The muscles of a professional marathon runner are most likely to have ________.

A. 80 percent fast-twitch muscle fibers and 20 percent slow-twitch muscle fibers

B. 20 percent fast-twitch muscle fibers and 80 percent slow-twitch muscle fibers

C. 50 percent fast-twitch muscle fibers and 50 percent slow-twitch muscle fibers

D. 40 percent fast-twitch muscle fibers and 60 percent slow-twitch muscle fibers

3. Which of the following statements is *true*?

A. Fast fibers have a small diameter.

B. Fast fibers contain loosely packed myofibrils.

C. Fast fibers have large glycogen reserves.

D. Fast fibers have many mitochondria.

4. Which of the following statements is *false*?

A. Slow fibers have a small network of capillaries.

B. Slow fibers contain the pigment myoglobin.

C. Slow fibers contain a large number of mitochondria.

D. Slow fibers contract for extended periods.

Critical Thinking Questions

1. What changes occur at the cellular level in response to endurance training?<

2. What changes occur at the cellular level in response to resistance training?

Glossary

angiogenesis
: formation of blood capillary networks

atrophy
: loss of structural proteins from muscle fibers

hypertrophy
: addition of structural proteins to muscle fibers

sarcopenia
: age-related muscle atrophy

Solutions

Answers for Review Questions

1. A
2. B
3. C
4. A

Answers for Critical Thinking Questions

1. Endurance training modifies slow fibers to make them more efficient by producing more mitochondria to enable more aerobic metabolism and more ATP production. Endurance exercise can also increase the amount of myoglobin in a cell and formation of more extensive capillary networks around the fiber.
2. Resistance exercises affect muscles by causing the formation of more actin and myosin, increasing the structure of muscle fibers.

70. 9.7 Interactions of Skeletal Muscles, Their Fascicle Arrangement, and Their Lever Systems

To move the skeleton, the tension created by the contraction of the fibers in most skeletal muscles is transferred to the tendons. The tendons are strong bands of dense, regular connective tissue that connect muscles to bones. The bone connection is why this muscle tissue is called skeletal muscle.

Interactions of Skeletal Muscles in the Body

To pull on a bone, that is, to change the angle at its synovial joint, which essentially moves the skeleton, a skeletal muscle must also be attached to a fixed part of the skeleton. The moveable end of the muscle that attaches to the bone being pulled is called the muscle's **insertion**, and the end of the muscle attached to a fixed (stabilized) bone is called the **origin**. During forearm **flexion**—bending the elbow—the brachioradialis assists the brachialis.

Although a number of muscles may be involved in an action, the principal muscle involved is called the **prime mover**, or **agonist**. To lift a cup, a muscle called the biceps brachii is actually the prime mover; however, because it can be assisted by the brachialis, the brachialis is called a **synergist** in this action (Figure 1). A synergist can also be a **fixator** that stabilizes the bone that is the attachment for the prime mover's origin.

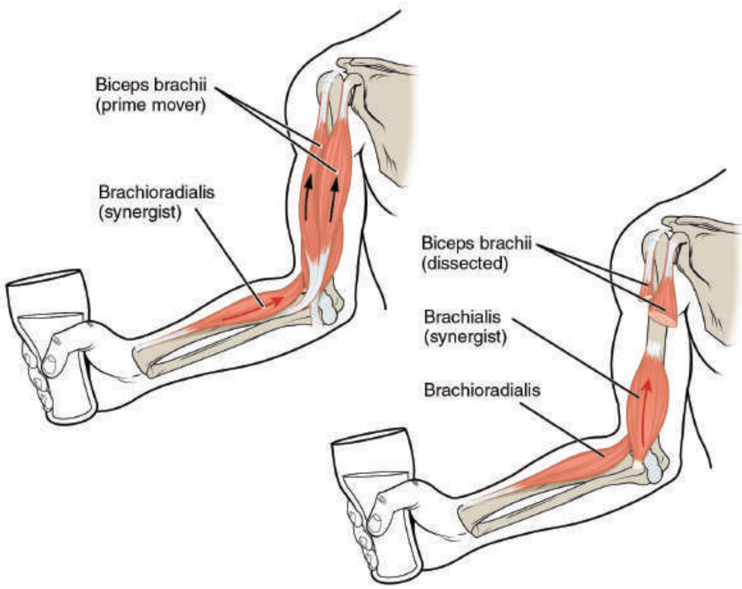

Figure 1. Prime Movers and Synergists. The biceps brachii flex the lower arm. The brachoradialis, in the forearm, and brachialis, located deep to the biceps in the upper arm, are both synergists that aid in this motion.

A muscle with the opposite action of the prime mover is called an **antagonist**. Antagonists play two important roles in muscle function: (1) they maintain body or limb position, such as holding the arm out or standing erect; and (2) they control rapid movement, as in shadow boxing without landing a punch or the ability to check the motion of a limb.

For example, to extend the knee, a group of four muscles called the quadriceps femoris in the anterior compartment of the thigh are activated (and would be called the agonists of knee extension). However, to flex the knee joint, an opposite or antagonistic set of muscles called the hamstrings is activated.

As you can see, these terms would also be reversed for the opposing action. If you consider the first action as the knee bending, the hamstrings would be called the agonists and the quadriceps femoris would then be called the antagonists. See Table 1 for a list of some agonists and antagonists.

Agonist and Antagonist Skeletal Muscle Pairs (Table 1)

Agonist	Antagonist	Movement
Biceps brachii: in the anterior compartment of the arm	Triceps brachii: in the posterior compartment of the arm	The biceps brachii flexes the forearm, whereas the triceps brachii extends it.
Hamstrings: group of three muscles in the posterior compartment of the thigh	Quadriceps femoris: group of four muscles in the anterior compartment of the thigh	The hamstrings flex the leg, whereas the quadriceps femoris extend it.
Flexor digitorum superficialis and flexor digitorum profundus: in the anterior compartment of the forearm	Extensor digitorum: in the posterior compartment of the forearm	The flexor digitorum superficialis and flexor digitorum profundus flex the fingers and the hand at the wrist, whereas the extensor digitorum extends the fingers and the hand at the wrist.

There are also skeletal muscles that do not pull against the skeleton for movements. For example, there are the muscles that produce facial expressions. The insertions and origins of facial muscles are in the skin, so that certain individual muscles contract to form a smile or frown, form sounds or words, and raise the eyebrows. There also are skeletal muscles in the tongue, and the external urinary and anal sphincters that allow for voluntary regulation of urination and defecation, respectively. In addition, the diaphragm contracts and relaxes

to change the volume of the pleural cavities but it does not move the skeleton to do this.
Everyday Connections

Exercise and Stretching
When exercising, it is important to first warm up the muscles. Stretching pulls on the muscle fibers and it also results in an increased blood flow to the muscles being worked. Without a proper warm-up, it is possible that you may either damage some of the muscle fibers or pull a tendon. A pulled tendon, regardless of location, results in pain, swelling, and diminished function; if it is moderate to severe, the injury could immobilize you for an extended period.

Recall the discussion about muscles crossing joints to create movement. Most of the joints you use during exercise are synovial joints, which have synovial fluid in the joint space between two bones. Exercise and stretching may also have a beneficial effect on synovial joints. Synovial fluid is a thin, but viscous film with the consistency of egg whites. When you first get up and start moving, your joints feel stiff for a number of reasons. After proper stretching and warm-up, the synovial fluid may become less viscous, allowing for better joint function.

Patterns of Fascicle Organization

Skeletal muscle is enclosed in connective tissue scaffolding at three levels. Each muscle fiber (cell) is covered by endomysium and the entire muscle is covered by epimysium. When a group of muscle fibers is “bundled” as a unit within the whole muscle by an additional covering of a connective tissue called perimysium, that bundled group of muscle fibers is called a **fascicle**. Fascicle arrangement by perimysia is correlated to the force generated by a muscle; it also affects the range of motion of the muscle. Based on the patterns of fascicle arrangement, skeletal muscles can be classified in several ways. What follows are the most common fascicle arrangements.

Parallel muscles have fascicles that are arranged in the same direction as the long axis of the muscle (Figure 2). The majority of skeletal muscles in the body have this type of organization. Some parallel muscles are flat sheets that expand at the ends to make broad attachments. Other parallel muscles are rotund with tendons at one or both ends. Muscles that seem to be plump have a large mass of tissue located in the middle of the muscle, between the insertion and the origin, which is known as the central body. A more common name for this muscle is **belly**. When a muscle contracts, the contractile fibers shorten it to an even larger bulge. For example, extend and then flex your biceps brachii muscle; the large, middle section is the belly (Figure 3). When a parallel muscle has a central, large belly that is spindle-shaped, meaning it tapers as it extends to its origin and insertion, it sometimes is called **fusiform**.

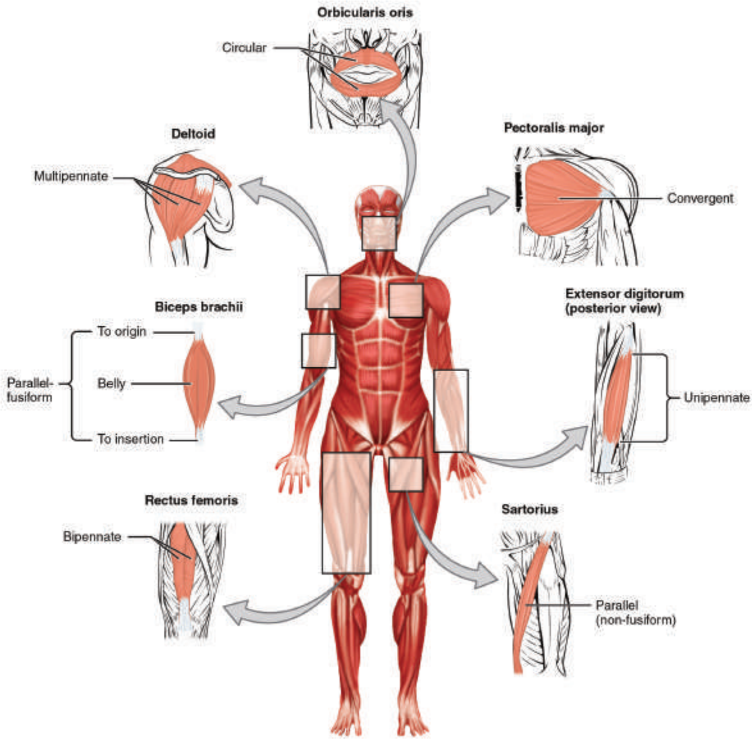

Figure 2. Muscle Shapes and Fiber Alignment. The skeletal muscles of the body typically come in seven different general shapes.

Figure 3. Biceps Brachii Muscle Contraction. The large mass at the center of a muscle is called the belly. Tendons emerge from both ends of the belly and connect the muscle to the bones, allowing the skeleton to move. The tendons of the bicep connect to the upper arm and the forearm. (credit: Victoria Garcia)

Circular muscles are also called sphincters (see Figure 2). When they relax, the sphincters' concentrically arranged bundles of muscle fibers increase the size of the opening, and when they contract, the size of the opening shrinks to the point of closure. The orbicularis oris muscle is a circular muscle that goes around the mouth. When it contracts, the oral opening becomes smaller, as when puckering the lips for whistling. Another example is the orbicularis oculi, one of which surrounds each eye. Consider, for example, the names of the two orbicularis muscles (orbicularis oris and oribicularis oculi), where part of the first name of both muscles is the same. The first part of orbicularis, orb (orb = "circular"), is a reference to a round or circular structure; it may also make one think of orbit, such as the moon's path around the earth. The word oris (oris = "oral") refers to the oral cavity, or the mouth. The word oculi (ocular = "eye") refers to the eye.

There are other muscles throughout the body named by their shape or location. The deltoid is a large, triangular-shaped muscle that covers the shoulder. It is so-named because the Greek letter delta looks like a triangle. The rectus abdomis (rector = "straight") is the straight muscle in the anterior wall of the abdomen, while the rectus femoris is the straight muscle in the anterior compartment of the thigh.

When a muscle has a widespread expansion over a sizable area, but then the fascicles come to a single, common attachment point, the muscle is called **convergent**. The attachment point for a convergent muscle could be a tendon, an aponeurosis (a flat, broad tendon), or a raphe (a very slender tendon). The large muscle on the chest, the pectoralis major, is an example of a convergent muscle because it converges on the greater tubercle of the humerus via a tendon. The temporalis muscle of the cranium is another.

Pennate muscles (penna = "feathers") blend into a tendon that runs through the central region of the muscle for its whole length, somewhat like the quill of a feather with the muscle arranged similar to the feathers. Due to this design, the muscle fibers in a pennate muscle can only pull at an angle, and as a result, contracting pennate muscles do not move their tendons very far. However, because a pennate muscle generally can hold more muscle fibers within it, it can produce relatively more tension for its size. There are three subtypes of pennate muscles.

In a **unipennate** muscle, the fascicles are located on one side of the tendon. The extensor digitorum of the forearm is an example of a unipennate muscle. A **bipennate** muscle has fascicles on both sides of the tendon. In some pennate muscles, the muscle fibers wrap around the tendon, sometimes forming individual fascicles in the process. This arrangement is referred to as **multipennate**. A common example is the deltoid muscle of the shoulder, which covers the shoulder but has a single tendon that inserts on the deltoid tuberosity of the humerus.

Because of fascicles, a portion of a multipennate muscle like the deltoid can be stimulated by the nervous system to change the direction of the pull. For example, when the deltoid muscle contracts, the arm abducts (moves away from midline in the sagittal plane), but when only the anterior fascicle is stimulated, the arm will **abduct** and flex (move anteriorly at the shoulder joint).

The Lever System of Muscle and Bone Interactions

Skeletal muscles do not work by themselves. Muscles are arranged in pairs based on their functions. For muscles attached to the bones of the skeleton, the connection determines the force, speed, and range of movement. These characteristics depend on each other and can explain the general organization of the muscular and skeletal systems.

The skeleton and muscles act together to move the body. Have you ever used the back of a hammer to remove a nail from wood? The handle acts as a lever and the head of the hammer acts as a fulcrum, the fixed point that the force is applied to when you pull back or push down on the handle. The effort applied to this system is the pulling or pushing on the handle to remove the nail, which is the load, or “resistance” to the movement of the handle in the system. Our musculoskeletal system works in a similar manner, with bones being stiff levers and the articular endings of the bones—encased in synovial joints—acting as fulcrums. The load would be an object being lifted or any resistance to a movement (your head is a load when you are lifting it), and the effort, or applied force, comes from contracting skeletal muscle.

Chapter Review

Skeletal muscles each have an origin and an insertion. The end of the muscle that attaches to the bone being pulled is called the muscle's insertion and the end of the muscle attached to a fixed, or stabilized, bone is called the origin. The muscle primarily responsible for a movement is called the prime mover, and muscles that assist in this action are called synergists. A synergist that makes the insertion site more stable is called a fixator. Meanwhile, a muscle with the opposite action of the prime mover is called an antagonist. Several factors contribute to the force generated by a skeletal muscle. One is the arrangement of the fascicles in the skeletal muscle. Fascicles can be parallel, circular, convergent, pennate, fusiform, or triangular. Each arrangement has its own range of motion and ability to do work.

Review Questions

1. Which of the following is unique to the muscles of facial expression?

A. They all originate from the scalp musculature.

B. They insert onto the cartilage found around the face.
C. They only insert onto the facial bones.
D. They insert into the skin.

2. Which of the following helps an agonist work?

A. a synergist
B. a fixator
C. an insertion
D. an antagonist

3. Which of the following statements is correct about what happens during flexion?

A. The angle between bones is increased.
B. The angle between bones is decreased.
C. The bone moves away from the body.
D. The bone moves toward the center of the body.

4. Which is moved the *least* during muscle contraction?

A. the origin
B. the insertion
C. the ligaments
D. the joints

5. Which muscle has a convergent pattern of fascicles?

A. biceps brachii
B. gluteus maximus
C. pectoralis major
D. rectus femoris

6. A muscle that has a pattern of fascicles running along the long axis of the muscle has which of the following fascicle arrangements?

A. circular
B. pennate
C. parallel
D. rectus

7. Which arrangement *best* describes a bipennate muscle?

A. The muscle fibers feed in on an angle to a long tendon from both sides.
B. The muscle fibers feed in on an angle to a long tendon from all directions.
C. The muscle fibers feed in on an angle to a long tendon from one side.
D. The muscle fibers on one side of a tendon feed into it at a certain angle and muscle fibers on the other side of the tendon feed into it at the opposite angle.

Critical Thinking Questions

1. What effect does fascicle arrangement have on a muscle's action?<

2. Movements of the body occur at joints. Describe how muscles are arranged around the joints of the body.

3. Explain how a synergist assists an agonist by being a fixator.

Glossary

abduct
: move away from midline in the sagittal plane

agonist
: (also, prime mover) muscle whose contraction is responsible for producing a particular motion

antagonist
: muscle that opposes the action of an agonist

belly
: bulky central body of a muscle

bipennate
: pennate muscle that has fascicles that are located on both sides of the tendon

circular
: (also, sphincter) fascicles that are concentrically arranged around an opening

convergent
: fascicles that extend over a broad area and converge on a common attachment site

fascicle
: muscle fibers bundled by perimysium into a unit

fixator
: synergist that assists an agonist by preventing or reducing movement at another joint, thereby stabilizing the origin of the agonist

flexion
: movement that decreases the angle of a joint

fusiform
: muscle that has fascicles that are spindle-shaped to create large bellies

insertion
: end of a skeletal muscle that is attached to the structure (usually a bone) that is moved when the muscle contracts

multipennate
: pennate muscle that has a tendon branching within it

origin
: end of a skeletal muscle that is attached to another structure (usually a bone) in a fixed position

parallel
: fascicles that extend in the same direction as the long axis of the muscle

pennate
: fascicles that are arranged differently based on their angles to the tendon

prime mover
: (also, agonist) principle muscle involved in an action

synergist
: muscle whose contraction helps a prime mover in an action

unipennate
: pennate muscle that has fascicles located on one side of the tendon

Solutions

Answers for Review Questions

1. D
2. A

3. B
4. A
5. C
6. C
7. A

Answers for Critical Thinking Questions

1. Fascicle arrangements determine what type of movement a muscle can make. For instance, circular muscles act as sphincters, closing orifices.
2. Muscles work in pairs to facilitate movement of the bones around the joints. Agonists are the prime movers while antagonists oppose or resist the movements of the agonists. Synergists assist the agonists, and fixators stabilize a muscle’s origin.
3. Agonists are the prime movers while antagonists oppose or resist the movements of the agonists. Synergists assist the agonists, and fixators stabilize a muscle’s origin.

PART X

CHAPTER 10: MECHANISM OF INJURY

Chapter Objectives

After this chapter, you will be able to:

- Define acute injury, bending, brittle, chronic injury, compliance, compression, creep, deformation, ductile, elastic deformation, failure, tolerance, fatigue, fracture, shear, sprain, stiffness, strain, strain energy, stress, tension, torsion, toughness, wear, yield point.
- Describe the different types of load on the body.
- Sketch a load-deformation curve and label the toe region, elastic region, yield point, plastic region, strength, stiffness and strain energy.
- Explain how material failure occurs.
- Describe the mechanism of injury for the muscle, tendon and bones
- Describe how injury affects the mechanics of the muscle and tendon.
- List the common injuries that occur to bones, tendons and ligaments.
- Describe the effects of disuse, aging, exercise on the mechanical properties of bones, tendons

and ligaments.

71. 10.1 Force-Deformation Curve

Forces can affect an object’s shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, **Hooke’s law** is obeyed. In equation form, Hooke’s law is given by

$$\boldsymbol{F = k\Delta L,}$$

where $\boldsymbol{\Delta L}$ is the amount of deformation (the change in length, for example) produced by the force $\boldsymbol{F}$, and $\boldsymbol{k}$ is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation $\boldsymbol{\Delta L}$ — it is not constant as a kinetic friction force is. Sometimes we use $\boldsymbol{\Delta x}$ instead of $\boldsymbol{\Delta L}$**.** The deformation can be along any axis. Rearranging this to

$$\boldsymbol{\Delta L = \frac{F}{k}}$$

makes it clear that the deformation is proportional to the applied force. Figure 1 shows the Hooke’s law relationship between the extension $\boldsymbol{\Delta L}$ of a spring or of a human bone. For metals or springs, the straight line region in which Hooke’s law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture. **Tensile strength** is the breaking stress that will cause permanent deformation or fracture of a material.

HOOKE'S LAW

$$\boldsymbol{F = k\Delta L,}$$

where $\boldsymbol{\Delta L}$ is the amount of deformation (the change in length, for example) produced by the force $\boldsymbol{F}$, and$\boldsymbol{k}$ is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

$$\boldsymbol{\Delta L = \frac{F}{k}}$$

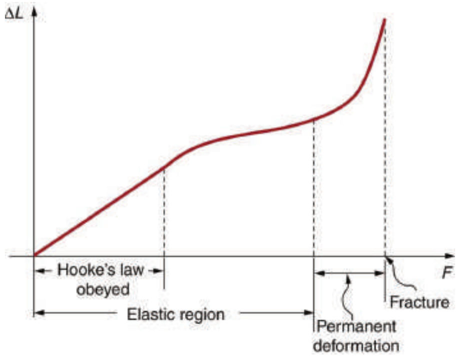

Figure 1. *A graph of deformation* ΔL *versus applied force* F*. The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is* $1/k$*. For larger forces, the graph is curved but the deformation is still elastic—*ΔL *will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force* F *is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in*F *is producing a large increase in* L *near the fracture.*

The proportionality constant k depends upon a number of factors for the material.

We now consider the type of deformation that will cause a change in length (tension and compression). There are also sideways forces that cause shear (stress), and changes in volume, but we will not go into detail about them in this course.

Changes in Length—Tension and Compression: Elastic Modulus

A change in length ΔL is produced when a force is applied to a muscle or tendon parallel to its length L_0, either stretching it (a tension) or compressing it. (See Figure 3.)

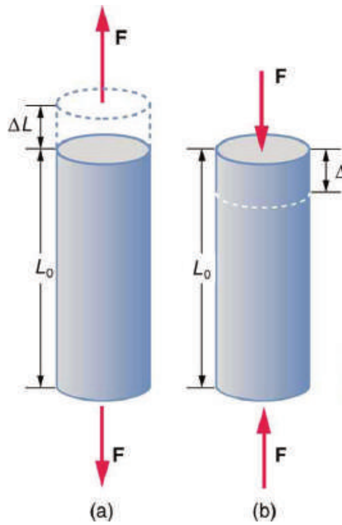

Figure 3. *(a) Tension. The muscle/tendon is stretched a length **ΔL** when a force is applied parallel to its length. (b) Compression. The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, **ΔL** is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.*

Experiments have shown that the change in length (**Δ*L***) depends on only a few variables. As already noted, **Δ*L*** is proportional to the force ***F*** and depends on the substance from which the material is made. Additionally, the change in length is proportional to the original length L_0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one. We can combine all these factors into one equation for **Δ*L***:

$$\boldsymbol{\Delta L = \frac{1}{Y}\frac{\vec{F}}{A}L_0},$$

where **Δ*L*** is the change in length, ***F*** the applied force, ***Y*** is a factor, called the elastic modulus or Young's modulus, that depends on the substance, ***A*** is the cross-sectional area, and L_0 is the original length. Table 3 lists values of ***Y*** for several materials—those with a large ***Y*** are said to have a large tensile stiffness because they deform less for a given tension or compression.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behaviour of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus the bone in the top of the femur is arranged in thin sheets separated by marrow while in other places the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Functionally, the tendon (the tissue connecting muscle to bone) must stretch easily at first when a force is applied, but offer a much greater restoring force for a greater strain. Figure 5 shows a stress-strain relationship for a human tendon. Some tendons have a high collagen content so there is relatively little strain, or length change; others, like support tendons (as in the leg) can change length up to 10%. Note that this stress-strain curve is nonlinear, since the slope of the line changes in different regions. In the first part of the stretch called the toe region, the fibers in the tendon begin to align in the direction of the stress—this is called *uncrimping*. In the linear region, the fibrils will be stretched, and in the failure region individual fibers begin to break. A simple model of this relationship can be illustrated by springs in parallel: different springs are activated at different lengths of stretch. Examples of this are given in the problems at end of this chapter. Ligaments (tissue connecting bone to bone) behave in a similar way.

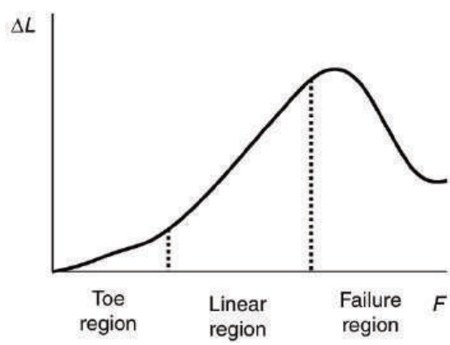

Figure 5. *Typical stress-strain curve for mammalian tendon. Three regions are shown: (1) toe region (2) linear region, and (3) failure region.*

Unlike bones and tendons, which need to be strong as well as elastic, the arteries and lungs need to be very stretchable. The elastic properties of the arteries are essential for blood flow. The pressure in the arteries increases and arterial walls stretch when the blood is pumped out of the heart. When the aortic valve shuts, the pressure in the arteries drops and the arterial walls relax to maintain the blood flow. When you feel your pulse, you are feeling exactly this—the elastic behaviour of the arteries as the blood gushes through with each pump of the heart. If the arteries were rigid, you would not feel a pulse. The heart is also an organ with special elastic properties. The lungs expand with muscular effort when we breathe in but relax freely and elastically when we breathe out. Our skins are particularly elastic, especially for the young. A young person can go from 100 kg to 60 kg with no visible sag in their skins. The elasticity of all organs reduces with age. Gradual physiological aging through reduction in elasticity starts in the early 20s.

Example 2: Calculating Deformation: How Much Does Your Leg Shorten When You Stand on It?

Calculate the change in length of the upper leg bone (the femur) when a 70.0 kg man supports 62.0 kg of his mass on it, assuming the bone to be equivalent to a uniform rod that is 40.0 cm long and 2.00 cm in radius.

Strategy

The force is equal to the weight supported, or

$$\boldsymbol{F = mg = (62.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 607.6\ \mathrm{N}},$$

and the cross-sectional area is $\pi r^2 = 1.257 \times 10^{-3}$ $\mathbf{m^2}$. The equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$ can be used to find the change in length.

Solution

All quantities except ΔL are known. Note that the compression value for Young's modulus for bone must be used here. Thus,

$$\Delta L = \left(\frac{1}{9 \times 10^9\ \mathrm{N/m^2}}\right)\left(\frac{607.6\ \mathrm{N}}{1.257 \times 10^{-3}\ \mathrm{m^2}}\right)(0.400\ \mathrm{m}) = 2 \times 10^{-5}\ \mathrm{m}.$$

Discussion

This small change in length seems reasonable, consistent with our experience that bones are rigid. In fact, even the rather large forces encountered during strenuous physical activity do not compress or bend bones by large amounts. Although bone is rigid compared with fat or muscle, several of the substances listed in Table 3 have larger values of Young's modulus Y. In other words, they are more rigid.

The equation for change in length is traditionally rearranged and written in the following form:

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}.$$

The ratio of force to area, $\frac{F}{A}$, is defined as **stress** (measured in

N/m²), and the ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as **strain** (a unitless quantity). In other words,

$$\textbf{stress} = Y \times \textbf{strain}.$$

In this form, the equation is analogous to Hooke's law, with stress analogous to force and strain analogous to deformation. If we again rearrange this equation to the form

$$F = YA\frac{\Delta L}{L_0},$$

we see that it is the same as Hooke's law with a proportionality constant

$$k = \frac{YA}{L_0}.$$

This general idea—that force and the deformation it causes are proportional for small deformations—applies to changes in length, sideways bending, and changes in volume.

STRESS

The ratio of force to area, $\frac{F}{A}$, is defined as stress measured in N/m^2.

STRAIN

The ratio of the change in length to length, $\boldsymbol{\frac{\Delta L}{L_0}}$, is defined as strain (a unitless quantity). In other words,

$$\mathbf{stress} = \boldsymbol{Y} \times \mathbf{strain}.$$

Section Summary

- Hooke's law is given by

  $$\boldsymbol{F = k\Delta L}, \text{ or } \boldsymbol{F = k\Delta x},$$

  where $\boldsymbol{\Delta L}$ is the amount of deformation (the change in length), $\boldsymbol{F}$ is the applied force, and $\boldsymbol{k}$ is a proportionality constant that depends on the shape and composition of the object and the direction of the force. The relationship between the deformation and the applied force can also be written as

  $$\boldsymbol{\Delta L = \frac{1}{Y}\frac{F}{A}L_0},$$

  where $\boldsymbol{Y}$ is *Young's modulus*, which depends on the substance, **A** is the cross-sectional area, and $\boldsymbol{L_0}$ is the original length.

- The ratio of force to area, $\frac{F}{A}$, is defined as *stress*, measured in N/m^2.
- The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as *strain* (a unitless quantity). In other words,
$$\textbf{stress} = Y \times \textbf{strain}.$$
-

Conceptual Questions

1: The elastic properties of the arteries are essential for blood flow. Explain the importance of this in terms of the characteristics of the flow of blood (pulsating or continuous).

2: What are you feeling when you feel your pulse? Measure your pulse rate for 10 s and for 1 min. Is there a factor of 6 difference?

3: Examine different types of shoes, including sports shoes and thongs. In terms of physics, why are the bottom surfaces designed as they are? What differences will dry and wet conditions make for these surfaces?

4: Would you expect your height to be different depending upon the time of day? Why or why not?

5: Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Problems & Exercises

1: During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.

2: During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex his already moribund adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius.

3: (a) By how much does a 65.0-kg mountain climber stretch her 0.800-cm diameter nylon rope when she hangs 35.0 m below a rock outcropping? (b) Does the answer seem to be consistent with what you have observed for nylon ropes? Would it make sense if the rope were actually a bungee cord?

Footnotes

1. 1 Approximate and average values. Young's moduli ***Y*** for tension and compression sometimes differ but are averaged here. Bone has significantly different Young's moduli for tension and compression.

Glossary

deformation
: change in shape due to the application of force

Hooke's law
: proportional relationship between the force ***F*** on a material and the deformation ***ΔL*** it causes, ***F = kΔL***

tensile strength
: the breaking stress that will cause permanent deformation or fraction of a material

stress
: ratio of force to area

strain
: ratio of change in length to original length

shear deformation
: deformation perpendicular to the original length of an object

Solutions

Problems & Exercises

1: $\mathbf{1.90 \times 10^{-3}\ cm}$

3: (a) $\boldsymbol{9\textbf{ cm}}$ (b) This seems reasonable for nylon climbing rope, since it is not supposed to stretch that much.