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# Financial Security and Household Life Insurance

Bridging the Gap

JOANNA DĘBICKA  
AND ANNA JĘDRZYCHOWSKA



# Financial Security and Household Life Insurance

This book offers a comprehensive understanding of a novel approach to household security following the loss of an adult member, introducing the concept of Bridging Household Life Insurance. It addresses the financial and functional instability that families face after the death of a spouse or partner by proposing a new type of insurance that compensates for the loss of unpaid domestic labour such as caregiving, household management, and other essential non-market activities.

Unlike traditional life insurance, the proposed contract focuses not only on income replacement but also on maintaining household stability by covering the cost of services that become necessary due to the loss of a family member's contributions. Thus, it helps ensure continuity in everyday life, reducing the burden on surviving family members and helping them adapt to new circumstances without significant disruptions.

This book develops an advanced multi-state model that considers the variability of the household life cycle, not just the future lifetime of the insured persons, which allows for more accurate contract valuation. Comparative analyses across EU countries are used to examine the impact of socio-demographic factors and the COVID-19 pandemic on the premium amount.

In addition, this book offers practical recommendations for implementing this insurance on the market and considers its applications in social and demographic policy, national accounts, and the fields of law and economics. The work enriches the existing literature with new tools and concepts that researchers, scholars, actuaries, and practitioners can use to manage risk better and protect households against financial instability.

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# Preface

*Joanna Dębicka and Anna Jędrzychowska*

Economic research should extend beyond merely describing reality or analysing existing markets. One of its essential tasks is also identifying new areas of intervention and designing solutions that respond to changing social and economic needs. This is particularly important in applied economics, including insurance and personal finance, where the practical application of scientific knowledge can truly contribute to increasing the economic security of individuals and households. In this context, the concept of assistance insurance was developed to offer organisational support for families after the death of an adult member of the household.

This book defines the concept of a household primarily through its functional role, rather than formal legal criteria. A household is understood as a unit composed of two adults, irrespective of their legal marital status, type of partnership (formal or informal), or gender composition. When these adults share responsibility for the care of young children, the unit is classified as a household with dependent children. This inclusive definition reflects contemporary social transformations and the growing diversity of family structures – realities that remain insufficiently acknowledged by existing national social security frameworks.

Having worked in the field of life insurance and personal finance for nearly two decades, the authors felt justified to consider the protection gap affecting households that depend on the unpaid work of one member, most commonly a woman. Modern insurance contracts provide minimal coverage for intangible losses such as those resulting from the sudden need to take on household and caregiving responsibilities. Meanwhile, the death of an adult, whether the primary breadwinner or the main provider of household labour, causes significant disruption for the entire family.

Reorganising daily life can pose a significant burden for those who are grieving, as it entails taking over childcare, preparing meals, and managing household responsibilities. The Bridging Household Life Insurance contract proposed in this book is designed to alleviate these challenges by providing targeted assistance during a critical period, thereby mitigating the risk of secondary economic losses. This support goes beyond emotional or symbolic

relief, offering the practical delegation of selected tasks to external service providers. Such an arrangement helps preserve family routines, for instance enabling children to continue attending school without disruption and allowing adults to maintain their employment and income. Thus, the proposed insurance functions as a preventative and supportive tool, as well as a stabilising mechanism that limits the ripple effect of a single disruptive event across multiple domains of family life.

The proposed insurance contract aligns with a significant trend in contemporary economic research that increasingly emphasises the value of unpaid domestic and care work. Analyses of satellite accounts and studies on personal injury valuation, e.g. those related to civil and compensation cases, demonstrate that while these activities are not reflected in the market, they possess real and measurable economic value. The introduced insurance can strengthen this argument by providing an actuarial valuation and converting it into specific service benefits. Additionally, it can raise public awareness and foster greater respect for those who perform these essential tasks for their household members.

The monograph begins by outlining the context and theoretical foundations related to unpaid domestic work and then proposes a new type of contract and presents an actuarial model designed to assess its value. The following part provides a comparative analysis of how this proposed insurance is valued internationally, as well as an exploration of the practical possibilities of applications of the Bridging Household Life Insurance contract within the insurance market and various economic, sociological, and legal fields. Lastly, possible directions for further research are discussed.

Chapter 1 explores how households adapt to new living conditions following the death of an adult member, with particular emphasis on the risk of poverty and lifestyle destabilisation for the survivors, especially widows and widowers. It highlights the need to secure access to services previously provided by the deceased, which may include both market and non-market contributions.

First, the chapter defines the household from a functional perspective, outlines key household types, and then analyses the range of losses incurred after the death of an adult household member, including the loss of income, unpaid personal contributions, increased care needs, and non-material harm such as pain and suffering. The central aim is to estimate compensation levels that reflect these multifaceted losses. In this context, particular attention is paid to unpaid domestic labour, which is typically captured in satellite accounts as the production of goods and services for own use.

Given that the type and timing of domestic contributions vary throughout a household's development, the chapter introduces the concept of household life-cycle phases. This framework helps explain how financial circumstances, resource allocation, and time use evolve over time depending on household

composition and the age of its members. Understanding these dynamics is essential for accurately assessing the economic consequences of the loss.

In the final section, the chapter introduces the concept of Bridging Household Life Insurance, a new type of insurance contract that combines elements of life insurance and personal injury compensation. The contract is designed to transfer the risk of temporary destabilisation following the death of an adult household member. Key parameters of the contract, including the subject of insurance, duration of coverage, and the scope of benefits, are presented and discussed.

In Chapter 2, to model the proposed insurance contract in Chapter 1, the authors used a multi-state model that gives the possibility to account for all possible events occurring in the lives of the insured individuals during the coverage period. As with any insurance, premiums and benefits are paid only when the general and specific insurance conditions (which are an integral part of the insurance policy) are met. These conditions are defined similarly to those in health insurance and are represented by ordered sets of parameters describing the realisation of different kinds of payments arising from the insurance contract. This allows the model to include such conditions as the maximum number of benefits paid, as well as payments made after the end of the insurance contract. This approach has enabled the development of the model for Bridging Household Life Insurance that is similar to the multi-state model used for marital life insurance. In the next phase of model development, it is expanded to incorporate the various stages of the household life cycle. Hence, the probabilistic structure of the model is determined not only based on the future lifetime of adult individuals (partners/spouses) but also based on the changing age and number of their children over the term of the policy. Due to practical applications, a discrete-time model is considered, where benefits and premiums are realised at the end of the time units into which the insurance period is divided. The chapter also specifies the financial structure of the model, including the benefits and the interest rate.

Given the unique nature of Bridging Household Life Insurance, which reflects the dynamics of the household life cycle rather than just the future lifetime of the adult family members, formulas for net premiums are applied that can be paid across multiple states.

Chapter 3 focuses on numerical illustrations that confirm the practical feasibility of the proposed solution. Specifically, the formulas introduced in Chapter 2 are applied to evaluate the Bridging Household Life Insurance contract across selected EU countries. Comparative analyses of net premium amounts are conducted, taking into account the diversities between the countries and the demographic structures of the households. Additionally, the impact of the COVID-19 pandemic on insurance pricing in European countries is examined.

This chapter also discusses the practical aspects of introducing the proposed insurance contract to the market, highlighting both the advantages and the challenges associated with this solution. The chapter indicates that the actuarial analysis of cash flows from the Bridging Household Life Insurance contract can be seen as a valuation of the household's own labour. As a result, other potential areas where this approach might be applicable are identified. The chapter concludes with considerations for future research directions related to the theoretical and applicational aspects of designing, modelling, and conducting actuarial analyses of the proposed insurance contract.

The individual parts of this monograph complement each other and together form a cohesive whole tailored to the needs of various target audiences.

Chapter 1 is suited for students specialising in disciplines such as insurance, economics, law, or management. It offers a solid theoretical foundation that may serve as a starting point for further academic exploration. This section may also be of interest to readers who are just beginning to engage with insurance-related topics or who wish to understand the connections between household insurance, unpaid domestic work, and the household life cycle. It may also appeal to insurance professionals seeking a more nuanced understanding of household insurance in its broader socio-economic context.

Chapter 2 is aimed at professionals in the actuarial and financial sectors, including actuaries and risk analysts, who are interested in the technical aspects of insurance risk modelling. This chapter provides theoretical frameworks that can be applied in practice when designing insurance products or assessing risk. It will also benefit Master's and PhD students in fields involving actuarial mathematics, insurance, and risk analysis, particularly those focused on modelling and quantitatively evaluating insurance contracts.

Lastly, Chapter 3 is intended for managers and product development specialists in insurance companies who are looking for real-world examples of how the proposed concepts and models are applied in different national contexts. This chapter also addresses the needs of decision-makers and industry leaders responsible for implementing innovative insurance products, policies, and strategies. Consultants and analysts working in advisory companies may also find this section valuable, especially those involved in drafting reports and/or strategies for developing insurance markets.

This book aims to provide guidance for developing insurance practices that align with international trends, evolving social needs, and the capabilities of modern financial and insurance institutions. The authors believe that creating insurance contracts to protect households from the risks associated with the loss of unpaid work is not only achievable but also essential from both an economic and social policy perspective. In this regard, the proposed Bridging Household Life Insurance illustrates how scientific knowledge can be transformed into innovative market instruments that benefit society.

# 1 Household Work and Insurance

## Concepts, Risks, and Coverage

*Anna Jędrzychowska*

### 1.1 Introduction

Unpaid work performed by household members, such as cleaning, cooking, childcare, and caregiving, constitutes a substantial contribution to family well-being and societal functioning. Although this labour does not receive direct monetary compensation, it remains essential for maintaining household operations and supporting economic productivity indirectly. According to Folbre (2012), “unpaid household work is a critical component of the economy that often remains invisible in traditional economic measures yet underpins the functioning of paid labour markets”. This type of work involves time-intensive and skill-demanding tasks, and despite its invisibility in formal labour statistics, it produces real economic value by sustaining human capital and enabling workforce participation.

It is also worth noting that unpaid domestic work has gained visibility in modern national accounting systems, largely due to time-use surveys and imputation methods used to estimate its monetary value. This growing recognition is particularly evident in the development of satellite accounts, which complement traditional economic measures by capturing the significance of non-market household labour. These satellite accounts utilise time-use surveys and imputed wage rates to assign monetary value to household production, thereby integrating unpaid labour into broader economic assessments. Ferrant et al. (2014) argued that including unpaid work in satellite accounts provides a more comprehensive understanding of economic welfare and labour inputs, uncovering dimensions of economic activity often obscured in conventional metrics. Similarly, Folbre (2006a) emphasised that recognising unpaid care work is crucial for capturing the full scope of labour that sustains both families and economies. Furthermore, Van de Ven, Zwijnenburg, and De Queljo (Van de Ven et al., 2018) provided a robust methodological framework for integrating unpaid household activities into national accounts and demonstrated that valuing such non-market production via time-use surveys and imputed wage rates substantially improves the accuracy and policy relevance of macroeconomic indicators by revealing the hidden contributions of domestic labour

to GDP and economic growth. Lastly, scientists also described the practical implications of incorporating unpaid household activities in national accounts, showing the significant impacts on GDP estimates and labour input statistics across OECD countries. Together, these studies underline the importance of integrating unpaid domestic work into economic frameworks to create more equitable and effective social and economic policies.

Moreover, it is essential to recognise that households function not only as units of consumption but also as producers of unpaid services – a role largely overlooked in insurance modelling – as they provide critical non-market contributions such as childcare, care for elderly, and home maintenance. Despite their macroeconomic importance, these functions are underrepresented in financial systems and remain difficult to insure. Unpaid domestic work, predominantly performed by women, has substantial value that is difficult to quantify and insure, yet it underpins the resilience of household functioning and intergenerational welfare transfer (Castellani et al., 2021).

Against this backdrop, the life-cycle concept becomes particularly relevant, offering a framework for understanding how household needs and vulnerabilities evolve over time. Household economic behaviour is undeniably shaped by the life course, influencing income patterns, debt accumulation, and insurance requirements. As established by Modigliani in his study on the life-cycle hypothesis (2005), individuals and households plan consumption and saving behaviour with foresight across different life stages. These evolving financial needs yet to be fully captured by traditional insurance products highlight a critical gap in economic policymaking and product design.

### **1.2 Household Classification in the Context of Life Cycle**

Contemporary definitions of the household, as employed by national statistical offices and international institutions such as Eurostat, are increasingly moving away from the traditional association of the household with the family. Instead, they prioritise functional and economic relationships, including cohabitation and shared budgeting. This shift reflects the real diversity of living arrangements, encompassing informal partnerships, unrelated individuals sharing accommodation, and cohabiting persons managing a household jointly.

An analysis of definitions used by statistical offices across Europe, including those in France, Finland, Belgium, and Poland, indicates that familial ties are not a necessary condition for recognising a collective unit as a household. For instance, Eurostat defines a household as any group of individuals living together and sharing resources or meals, regardless of their kinship. Similarly, Statistics Poland defines a household as an economic unit that may consist of a single person or several people jointly supporting themselves, irrespective of legal or family relationships.

In this monograph, a functional (rather than legal) definition of the household is adopted. The household is considered to comprise two adults, regardless of

marital status, gender composition, or the formality of their partnership. If the adults are responsible for caring for dependent children, the unit is classified as a household with children. This approach reflects evolving societal norms and the increasing diversity of family forms, which remain insufficiently acknowledged in national social protection systems.

In many countries, access to social insurance and welfare support depends on having a formal legal relationship, typically marriage. As a result, a second adult who is actively involved in childrearing and financial support may not be recognised by the system. This exclusion becomes particularly problematic when the legally recognised partner, such as the only insured party or the sole legal guardian, dies. In such cases, institutional gaps may lead to increased economic insecurity and unequal treatment of households that do not conform to the traditional family model.

The demographic and social relevance of informal partnerships involving children has grown markedly. According to Eurostat data, the number of children raised in such unions is increasing steadily in most European countries. In Sweden, France, and the Netherlands, over half of all children are now born outside of marriage. A similar, although slower, trend can be observed in Poland and other Central and Eastern European countries.

There is also a rising number of children growing up in non-traditional family structures, such as those raised by a biological parent and that parent's partner. This shift highlights the need to adapt social security and insurance mechanisms to the realities of contemporary households. In addition, modern legal approaches increasingly move beyond strictly formal definitions of family and instead embrace broader interpretations of social and emotional ties, particularly in the context of compensation for the loss of a loved one.

There is growing recognition that what matters in determining eligibility for support is not legal status, such as marriage or kinship, but the existence of a stable emotional bond and shared responsibilities. Courts in various jurisdictions are more frequently acknowledging that people who live together, share caregiving duties, and offer each other daily support may be considered 'close persons' for legal purposes. This aligns with the evolution of family and social life, where legal protection is increasingly grounded in actual closeness and interdependence, rather than formal or biological ties alone.

In response to these developments, the insurance solution proposed in this study expands beyond the conventional boundaries of social insurance. It is designed to protect households formed not only through formal relationships but also informal unions, including same-sex partnerships. As society evolves and family structures become more diverse, traditional protection systems often fall short of recognising this variety. The proposed approach offers a practical way of addressing this gap, contributing to the economic resilience and long-term security of contemporary households.

To develop effective insurance and policy mechanisms, it is necessary to understand not only how households are defined and recognised in legal or statistical terms but also how they evolve over time. Their internal dynamics,

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life-course patterns, and changing risk exposures are crucial for designing appropriate financial instruments and social protections. Households evolve through identifiable stages that fundamentally shape their composition, allocation of work, financial behaviour, and exposure to economic risk. The family life-cycle framework, initially introduced by Glick (1947), has since been substantially developed to incorporate changing social structures and consumption patterns (Hawks & Ackerman, 1990; Murphy & Staples, 1979). These models describe phases, including formation, expansion, stabilisation, and disintegration, each aligned with specific demographic transitions (e.g. marriage, childbirth, and retirement) and functional transformations (e.g. income source shifts and caregiving responsibilities). The progression through these stages is not uniform across societies, particularly when local institutions and norms mediate familial and financial behaviour.

While these classical models provide a useful heuristic in Western contexts, they may not fully capture the diversity of household trajectories globally. In rural and developing economies, family life cycles tend to deviate from Western models, reflecting intergenerational dependencies and informal safety nets. Liang et al. (2015) analysed the Chinese family system and identified five distinct stages characterised by dynamic income flows and evolving caregiving responsibilities. Their study highlighted the re-entry of older family members into multigenerational households due to insufficient pension systems and welfare support. This reintegration alters household labour structures, dependency ratios, and financial strategies, emphasising the importance of cultural context in designing household-level economic interventions.

These culturally and institutionally shaped life-cycle patterns directly influence household engagement with financial markets and instruments, and their temporal dynamics have critical implications for household interaction with credit, insurance, and savings markets. Chantarat et al. (2020) demonstrated that household indebtedness follows an inverted U trajectory across the life cycle, with debt accumulation peaking during middle adulthood – when consumption and housing investments are highest – and gradually declining in retirement. However, many older households continue to carry debt, particularly in the absence of adequate retirement income or due to health shocks. Understanding these life stage transitions is essential for aligning financial instruments with household risk preferences, especially as the burden of long-term care, medical expenses, and income volatility shifts over time (Jian Xiao & Yao, 2014).

Beyond temporal variation, the structural composition of households plays an equally crucial role in shaping economic resilience and vulnerability. Complementing life-cycle approaches, recent literature has advanced more granular frameworks for household classification based on structural and functional criteria. A pivotal distinction is made according to employment structure, leading to the categorisation of households as zero-earner, single-earner, or dual-earner units. These types are frequently utilised in macroeconomic labour models to simulate household-level labour supply, consumption smoothing, and participation decisions (Ljungqvist & Sargent, 2018). In dual-earner

households, joint labour force participation increases bargaining power and reduces vulnerability to income shocks through enhanced intra-household risk-sharing mechanisms.

However, household vulnerability is not determined by labour structure alone. Composition-based typologies – such as single-parent or multigenerational households – introduce additional layers of economic complexity and policy relevance. Beyond labour status, household typologies are further stratified by composition, including single-adult, couple-based, single-parent, and multi-generational households. These configurations are directly linked to socioeconomic outcomes, for example poverty risk and benefit dependency. Azzollini et al. (2025) found that single-parent households without any employed adult consistently experience elevated poverty rates across high-income countries, whereas dual-earner households with children show significantly more resilience to economic adversity. These insights are vital for tailoring tax-benefit policies, in-work subsidies, and universal basic service frameworks to the diversity of household configurations.

While structural classifications highlight external characteristics, microeconomic analysis reveals how internal household arrangements, particularly the distribution of income, shape financial well-being and satisfaction. Empirical findings by Bonke and Browning (2009) indicated that partners' relative income contributions play a decisive role in determining their own and their spouse's financial satisfaction, suggesting that a more balanced distribution of earnings within couples is associated with improved subjective well-being. This relationship is particularly salient in dual-earner households, where income parity may enhance feelings of fairness and reduce intra-household tension. Complementary research underlines that income arrangements are not merely economic outcomes but are intertwined with broader social constructs, including gender norms and intra-household bargaining processes. Thus, financial equality between partners contributes not only to economic resilience but also to relational satisfaction and household-level stability (Bennett, 2013).

Beyond financial well-being, household structure also has significant implications in applied fields, e.g. transportation and urban planning and employment status are crucial for modelling travel demand and planning infrastructure. Empirical studies demonstrated that these demographic characteristics significantly shape trip generation patterns and mobility behaviour. For instance, Zhang et al. (2019) showed that while higher-density environments influence the volume and type of household trips, household-level attributes such as car ownership and income remain dominant predictors of travel frequency and mode choice. Similarly, Koszowski et al. (2024) presented a methodology for enhancing travel surveys by incorporating detailed stage-level information, stressing the need to consider household-specific structures in transportation planning. These findings suggest that household heterogeneity plays a central role in shaping urban resource allocation and mobility systems, reinforcing the relevance of microeconomic classification in broader spatial and policy contexts.

This growing awareness of household heterogeneity across domains, from income distribution to mobility patterns, underlined the need for precise and adaptable classification systems, and as a result, statistical and financial institutions have developed diverse household typologies tailored to their analytic and policy objectives (from demographic analysis to financial stability and cross-country comparability). In Poland, Statistics Poland (GUS) employs definitions rooted in the Household Budget Survey, where households are defined as people sharing living arrangements and/or meals and are classified by size (e.g. single person, two to five persons), structure (e.g. couple with/without children, single-parent, multi-generational), and income type. This enables precise measurement of poverty, inequality, and the impact of benefit programmes, including ‘Family 500+’ (actually ‘Family 800+’). The National Bank of Poland approaches classification from a financial-stability perspective. It segments households based on income source (e.g. wage-earners and pensioners), debt-service-to-income metrics, and demographic markers such as age, education, and rural vs. urban location. This typology supports macro-prudential oversight, pinpointing vulnerable segments, particularly indebted multi-person, rural households, at risk from economic shocks. International comparisons are enabled through OECD and Eurostat classifications. The OECD aligns households with the institutional ‘household sector’ from the System of National Accounts (SNA) – a grouping used to compare consumption, wealth, labour participation, and digital inclusion across countries. Eurostat further disaggregates household types: single adults, couples (with/without children), single parents, and multi-person households, additionally classifying them by the employment status of adult members. This granularity is essential for the EU-wide labour market and poverty analyses. While not directly classifying households, the World Bank categorises economies based on gross national income per capita, applying the Atlas method to smooth exchange-rate fluctuations. Countries are assigned to low, lower-middle, upper-middle, or high-income categories, updated annually on 1 July, and this schema informs about their lending eligibility and provides a framework for comparative household survey analyses, for example Living Standards Measurement Study and Poverty & Inequality Platform. Beyond these well-established bodies, several other entities use household classification for specialised purposes. The United Nations Department of Economic and Social Affairs publishes standardised typologies (e.g. nuclear, extended, single-person, multi-person non-kin) to monitor global demographic trends. Similarly, the Bank for International Settlements aggregates household and Non-Profit Institutions Serving Households data in financial statistics such as debt-service ratios, contributing a sectoral view essential for macroeconomic risk assessments. Additionally, central banks like the Bank of Italy conduct detailed household finance surveys (Survey on Household Income and Wealth) examining the number of earners, income sources, assets, and liabilities for microsimulation and policy modelling.

These diverse classification systems enable the monitoring and interpretation of demographic patterns across countries, while in the European Union (EU) such classifications help capture the wide-ranging household structures reflecting regional demographic and cultural dynamics. The structure of households across the EU reveals significant demographic and cultural diversity, which in turn shapes economic behaviour, social policies, and welfare needs. Single-person households constitute a substantial share of all households in many EU countries, particularly in Northern and Western Europe, where Sweden, Finland, Denmark, and Germany exhibit single-person household rates ranging from approximately 30% to 40%. This trend correlates with higher urbanisation, ageing populations, and social changes, including delayed family formation and increased longevity (Eurostat, 2023). The predominance of one-person households highlights emerging challenges related to social isolation, housing demand and tailored social services. Two-person households, often representing couples without children or with one child, form another large segment of the EU's household landscape. In Southern and Eastern European countries – including Poland, Hungary, and Romania – there remains a higher prevalence of larger, multi-person households, frequently encompassing families with young children or multigenerational arrangements. These patterns reflect enduring traditional family structures, stronger intergenerational ties, and the role of informal support networks, which influence household economic strategies and social risk management (Eurostat, 2023; OECD, 2023). Such diversity underlines the necessity of regionally sensitive social and economic policies that address differing household compositions and their specific needs. Households with young children (typically defined as those under six years of age) constitute about 15–20% of family households across most EU member states. Countries with higher fertility rates, such as France and Ireland, tend to report a larger proportion of such households, while nations facing demographic decline and ageing populations, for example Germany and Italy, show comparatively lower shares. Understanding these demographic distributions is crucial for the design of family policies, childcare infrastructure, and social support mechanisms that adequately reflect the evolving needs of diverse household types.

To interpret these structural patterns more dynamically, scholars often turn to the concept of the household life cycle, which provides a temporal framework for understanding how household composition and economic behaviour change over time. The household life cycle is a foundational concept in both economics and demography, describing how household structure, income patterns, consumption needs, and financial behaviour evolve as individuals age and experience key life transitions. Originating in the work of Modigliani and Brumberg (1954), the life-cycle hypothesis suggests that individuals seek to smooth consumption over time, saving during high-earning years and ‘dissaving’ during retirement. Glick (1947) was among the first to systematically describe how households transition through sequential stages: formation,

*Table 1.1* Household structure in EU Countries (in 2023) – selected categories (percentage)

| <i>Country</i> | <i>1-Person</i> | <i>2-Person</i> | <i>With young children (&lt;6 years)</i> | <i>Multiperson without children</i> | <i>Multigenerational households</i> |
|----------------|-----------------|-----------------|------------------------------------------|-------------------------------------|-------------------------------------|
| Austria        | 33.1            | 27.5            | 13.2                                     | 15.0                                | 11.2                                |
| Belgium        | 33.4            | 28.1            | 15.5                                     | 12.0                                | 11.0                                |
| Bulgaria       | 23.5            | 30.2            | 16.0                                     | 20.3                                | 10.0                                |
| Croatia        | 25.0            | 28.0            | 14.0                                     | 20.0                                | 13.0                                |
| Cyprus         | 21.0            | 29.0            | 18.0                                     | 22.0                                | 10.0                                |
| Czechia        | 28.5            | 31.0            | 16.5                                     | 14.0                                | 10.0                                |
| Denmark        | 38.0            | 27.5            | 13.5                                     | 12.0                                | 9.0                                 |
| Estonia        | 34.0            | 27.0            | 15.0                                     | 13.0                                | 11.0                                |
| Finland        | 39.0            | 28.0            | 12.0                                     | 11.0                                | 10.0                                |
| France         | 34.0            | 27.5            | 19.0                                     | 11.0                                | 8.5                                 |
| Germany        | 39.5            | 26.5            | 13.0                                     | 12.0                                | 9.0                                 |
| Greece         | 22.0            | 29.0            | 15.0                                     | 19.0                                | 15.0                                |
| Hungary        | 26.0            | 30.5            | 15.5                                     | 18.0                                | 10.0                                |
| Ireland        | 31.5            | 26.0            | 21.0                                     | 13.5                                | 8.0                                 |
| Italy          | 27.0            | 28.0            | 14.0                                     | 20.0                                | 11.0                                |
| Latvia         | 35.0            | 27.0            | 15.0                                     | 13.0                                | 10.0                                |

*Source:* Own work based on (Eurostat 2023).

expansion (childbearing and childrearing), consolidation, and dissolution (widowhood or empty-nest), with each phase marked by distinct demographic and economic features. Later refinements Murphy and Staples (1979), Yan and Zhou (2016) acknowledged the increasing diversity of family forms and labour arrangements, incorporating single-parent families, dual-earner couples, and multi-generational living situations into more flexible life-cycle models.

These models allow to trace household behaviour across distinct stages. In the early phases, typically encompassing young adulthood and household formation, specific financial and demographic patterns begin to emerge. In the early stages of the household cycle, typically characterised by young adulthood and household formation, incomes are relatively low while consumption demands may be high, often requiring credit to bridge shortfalls. Households in this phase are more likely to experience financial instability due to high educational and/or housing debt and limited wealth accumulation. As noted by Chantarat et al. (2020), “household debt follows an inverted-U trajectory, peaking in mid-life when income is highest and financial obligations, such as mortgages, childcare, and education costs, are most intense”. Liang et al. (2015) further argued that in developing economies this structure is complicated by intergenerational dependencies, where elder household members may return during retirement, affecting both dependency ratios and resource allocation.

While households move through their life course, financial pressures shift in nature rather than disappear. As households transition into the consolidation

and pre-retirement phase, incomes typically peak, assets accumulate, and borrowing slows. However, family demands often persist. Research by Kowalski et al. (2023) demonstrated that households with dependent children, regardless of income level, show elevated debt-to-income ratios and a greater likelihood of financial distress. Children are an important factor contributing to debt delinquency, they argue, emphasising the multidimensional burden of caregiving. At the same time, research has drawn attention to how intra-household income dynamics and gender roles affect well-being and satisfaction, particularly during the consolidation phase, when income peaks and family responsibilities remain high. It should be noted, however, that these roles have been slowly shifting over the years, with men's involvement in domestic work and childcare gradually increasing (England, 1986). Navarro and Salverda (2019) suggested that the share of household income contributed by each partner is associated with differential levels of satisfaction, particularly when moderated by gender and occupational context. Their analysis revealed that individuals tend to report higher job and life satisfaction when they contribute a substantial, though not necessarily equal, share of household income. However, the relationship is nuanced: while equal income contributions are positively associated with satisfaction among men in higher-status occupations, women's well-being appears less dependent on income parity and more influenced by broader employment and family dynamics (Addabbo et al., 2010). These findings challenge assumptions about the universal benefits of income equality within households and highlight the complex interplay between economic arrangements, gender roles, and personal well-being.

In the later stages of the household life cycle, particularly post-retirement, household income typically declines while medical, caregiving and housing needs rise. Although classical theory predicts asset decumulation at this stage, many older adults retain debt, especially in the contexts of high housing costs and incomplete pension coverage. Some authors (Bédard & Michaud, 2021; Mudrazija & Butrica, 2023) stated that older households in Canada and the United States were increasingly likely to carry mortgage or consumer debt into retirement. Moreover, longitudinal health and finance studies suggest that long-term indebtedness in older age is associated with chronic stress, poorer health outcomes, and higher mortality risk (cf. Sweet et al., 2013). These patterns stress the importance of aligning retirement, credit, and social insurance policies with the real trajectories of household financial ageing.

The household life-cycle framework, rooted in the seminal work of Modigliani and Brumberg (1954), posits that individuals strive to maintain a stable consumption path by borrowing during early adulthood and saving during peak earning years before drawing on assets in retirement. This consumption-smoothing behaviour is supported by household surveys, for example Canada's Survey of Consumer Finances, which indicates that median net worth tends to peak during late working-age before tapering off in retirement. Another study in Poland (Anioła-Mikołajczak, 2017) affirms this pattern,

finding that younger household heads are the most active borrowers, consistent with the life-cycle hypothesis, and are more prone to indebtedness relative to older cohorts. During child-rearing and household expansion stages, financial obligations often intensify. Studies across multiple countries demonstrate that the addition of children increases household debt and incidence of delinquency. Xiao and Yao (2014) observed that children are an important factor contributing to debt delinquency among families in middle age. Similarly, research from the Asian Development Review (Chantarat et al., 2020) documented a clear inverted-U shape in indebtedness across life phases, with the peak occurring during middle adulthood due to cumulating responsibilities. These findings are echoed in national studies in Thailand, where granular credit bureau data also revealed a midlife peak in debt levels and delinquency.

In the retirement stage, life-cycle theory predicts asset decumulation; however, evidence indicates that many older households carry debt well into their senior years. Lee et al. (2007) documented persistent mortgage holdings among US retirees, a trend confirmed by Crawford and Faruqi (2011) in Canada. Polish research further signals that over-indebtedness in old age is significantly associated with low income and limited financial literacy (cf. Świecka, 2016; Wałęga & Wałęga, 2021). This sustained indebtedness correlates with increased stress and poorer health outcomes among older individuals. Collectively, these findings suggest the need for tailored financial policies that address retirement-stage debt, including reverse mortgages, targeted debt relief, and enhanced social pensions.

Based on the detailed Eurostat and UNECE data, a nuanced cross-national analysis reveals substantial variability in the timing of key life-course transitions across EU member states. The differences between men and women, and between regions, point to deep-seated socio-economic and cultural structures that shape demographic behaviour – particularly in terms of leaving the parental household, entering parenthood, and forming formal unions. These stages are increasingly being postponed, yet the pace and structure of this delay vary considerably by gender and geography.

Across Europe, the transition to adulthood, marked by leaving the parental household, entering into marriage, and becoming a parent, is occurring at increasingly later ages, with notable gender and regional disparities. On average across the EU-27, men leave home at 27.2 years, while women do so at 25.4 years, maintaining a consistent gender gap of around two years. This gap reflects broader socio-economic expectations around male financial independence and labour market stability. In Croatia (33.4 for men, 30.3 for women) and Slovakia (32.4 vs. 29.6), home-leaving is markedly delayed for both genders, suggesting structural barriers such as housing constraints, high youth unemployment, and persistent intergenerational co-residence. In contrast, Nordic countries like Sweden (22.0 vs. 21.6) and Finland (22.0 vs. 20.7) exhibit the earliest departures from the parental home, supported by institutional frameworks that facilitate early autonomy, such as student housing and generous social support.

Table 1.2 Demographic milestones across the EU

| <i>Country</i> | <i>Average age of leaving parental household – males</i> | <i>Mean age at first marriage – males</i> | <i>Average age of leaving parental household – females</i> | <i>Mean age at first marriage – females</i> | <i>Mean age of women at childbirth</i> | <i>Mean age of women at birth of first child</i> | <i>Mean age of women at birth of second child</i> | <i>Mean age of women at birth of third child</i> | <i>Mean age of women at birth of fourth and higher order child</i> |
|----------------|----------------------------------------------------------|-------------------------------------------|------------------------------------------------------------|---------------------------------------------|----------------------------------------|--------------------------------------------------|---------------------------------------------------|--------------------------------------------------|--------------------------------------------------------------------|
| EU             | 27.2                                                     | –                                         | 25.4                                                       | –                                           | 31.2                                   | 29.8                                             | 32.0                                              | 33.0                                             | 34.2                                                               |
| Austria        | 26.0                                                     | 35.3                                      | 24.5                                                       | 32.7                                        | 31.2                                   | 29.9                                             | 31.9                                              | 33.0                                             | 34.5                                                               |
| Belgium        | 27.0                                                     | 35.2                                      | 25.7                                                       | 32.9                                        | 31.1                                   | 29.5                                             | 31.6                                              | 33.1                                             | 34.9                                                               |
| Bulgaria       | 31.8                                                     | 31.8                                      | 28.2                                                       | 28.9                                        | 28.2                                   | 26.9                                             | 29.6                                              | 29.5                                             | 30.2                                                               |
| Cyprus         | 28.2                                                     | –                                         | 26.8                                                       | –                                           | 31.5                                   | 29.8                                             | 32.8                                              | 33.7                                             | 33.8                                                               |
| Czechia        | 26.8                                                     | 33.0                                      | 24.1                                                       | 30.4                                        | 30.4                                   | 28.9                                             | 31.4                                              | 33.1                                             | 33.5                                                               |
| Germany        | 24.7                                                     | –                                         | 23.1                                                       | –                                           | 31.3                                   | 29.8                                             | 32.2                                              | 33.2                                             | 34.3                                                               |
| Denmark        | 22.2                                                     | –                                         | 21.3                                                       | –                                           | 31.8                                   | 30.1                                             | 32.3                                              | 34.2                                             | 36.2                                                               |
| Estonia        | 23.6                                                     | 33.2                                      | 22.0                                                       | 30.7                                        | 31.0                                   | 28.7                                             | 31.5                                              | 33.6                                             | 35.8                                                               |
| Greece         | 32.3                                                     | 34.1                                      | 28.8                                                       | 31.2                                        | 32.1                                   | 31.0                                             | 33.0                                              | 33.3                                             | 32.7                                                               |
| Spain          | 31.3                                                     | 37.0                                      | 29.4                                                       | 34.9                                        | 32.6                                   | 31.5                                             | 33.5                                              | 34.0                                             | 35.2                                                               |
| Finland        | 22.0                                                     | 34.4                                      | 20.7                                                       | 32.3                                        | 31.5                                   | 30.0                                             | 31.9                                              | 33.2                                             | 35.1                                                               |
| France         | 24.4                                                     | –                                         | 23.0                                                       | –                                           | 31.0                                   | 29.1                                             | 31.6                                              | 33.1                                             | 34.9                                                               |
| Croatia        | 33.4                                                     | 31.7                                      | 30.3                                                       | 29.3                                        | 30.8                                   | 29.3                                             | 31.6                                              | 32.8                                             | 33.4                                                               |
| Hungary        | 28.4                                                     | 32.9                                      | 25.8                                                       | 30.2                                        | 30.2                                   | 28.8                                             | 30.9                                              | 31.8                                             | 32.2                                                               |
| Ireland        | 28.5                                                     | –                                         | 27.5                                                       | –                                           | 33.2                                   | 31.6                                             | 33.8                                              | 35.0                                             | 35.5                                                               |
| Italy          | 30.9                                                     | 36.5                                      | 29.2                                                       | 33.9                                        | 32.5                                   | 31.8                                             | 33.2                                              | 33.4                                             | 33.5                                                               |
| Lithuania      | 25.0                                                     | –                                         | 23.2                                                       | –                                           | 30.4                                   | 28.4                                             | 31.5                                              | 33.6                                             | 35.2                                                               |
| Luxembourg     | 27.1                                                     | 35.2                                      | 26.3                                                       | 33.0                                        | 32.4                                   | 31.4                                             | 33.3                                              | 34.5                                             | 35.0                                                               |
| Latvia         | 26.8                                                     | 32.9                                      | 25.5                                                       | 30.5                                        | 30.4                                   | 28.0                                             | 31.2                                              | 33.3                                             | 34.8                                                               |
| Malta          | 28.0                                                     | –                                         | 29.2                                                       | –                                           | 31.2                                   | 29.8                                             | 32.6                                              | 33.6                                             | 33.9                                                               |
| Netherlands    | 23.8                                                     | –                                         | 22.5                                                       | –                                           | 31.8                                   | 30.4                                             | 32.5                                              | 33.8                                             | 35.4                                                               |
| Poland         | 28.3                                                     | 30.8                                      | 26.1                                                       | 28.5                                        | 30.1                                   | 28.4                                             | 30.9                                              | 32.5                                             | 34.3                                                               |
| Portugal       | 29.7                                                     | 35.0                                      | 28.3                                                       | 33.2                                        | 31.7                                   | 30.2                                             | 33.1                                              | 34.1                                             | 34.8                                                               |
| Romania        | 30.0                                                     | 31.5                                      | 25.3                                                       | 28.3                                        | 28.3                                   | 27.1                                             | 29.1                                              | 29.8                                             | 31.4                                                               |
| Sweden         | 22.0                                                     | 37.0                                      | 21.6                                                       | 34.8                                        | 31.7                                   | 30.0                                             | 32.2                                              | 33.9                                             | 35.8                                                               |
| Slovenia       | 30.3                                                     | 34.3                                      | 27.8                                                       | 31.9                                        | 30.5                                   | 29.1                                             | 31.2                                              | 32.8                                             | 34.3                                                               |
| Slovakia       | 32.4                                                     | 31.9                                      | 29.6                                                       | 29.3                                        | 28.8                                   | 27.3                                             | 29.8                                              | 30.4                                             | 30.5                                                               |
| <i>Min</i>     | <i>22.0</i>                                              | <i>30.8</i>                               | <i>20.7</i>                                                | <i>28.3</i>                                 | <i>28.2</i>                            | <i>26.9</i>                                      | <i>29.1</i>                                       | <i>29.5</i>                                      | <i>30.2</i>                                                        |
| <i>Max</i>     | <i>33.4</i>                                              | <i>37.0</i>                               | <i>30.3</i>                                                | <i>34.9</i>                                 | <i>33.2</i>                            | <i>31.8</i>                                      | <i>33.8</i>                                       | <i>35.0</i>                                      | <i>36.2</i>                                                        |

Source: Own work based on (Eurostat, 2025).

The timing of first childbirth continues to rise, particularly in Western and Southern Europe. The EU average age of women at the birth of their first child is now 29.8 years, with Ireland (31.6), Italy (31.8), and Spain (31.5) exhibiting the most advanced maternal ages. These postponements are strongly correlated with increased female educational attainment, delayed labour market entry, and evolving partnership models. In comparison, Romania (27.1), Bulgaria (26.9), and Slovakia (27.3) report some of the youngest average ages at first birth, reflecting cultural norms and divergent economic contexts. Moreover, in high-income countries, postponement is often paired with lower fertility and higher rates of non-marital childbearing, highlighting the decoupling of childbirth from traditional family structures.

Marriage, like childbirth, is increasingly delayed across the EU and continues to show persistent gender gaps. In nearly all countries with available data, men marry later than women – by approximately two to 2.5 years. For example, in Italy, men marry at 36.5 on average compared to 33.9 for women, whereas in Sweden the difference is even more pronounced (37.0 vs. 34.8). This aligns with social expectations around male readiness and financial security before formalising unions. The earliest marriages are noted in Poland (30.8 for men, 28.5 for women) and Romania (31.5 vs. 28.3), indicative of more traditional sequencing of life-course transitions in Central and Eastern Europe.

As seen in Figure 1.1, there is a clear and consistent increase in the mean age of women at childbirth with each successive child across the EU-27 countries in 2020. The average age of women at the birth of their first child typically ranges between 28 and 32 years, with some variation among countries, while for the second child the average age increases by approximately two years, and this upward trend continues for third and higher-order births.

The pattern is particularly evident in countries such as Italy, Spain, and Greece, where delayed childbearing is more common, and the mean age for all birth orders tends to be higher than the EU average. In contrast, Central and Eastern European countries like Bulgaria, Romania, and Slovakia exhibit noticeably lower average maternal ages, reflecting different socio-cultural and economic dynamics related to family formation and fertility behaviour.

Interestingly, while the age gap between first and second births is relatively stable across most countries, the variation becomes more pronounced at higher birth orders. The average age at the birth of the fourth or higher-order child shows significant cross-country disparities, which may be attributed to differences in family policies, childbearing preferences and access to reproductive healthcare.

Another notable feature is the relatively flat trajectory of age increase in some countries for higher-order births, suggesting either a smaller number of such births or a more compressed timing between children. The Nordic countries, for instance, show a more uniform age pattern, potentially due to supportive family policies that encourage both early and sustained fertility.

Overall, the graph reflects the broader demographic trend of postponed motherhood in Europe, particularly for initial births, while also highlighting

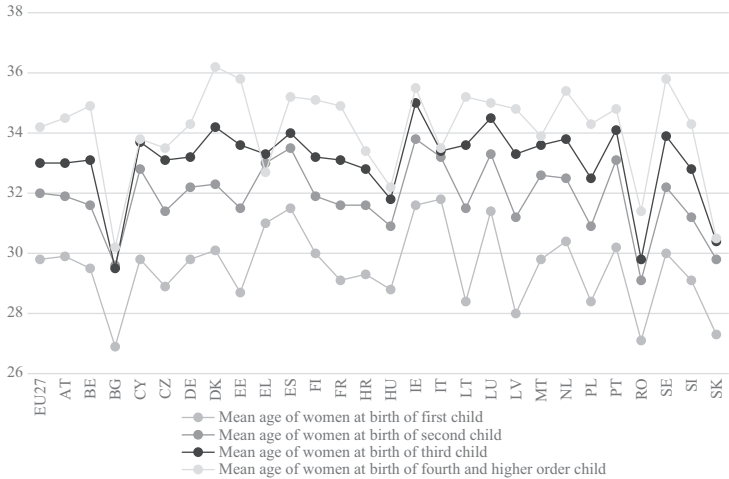


Figure 1.1 Timing of marriage and childbearing in the EU: gender gaps and birth order patterns.

Source: Own work based on (Eurostat, 2025).

national-level differences in fertility timing. These variations have implications for demographic ageing, labour market participation, and policy responses aimed at supporting family life and gender equality in the region.

A regional divide remains evident in the structure and sequencing of these life events. In Northern and Western Europe, individuals tend to leave home earlier, marry later, and delay childbirth, often forming cohabiting partnerships prior to marriage or childbearing. This ‘extended transition’ is supported by strong welfare systems, gender-equality norms, and flexible housing and education policies. Conversely, Southern Europe displays a ‘delayed but compressed’ pattern – individuals remain in the parental home longer (often into their 30s), yet once they leave, transitions to marriage and childbirth occur relatively quickly. Meanwhile, Central and Eastern Europe retains younger ages at childbirth and marriage, although convergence with Western patterns is visible in some countries.

As shown in the graph, there is a noticeable gender gap in the average age of leaving the parental home across the EU-27 countries in 2020. In nearly all cases, women tend to leave earlier than men, although the extent of this difference varies significantly across countries. This pattern reflects broader socio-cultural norms and economic conditions influencing young adults’ transition to independence. In Bulgaria, Romania, and Slovakia, the gender gap is particularly wide, with men leaving home significantly later than women, often by more than four years. These differences may be linked to traditional family structures, labour market barriers, and limited access to affordable housing, especially for young men. In contrast, Northern and Western European

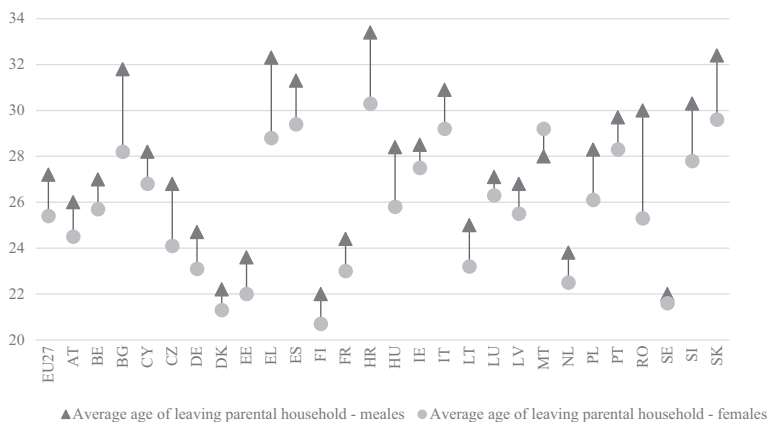


Figure 1.2 Gender differences in leaving the parental home across the EU: age gaps and regional patterns.

Source: Own work based on (Eurostat, 2025).

countries such as Sweden, Denmark, and the Netherlands exhibit much smaller gender gaps and lower overall ages of leaving the parental home, suggesting a more egalitarian and supportive socio-economic context for young adults.

Furthermore, Southern European countries, that is Italy, Spain, and Greece show high average ages for both sexes, with men often leaving home close to the age of 32. This may reflect prolonged dependence on family support in the context of high youth unemployment and housing costs, combined with strong family ties and cultural expectations regarding co-residence.

The consistent tendency for females to leave earlier may be influenced by factors such as earlier union formation, differing labour market expectations, and educational mobility. However, these patterns also raise questions about structural gender inequalities, as earlier household departure among women may coincide with caregiving responsibilities and/or economic vulnerability.

These life-course shifts carry significant implications for demographic trends and policy. Delayed transitions affect fertility rates, intergenerational support systems, housing markets, and pension planning. As the timing of adulthood becomes more protracted and heterogeneous, particularly along gendered and regional lines, European social and economic policies must adapt. Tailored interventions, including affordable housing, accessible childcare, and inclusive labour markets, are essential to ensure equitable life-course opportunities across diverse demographic contexts. The European demographic landscape is undergoing a synchronised trend toward later transitions, yet this postponement is deeply stratified along both gender and regional lines. Men consistently delay these transitions more than women, while Southern and Eastern Europeans face structural and cultural barriers that compress or prolong life-course events. These

findings have critical implications for long-term financial planning, pension timing, labour market participation and social investment strategies. As delayed transitions become the norm, policy frameworks, particularly around housing, childcare, and employment, will need to be recalibrated to reflect the changing timelines and needs of Europe's increasingly heterogeneous adult population.

These demographic shifts and their ramifications highlight the importance of understanding household dynamics at a more granular level, particularly when assessing economic contributions within families. In this context an essential consideration for valuing non-market household labour is the specific life-cycle phase of the household at the moment of loss. Focusing on a target group of households with two adults and at least one child, the analysis draws on EU time-use data to distinguish five key household configurations based on the age of members and the presence of children, enabling a nuanced approach aligned with the demographic realities outlined above. An essential consideration in estimating the value of non-market household labour relates to the life-cycle phase of the household at the moment of death of an adult member. For the purpose of the proposed insurance product, the analysis is restricted to the target group of households composed of two adults and at least one child. Drawing from time-use data across the EU, and for analytical consistency, five main household configurations are distinguished according to the age of household members and the presence or absence of children:

- HH1. Households with two adults under the age of 45 and no children under 18,
- HH2. Households with two adults and the youngest child aged 0–6,
- HH3. Households with two adults and the youngest child aged 7–17,
- HH4. Households with two adults aged 45 or older and no children under 18,
- HH5. Households with two adults aged 65 or more.

It is acknowledged that not all households will transition through all stages (e.g. some will remain childless), but the actuarial multi-state life model for the proposed insurance product can accommodate such heterogeneity by assigning transition probabilities between states. The phases listed above provide the foundation for modelling variations in the economic value of non-market household services, with attention to changes over time and to gender-differentiated roles (Bettio et al., 2013).

To assess the personal contribution (human capital) of household members, the analysis distinguishes between five representative demographic profiles (types), allowing for more precise simulations of household labour loss following the death of a member:

- 1 Type I: Total – all categories aggregated (used as a general benchmark),
- 2 Type II: A person in a couple with the youngest child under the age of six,

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- 3 Type III: A person in a couple with the youngest child aged 7–17,
- 4 Type IV: A person under 45 in a couple, with no children under 18,
- 5 Type V: A single parent with the youngest child under 18 (to simulate the effect of one parent's death).

Within these categories, time contributions are further broken down into three main domains of unpaid household work, each reflecting significant components of domestic production:

- 1 Home maintenance: food preparation (excluding dishwashing), dishwashing, household cleaning, household upkeep (excluding cleaning), laundry, ironing, textile care and handicraft, gardening, pet care, construction and repairs, shopping and services, travel associated with these tasks,
- 2 Childcare: non-instructional care, reading and conversation with a child, educational interaction, and transporting children,
- 3 Other household tasks: household management, assisting other family members, and related travel.

Taken together, the data support a strong argument for incorporating gender-specific and household life-cycle – specific parameters into insurance models covering non-market household labour loss. The observed patterns underline the relevance of gender, caregiving status, and household configuration for actuarial calculations, compensation policies, and public support mechanisms (Błaszczak-Przybycińska & Marszałek, 2019; Marszałek, 2025; Schwanitz et al., 2017, 2021).

### 1.3 **Unpaid Domestic Work and Its Valuation**

Unpaid domestic work, including activities such as cooking, cleaning, childcare, and care for the elderly, constitutes a substantial yet often unrecognised or almost invisible component of overall economic activity. Despite its substantial contribution to social reproduction and household well-being, this form of work remains excluded from national accounts and receives no direct financial protection in systems like household insurance. Its invisibility reflects the historical and institutional undervaluation of care and domestic work, rooted in gendered labour divisions and the structure of modern economic metrics.

Chayanov (1966) was among the first to conceptualise household economies as labour-consumer units, emphasising how internal labour supply is governed by consumption needs rather than profit motives. This framing resonates with later theories of the moral economy (Scott, 1977) and substantive economics (Polanyi, 1944), which argue that household decisions – such as retaining farming and caregiving roles – are embedded in social contexts that prioritise stability, care, and survival over market efficiency. These frameworks help explain why many households continue to perform unpaid work

despite significant opportunity costs, especially in terms of lacking adequate social protection.

From a gender perspective, the distribution of unpaid work is deeply asymmetric and remains one of the most persistent forms of economic inequality. Globally, women continue to shoulder most domestic and caregiving responsibilities, often performing more than three-quarters of all unpaid care work. This disproportionate burden cuts across regions, income levels, and cultures and is particularly pronounced in societies with limited access to formal childcare, care for the elderly, and parental leave systems (Charmes, 2019). The absence of monetary compensation or pension entitlements for such work leaves many women economically vulnerable, particularly in later years, when their lifetime earnings and savings are significantly lower than those of men. According to Charmes, women perform approximately 76.2% of total unpaid care work worldwide, devoting over 3.4 times more hours than men in regions like sub-Saharan Africa. Although the trends indicate a slight narrowing of the gender care gap, only by about 7 minutes per day over the last two decades, it would still take around 210 years at this pace to achieve equal distribution. These global figures mask important regional variations – in Asia and the Pacific, women perform more than 80% of unpaid care work, while men shoulder only about 20%, thus underlining the depth of gender inequities in care responsibilities.

Geographic disparities and economic contexts further shape both the burden of care and its labour market consequences. In Africa, female unpaid caregivers – often with low education, multiple children, informal jobs, and scant access to formal care systems – experience significant ‘jobquality penalties’ as they lose nearly an hour of paid work per week, whereas men in similar circumstances work 18 minutes longer. Similar burdens occur in Asia, Latin America, and the Pacific, where social protection systems often fail to support caregivers. The International Labour Organization (Charmes, 2019) warns that roughly 708 million women globally are excluded from the labour force due to care responsibilities compared to just 40 million men with care obligations identified as the primary labour market barrier across multiple regions. These findings stress the urgent need for investment in care infrastructure, such as childcare, care for the elderly, and parental leave, to open up women’s economic participation and promote gender equity in employment.

It should therefore be emphasised that unpaid work, although absent from conventional market statistics, generates immense economic value, supporting the reproduction of the labour force, sustaining household welfare, and filling the gaps left by underfunded public services. Feminist economists, for example M. Waring (1988), have long argued that the exclusion of unpaid work from national accounting systems distorts economic analysis and marginalises women’s contributions. In response, satellite accounts have been developed to measure the economic value of household production. These accounts, aligned with the SNA, offer methodologies to quantify unpaid care and domestic work using replacement cost or opportunity cost approaches

(Van de Ven et al., 2018). These standardised frameworks allow non-market household production to be incorporated within national accounting systems, thereby revealing substantial contributions that affect welfare, labour, and gender-equity policymaking.

Moreover, the unequal distribution of unpaid work reinforces broader structural inequalities. Women's time poverty, caused by the dual demands of paid and unpaid work, limits their access to full-time employment, education, and political participation. Recognising and valuing unpaid work is therefore a matter not only of statistical accuracy but also of social justice. Integrating satellite accounts into policymaking enables governments to design better-targeted interventions, such as caregiver allowances, pension credits for unpaid care, or public investment in care infrastructure, which are critical to achieving gender equity in both economic and social domains.

In the United States, Suh and Folbre (2016) proposed a prototype satellite account for childcare using the American Time Use Survey (ATUS). Their approach was notable for including not just direct care but also supervisory care, the caregiver-to-child ratio, and the education level of caregivers. They found that acknowledging the value of childcare alone could raise GDP by over 40%, a significant increase over earlier estimates of 20–26%.

A key methodological debate concerns the choice of wage rate in valuation:

- 1 Opportunity Cost Method – uses the individual's own market wage (Folbre, 2006b).
- 2 General Replacement Cost Method – uses the wage of a generic domestic worker (Suh & Folbre, 2016).
- 3 Specialist Replacement Cost Method – assigns distinct market wages to specific tasks (e.g. nanny, cook, cleaner).

The opportunity cost method may inflate estimates for non-employed individuals, while the general replacement cost method can understate the value of more skilled care. Suh and Folbre's use of specialist wages offered a nuanced alternative, better capturing the heterogeneity of unpaid work.

More recently, Jokubauskaitė and Schneebaum (2022) introduced a novel valuation technique based on real labour market data from online job platforms in Austria. By incorporating actual wage offers for domestic tasks, they estimated that unpaid household work accounted for roughly 22% of GDP. A Polish study by Błaszczak-Przybycińska and Marszałek (2019) based on Eurostat guidelines, constructed a national household satellite account and found that unpaid work constituted 60%–80% of total household production, which illustrates how deeply household economies depend on invisible labour contributions.

Beyond economic valuation, environmental research has drawn attention to the material intensity of domestic work. Castellani et al. (2021) used life-cycle assessment (LCA) to analyse the environmental footprint of household consumption, linking it to the use of products such as detergents,

personal care goods and furniture. These findings reinforce the argument that unpaid domestic work is both economically and ecologically consequential. Historical and comparative research deepens the understanding of unpaid labour's invisibility; for instance, Martini and Bellavitis (2014) documented how domestic work has been consistently undervalued over centuries, while Amarante and Rossel (2018) showed how cultural and social norms shape time use and gender roles across Latin America.

At a governance level, DeRock (2021) argued that GDP's exclusion of unpaid work is not merely technical but reflects broader institutional and political decisions. GDP serves as a tool of governance, guiding investment, taxation, and international rankings. Including unpaid work through satellite accounts would radically reshape how nations are evaluated and how resources are distributed. Marszałek (2025) reinforced this policy perspective by reviewing satellite accounting methodologies and their implications for social programmes, such as pensions and gender-responsive budgeting. The findings showed that the value of unpaid work ranges from 60% to 80% of market-measured output, depending on country and method. To sum up, while unpaid domestic work remains economically undervalued in mainstream indicators, methodological advances and satellite accounts provide the tools to rectify this gap. Standardisation and policy integration remain critical challenges, but the evidence consistently stresses that unpaid work, especially that performed by women, is fundamental to economic functioning and social resilience. Recognising and valuing it is a statistical and moral imperative.

Therefore, while unpaid domestic work remains economically undervalued in mainstream indicators, methodological advances and satellite accounts provide the tools to rectify this gap. Standardisation and policy integration remain critical challenges, but the evidence consistently underlines that unpaid labour, especially that performed by women, is fundamental to economic functioning and social resilience. Acknowledging and valuing this labour constitutes a statistical and moral imperative, as it remains an essential yet undervalued component of national economies. Despite its essential role in sustaining well-being and supporting the formal labour market, it is systematically excluded from conventional macroeconomic indicators such as GDP. A growing body of research has stressed the need to integrate unpaid work into economic assessments and policy frameworks. Van de Ven, Zwi-jnenburg, and De Queljoe (2018) estimated the inclusion of unpaid household production could increase GDP by up to 40% in some G7 countries, demonstrating its massive yet invisible economic footprint. Recent studies offer both conceptual and methodological advancements in measuring and valuing household production. Sambt, Donehower and Verbič (2016), in their study on Slovenia, integrated household labour into the National Transfer Accounts system, highlighting the intergenerational transfer of non-market labour and its implications for economic sustainability. Similarly, Hozer-Koćmiel et al. (2020) modelled unpaid housework time in Poland using national time-use

surveys, demonstrating how micro-level data can be aggregated to estimate macroeconomic value.

The valuation of unpaid work is also crucial in estimating societal costs and healthcare economics. Pradelli and Ghetti (2017) presented a model for estimating lost productivity and informal care in Italy, effectively linking unpaid labour to opportunity costs in the healthcare sector. In turn, Cadilhac et al. (2011, 2015) quantified the economic benefits of reducing physical inactivity and domestic violence in Australia, using household productivity loss as a key input. Time allocation studies further reveal the gendered structure of unpaid work. Zhang and Xu (2022) explored how gender norms and class differentiation shape time-use patterns in transitional China, revealing persistent inequalities in household labour. Matteazzi and Scherer (2021) found that gender wage gaps correlate with unequal involvement in household work, reinforcing economic disparities between men and women. Thébaud et al. (2021) also highlighted the role of cultural expectations and social norms in shaping the division of domestic labour, particularly in relation to 'house-keeping standards' and gendered responsibilities. Moreover, the methodological toolkit for valuing unpaid work is expanding. An innovative study by Yoon (2014) proposed a policy framework to value childcare and care for the elderly in Korea, reinforcing the relevance of care work in social security and labour policy. Another approach by Jokubauskaitė and Schneebaum (2022) applied platform-based wage benchmarks to estimate the monetary value of housework, thereby anchoring unpaid labour in real-world market substitutes.

Thus contemporary research clearly affirms that unpaid household work is not only economically significant but also deeply entwined with broader issues of gender equality, social policy, and macroeconomic measurement. Formalising this sector, as proposed by OECD (2021), could serve both to improve economic accuracy and to support more inclusive policymaking. As calls grow for more holistic well-being indicators, incorporating unpaid production is no longer a technical suggestion but an ethical and social imperative.

Unpaid domestic work constitutes a crucial yet often overlooked component of economic activity, whose recognition remains limited in conventional national accounting frameworks. As argued by Sambt et al. (2016), incorporating household production into systems such as the National Transfer Accounts offers a more comprehensive representation of societal welfare and economic contributions. Such integration enables a more accurate estimation of the real value generated within households, shedding light on the magnitude of unpaid work and facilitating cross-country as well as demographic comparisons. This recognition is crucial for understanding the full scope of economic production beyond market transactions.

The valuation of unpaid domestic work, however, presents methodological challenges due to its intangible and nonmarket nature. Studies by Huffman (2011) and Francavilla and Giannelli (2019) employed approaches based on time-use data combined with market equivalence methods such as replacement

cost or opportunity cost valuations to approximate the economic worth of domestic tasks. These tasks encompass a wide spectrum, including meal preparation, cleaning, caregiving, and informal family care, all of which contribute substantively to household well-being. The accurate measurement of this work is essential not only for economic accounting but also for shaping policies that address the often invisible yet vital work sustaining families and communities. A salient feature consistently identified in the literature is the marked gender disparity in the allocation of unpaid domestic labour. Yavorsky et al. (2015) demonstrated that women disproportionately bear the burden of household and caregiving responsibilities, a pattern reinforced by entrenched social norms, cultural expectations, and structural economic factors. The study also further elucidated that this gendered division intensifies following the transition to parenthood, entrenching inequalities in time use and limiting women's access to paid employment and leisure. These persistent imbalances reflect as well as perpetuate broader gender inequalities within the labour market and society at large.

The socioeconomic consequences of this unequal division of unpaid work are profound, particularly in terms of women's labour market participation and economic security. Craig et al. (2012) revealed that women frequently reduce their paid working hours or exit the workforce altogether to accommodate unpaid domestic duties, resulting in lower lifetime earnings, restricted career advancement, and diminished pension accumulation. This dynamic contributes to the feminisation of poverty and exacerbates financial vulnerability, underlining the need to address the issue of unpaid domestic work in discussions on economic inequality and social policy. Given these complexities, a nuanced understanding and accurate valuation of unpaid domestic work, especially regarding gender differences, are imperative for the development of effective policy interventions. Policies promoting a more equitable sharing of domestic responsibilities, such as inclusive parental leave schemes and expanded access to formal childcare, hold promises for mitigating gender disparities and enhancing family well-being. Furthermore, continued empirical research is necessary to refine measurement techniques, monitor evolving patterns in the division of domestic work, and assess the impact of policy measures across diverse sociocultural and economic contexts. Such efforts will contribute to more equitable and comprehensive economic frameworks that fully acknowledge the indispensable role of unpaid domestic work.

The valuation of household work is also used in personal injury and wrongful death cases, where economic loss assessments have traditionally centred on lost market earnings. However, this narrow focus systematically overlooks the economic value of unpaid household production, that is the non-market labour that supports the daily functioning of families and households. Tasks such as cleaning, cooking, childcare, care for the elderly, and household management represent substantial economic contributions, even though they are not monetised in conventional labour markets. Some economists (e.g. Suh & Folbre, 2016) and forensic economists (e.g. Tinari, 2016) have long argued

that this type of work should be systematically included in loss assessments, yet when the deceased was not employed in the formal labour market but instead contributed labour within the household, the loss to surviving dependents is not less real and is often significant.

From an economic standpoint, unpaid household labour produces services that would otherwise need to be purchased in the market. The economic value of these services can be estimated using time-use data and market proxies and is recognised in satellite accounts of several OECD countries. The SNA encourages countries to produce household satellite accounts, acknowledging the economic significance of unpaid production. The theoretical framework for valuing lost domestic production is grounded in replacement cost, opportunity cost, or specialist wage approaches (Folbre & Wagman, 1993). In the forensic economic literature some studies (e.g. Tinari, 2016) argue for the inclusion of both routine household work and non-routine caregiving services, stressing their tangible economic replacement costs. Similarly, Cushing and Rosenbaum (2012) stressed the long-term consequences of undervaluing non-market household contributions, especially by women.

In some jurisdictions, for example the United Kingdom, the value of lost household services is routinely included in compensation calculations. The Ogden Tables (Government Actuary's Department, 2020) provide detailed guidance for estimating dependency losses not only on earnings but also on household services. The courts have accepted these estimates for various household roles, including homemakers and carers, using actuarial methods based on life expectancy, work-life expectancy and wage proxies. In the United States, forensic economists frequently present household production loss estimates to courts, drawing on US Bureau of Labor Statistics wage data and on the ATUS. The National Association of Forensic Economics supports these methodologies and provides standardised frameworks for their application (Ward, 2014).

In the Polish legal context there is no explicit framework akin to the Ogden Tables or ATUS-based evaluations; however, the courts have implicitly recognised the economic value of unpaid labour under the category of increased household needs or loss of personal contribution to household work. For example, the judgement of the Supreme Court of 26 January 2011 (III CSK 57/10) upheld the right of a widower to claim compensation for increased needs resulting from the necessity to hire help after the death of his wife, who had performed household and caregiving tasks. The verdict emphasised that these services had tangible economic value and their loss warranted monetary compensation. Similarly, the Court of Appeal in Katowice on 12 January 2018 (IACa 715/17) awarded compensation for the loss of household services and childcare provided by the deceased mother. The court explicitly acknowledged that the surviving father was forced to reduce his working hours and incur additional costs to secure external help, indicating that the loss of unpaid labour created measurable economic harm. The ruling of the Supreme Court on 3 June 2021 (II CSKP 104/21) reiterated that household

labour may be considered an element of dependency loss and must be considered in the context of compensation for wrongful death; it also acknowledged that even non-working individuals contribute economically to the family through unpaid work.

These rulings reflect an evolving understanding of the role unpaid work plays in the household economy, although Polish courts continue to adjudicate such matters on a case-by-case basis, without a standard valuation framework.

Drawing on the best international practices and on economic theory, three primary valuation approaches are used to estimate the economic value of lost household production:

- 1 Replacement Cost Method: values based on the market cost of hiring paid workers (e.g. cleaners, caregivers). This method is consistent with judicial reasoning in Polish courts and with international standards, such as the Ogden Tables.
- 2 Opportunity Cost Method: values unpaid work based on the income the deceased could have earned in the labour market, assuming time otherwise spent on unpaid work would have been used for employment. This method is conceptually useful but more difficult to apply in litigation contexts, especially in jurisdictions with rigid evidentiary standards.
- 3 Specialist Wage Method (Hybrid): assigns task-specific wages to various forms of unpaid work. For example, childcare is priced using professional nanny rates, while cooking and cleaning use market wages of household staff. This method has been recommended by Tinari (2016) and employed in US and Canadian practice.

All three methods benefit from the use of time-use surveys, which are increasingly available in EU countries, including Poland (e.g. Główny Urząd Statystyczny, 2023). Where no national satellite accounts exist, time-use data can serve as a practical basis for estimating the scale and nature of unpaid contributions. Between 2013 and 2023, the organisation of time use in Polish households underwent significant changes, reflecting broader socio-economic transformations. One notable trend is the gradual reduction in time dedicated to unpaid domestic work, particularly among women, which may be attributed to increased labour market participation and changing gender roles. Additionally, the growing availability of household appliances and technological innovations has improved the efficiency of domestic tasks, potentially reducing the overall time burden. The role of informal care has also shifted, with a growing emphasis on professional and institutional support, particularly for care for the elderly, altering the allocation of unpaid caregiving time within families. Moreover, the 2023 survey highlighted the rise in voluntary activities and social engagement outside the household, reflecting increased civic participation and community involvement among various age groups. This shift indicates a diversification of non-market activities beyond traditional

domestic work. However, despite some reduction in the unpaid labour burden, gender disparities in time use persist, with women continuing to shoulder a disproportionate share of household responsibilities. These findings underline the need for targeted social policies to address gender inequalities and support the work-life balance, especially in the context of Poland's evolving demographic and economic landscape.

The systematic exclusion or undervaluation of unpaid labour in compensation law has profound gendered consequences, as women disproportionately perform such work. As Suh and Folbre (2016) noted, failure to account for unpaid work in damage valuations reflects a broader societal undervaluation of caregiving and domestic labour. Integrating household production into damage assessments ensures fairer compensation and also affirms the economic importance of care work. Hence, there is also growing interest in insurance instruments that cover the loss of household labour (e.g. household support insurance), as well as calls for the development of standardised valuation frameworks in civil law jurisdictions, following the example of the United Kingdom and the United States. Table 1.3 presents the average daily time allocated to selected categories of household activities across different household types, based on data from 15 European countries that participated in the 2010 Eurostat Harmonised European Time Use Survey (HETUS) survey.

First, it is evident that time spent on domestic work and childcare increases significantly in households with children, particularly for women. Comparing households without children under 18 with those raising young children or school-aged children reveals notable differences. For example, in childless households, women spend an average of 176 minutes per day on household tasks compared to 96 minutes for men. In households with a young child, women dedicate 217 minutes per day, while men contribute 92 minutes. In households with a child aged 7–17, women allocate 242 minutes daily, and men 105 minutes.

Importantly, these data refer to the age of the youngest child in the household, without indicating the number of children; therefore, cross-group comparisons must be approached cautiously. Nonetheless, the figures clearly show that women perform roughly twice as much unpaid household labour as men. A few task categories, such as household upkeep (excluding cleaning), gardening, pet care and minor repairs, are more frequently carried out by men. In terms of childcare, women also dedicate approximately twice as much time as men. In the third category, other domestic activities, time allocations between the genders are relatively similar and generally low.

Annually, this unpaid work represents a substantial time investment: in households without children, men contribute around 815 hours per year (9.3% of total annual hours), while women contribute 1,600 hours (18.3%). These figures illustrate the essential role that adult household members play in sustaining the domestic economy, contributions that would be lost in the event of their death. During the period when a child under six is present in the household, total domestic labour peaks at 2,427 hours for women (27.7% of the year) and 1,089 hours for men (12.4%).

Table 1.3 The average daily time allocated to selected categories of household activities across different household types

| Type of household                |                                                           | Type I |     | Type II |     | Type III |     | Type IV |     | Type V |     |
|----------------------------------|-----------------------------------------------------------|--------|-----|---------|-----|----------|-----|---------|-----|--------|-----|
| Gender                           |                                                           | M      | F   | M       | F   | M        | F   | M       | F   | M      | F   |
| Classification of domestic tasks |                                                           |        |     |         |     |          |     |         |     |        |     |
| Household chores                 | Food management except dish washing                       | 20     | 67  | 17      | 71  | 19       | 76  | 17      | 52  | 31     | 59  |
|                                  | Dishwashing                                               | 6      | 23  | 5       | 24  | 5        | 26  | 5       | 18  | 9      | 20  |
|                                  | Household cleaning                                        | 10     | 38  | 9       | 41  | 9        | 44  | 8       | 29  | 16     | 35  |
|                                  | Household upkeep, except cleaning                         | 16     | 14  | 13      | 13  | 16       | 15  | 11      | 11  | 28     | 14  |
|                                  | Laundry                                                   | 1      | 10  | 1       | 12  | 0        | 13  | 1       | 7   | 4      | 10  |
|                                  | Ironing                                                   | 0      | 9   | 0       | 10  | 0        | 13  | 0       | 7   | 0      | 8   |
|                                  | Handicraft and making clothes and other care for textiles | 0      | 5   | 0       | 2   | 0        | 3   | 0       | 2   | 0      | 2   |
|                                  | Gardening; other pet care                                 | 12     | 8   | 6       | 4   | 10       | 7   | 7       | 5   | 6      | 4   |
|                                  | Construction and repairs                                  | 14     | 2   | 14      | 2   | 16       | 2   | 14      | 3   | 8      | 1   |
|                                  | Shopping and services                                     | 22     | 29  | 18      | 26  | 20       | 30  | 23      | 28  | 25     | 29  |
|                                  | Travel related to shopping and services                   | 11     | 14  | 9       | 12  | 10       | 13  | 10      | 14  | 13     | 14  |
| Total                            |                                                           | 112    | 219 | 92      | 217 | 105      | 242 | 96      | 176 | 140    | 196 |
| Childcare                        | Childcare, except teaching, reading and talking           | 7      | 21  | 40      | 112 | 6        | 16  | 1       | 2   | 17     | 48  |
|                                  | Teaching, reading and talking with child                  | 7      | 11  | 35      | 49  | 9        | 17  | 0       | 0   | 20     | 28  |
|                                  | Transporting a child                                      | 2      | 4   | 7       | 15  | 4        | 8   | 0       | 0   | 6      | 12  |
| Total                            |                                                           | 16     | 36  | 82      | 176 | 19       | 41  | 1       | 2   | 43     | 88  |
| Other domestic activities        | Household management and helping family members           | 4      | 7   | 3       | 5   | 4        | 7   | 4       | 5   | 5      | 5   |
|                                  | Travel related to other household purposes                | 2      | 1   | 2       | 1   | 2        | 1   | 2       | 1   | 3      | 0   |
| Total                            |                                                           | 6      | 8   | 5       | 6   | 6        | 8   | 6       | 6   | 8      | 5   |
| Overall per day                  |                                                           | 134    | 263 | 179     | 399 | 130      | 291 | 103     | 184 | 191    | 289 |

Source: Own work based on (Eurostat, 2010).

Table 1.3 also reveals the shifting dynamics of household time use following the loss of a partner. For instance, single fathers spend more time on domestic tasks than partnered fathers: 140 minutes per day versus 92–105 minutes, depending on the child's age, whereas for women the pattern is reversed: single mothers tend to spend less time on household tasks than partnered mothers. Regarding childcare, time-use differences are equally striking. Based on weighted averages for both age groups (under 6 and 7–17), partnered fathers spend approximately 30 minutes daily on childcare, compared to 89 minutes for mothers. In contrast, single mothers dedicate roughly the same amount of time (89 minutes), while single fathers must nearly double their involvement, contributing around 43 minutes daily.

Across the EU, time-use data consistently demonstrate that women bear a disproportionate share of unpaid domestic labour. Based on the 2010 HETUS data from 15 EU countries, women spend an average of 3 to 4 hours per day on household and caregiving tasks, while men contribute only 1.5–2.5 hours. This gender disparity is evident across all countries but varies in magnitude. Southern European countries such as Italy, Greece, and Turkey show some of the widest gaps, where women's unpaid work exceeds men's by over 2 hours daily, as opposed to Sweden, Norway, and the Netherlands with smaller but still significant gender differences of about 47–70 minutes per day. Along with the time invested, the type of unpaid work differs markedly between genders. Women overwhelmingly perform routine and repetitive tasks such as cooking, cleaning, laundry, and childcare, whereas men are more likely to engage in occasional, task-specific activities, for example household maintenance, repairs, gardening, and building work. For instance, in Belgium and Germany, men demonstrate higher participation in home repairs and pet care, while women are nearly exclusively responsible for ironing and food preparation. Even in more gender-equal societies, these patterns of task specialisation persist, although to a lesser degree.

An important but underappreciated dimension of gender inequality is the intensity of multitasking. Women are more likely to perform unpaid household work alongside other primary or secondary tasks referred to in time-use studies as 'secondary activities', which include supervising children while cooking or doing laundry while helping with their homework. The Eurostat (2020) report showed that women spend substantially more time in such multitasking roles than men, especially in childcare. For example, women in Germany and Spain average over 1.5 hours of secondary activities per day, while men average less than an hour. This time compression exacerbates mental and emotional labour burdens for women, reinforcing their 'time poverty'.

Geographic variation in time-use patterns is closely tied to institutional and policy environments. Countries with comprehensive public childcare, generous parental leave schemes, and policies promoting work-life balance (e.g. the Nordics) present more equitable time-use distributions between genders, for example in Sweden and Denmark the difference in time spent on household chores between men and women is relatively narrow, and paternal

involvement in caregiving is significantly higher than in Eastern or Southern Europe. Conversely, in countries where traditional family roles dominate and care infrastructure is weak, women continue to shoulder the majority of unpaid labour without formal recognition or compensation.

Although the HETUS wave set the benchmark for comparative time-use data, more recent evidence, including national surveys and partial updates, suggests that gender roles have evolved slowly but not uniformly. In some Western and Nordic countries, male participation in domestic and caregiving tasks has modestly increased, particularly in childcare and food-related activities; however, the overall gender gap in unpaid work has not significantly declined across the EU as a whole. The COVID-19 pandemic temporarily intensified domestic labour burdens, disproportionately affecting women, which may have reinforced traditional roles rather than shifting them.

The persistent undervaluation of unpaid domestic work has profound implications for economic analysis and public policy. Women's disproportionate contribution to non-market production inflates their 'invisible workload' while artificially lowering official estimates of national productivity. Incorporating time-use data into national satellite accounts – according to Waring (1988), Suh and Folbre (2016), and Eurostat's own guidelines – could provide a more accurate representation of economic activity. Moreover, recognising these gendered imbalances is essential for designing targeted interventions such as tax credits for caregivers, pension entitlements linked to unpaid work, and expanded investment in care infrastructure. Without such measures, economic inequalities rooted in domestic labour will persist intergenerationally. These findings suggest that the death of an adult household member, particularly a woman, generates considerable shifts in household time allocation, especially in households with dependent children. This reinforces the argument that unpaid work is a vital yet vulnerable pillar of household functioning and economic sustainability.

Based on available secondary data and reports from Eurostat and the OECD, the changes in men's and women's involvement in unpaid household work between 2010 and 2020 have not been dramatic, although there is a gradual trend of increasing male participation, particularly in Nordic and Western European countries. The COVID-19 pandemic in 2020 had a temporary impact – numerous studies indicate that men assumed a slightly greater share of household and childcare responsibilities during lockdown periods, yet the evidence suggests that many of these changes were reversed once normal routines resumed.

The most notable shifts between 2010 and 2020 included:

- more time spent by fathers on childcare, with increases reported in some countries,
- greater female labour market participation, without a proportional reduction in their unpaid workload,

- rising political awareness of gender imbalances in unpaid labour, leading to policy measures such as paid paternity leave and investment in care infrastructure.

Although the 2020 edition of the HETUS has been completed, publicly available analyses still rely primarily on the 2010 wave, as micro-level ('raw') data from 2020 are not yet fully accessible for independent calculation and cross-country comparison.

## 1.4 General Insurance Terms and Coverage for Households

This section presents a novel insurance concept designed to address a critical gap in household protection (assistance), specifically the loss of unpaid domestic work following the death or serious illness of a household member. Traditional insurance products focus primarily on replacing lost income, yet they systematically overlook the economic value of non-market household contributions such as childcare, care for the elderly, cooking, cleaning, and household management. The proposed *Bridging Household Life Insurance* (BHLI) product aims to provide temporary support during the crucial adjustment period following such losses, recognising that household stability depends not only on financial income but also on the continuous provision of essential domestic services.

To advance the aim of this subsection, it is necessary to assess whether the loss of unpaid domestic work due to the death or incapacitation of a household member satisfies the standard criteria of insurability as defined within commercial insurance frameworks. A careful analysis suggests that this risk, although traditionally excluded from formal insurance markets, meets the essential preconditions for insurability and thus merits serious consideration for product development.

First, from an insurance standpoint, the loss of unpaid domestic work constitutes a measurable and economically significant risk, manifested unpredictably across a large segment of the population. As demonstrated in preceding sections, the combined insights from national time-use surveys, satellite accounts, and personal injury valuation methodologies offer a robust foundation for quantifying the economic value of household services. These approaches provide a means of assessing the *aggregate volume* of unpaid labour and enable the translation of these tasks into market-equivalent replacement costs, thereby satisfying the actuarial requirement for monetary estimation of risk exposure. Although the proposed insurance model centres on *service provision rather than cash indemnities*, the insurer must still engage in economic valuation of the services in order to price premiums, forecast liabilities, and subcontract service delivery to third-party providers. Thus the economic measurability of unpaid labour loss is both theoretically and practically established.

Second, the temporal concentration of support needs associated with such loss further strengthens its insurability. Empirical data indicate that the most acute household disruption typically occurs within the first three years following the event, particularly in households with dependent children. This defined timeframe enables insurers to construct time-limited benefit structures, reducing the duration of financial exposure and facilitating clear policy pricing. The bounded nature of the risk horizon, unlike open-ended disability and long-term care insurance, supports actuarial predictability and ensures cost containment.

Third, the risk in question is random and beyond the control of the insured, fulfilling a fundamental principle of risk pooling. In its basic form, the insurance would be triggered by the death of an adult household member – an event governed by established mortality patterns and subject to stochastic variation. Should the coverage be extended to encompass permanent incapacitation due to severe illness, the risk remains insurable, provided clear medical criteria and exclusion clauses are applied. Both contingencies, namely death and debilitating illness, are inherently aleatory, non-manipulable and objectively verifiable, thus minimising moral hazard and adverse selection when appropriate underwriting mechanisms are in place.

Fourth, the prevalence of exposure across a sufficiently large risk portfolio further supports the viability of such insurance. The widespread nature of unpaid labour, particularly among primary caregivers and parents of young children, ensures that a broad base of households would be both *eligible* and *motivated* to purchase protection. From a risk finance perspective, this breadth of exposure is essential to constructing an insurance fund capable of distributing losses and covering benefit payouts. Moreover, given demographic shifts such as delayed parenthood and the ageing population, the societal demand for such coverage is likely to rise.

Fifth, the proposed risk is of a non-catastrophic character, distinguishing it from high-severity, low-frequency events such as natural disasters and pandemics. Mortality and morbidity risks, especially in working-age populations, exhibit relatively smooth and predictable incidence curves that have been reliably modelled by insurers for decades. This stability in frequency and severity allows for the accurate calibration of premiums and reserves. Additionally, the design features of the insurance product, including eligibility restrictions, exclusion clauses, and underwriting protocols, can be further leveraged to manage potential volatility and maintain portfolio solvency.

Taken together, these considerations strongly indicate that the loss of unpaid domestic work constitutes an insurable risk under conventional actuarial and underwriting standards. Its measurable economic value, time-limited impact, randomness, prevalence, and predictable structure make it a promising candidate for private insurance solutions. Moreover, product innovations that offer structured support services in lieu of cash compensation provide a socially responsive and operationally feasible method for addressing this protection gap. In this context, bridge insurance not only meets the technical

criteria of insurability but also responds to evolving household vulnerabilities that have long been underestimated or overlooked in insurance design.

Building on the economic rationale for recognising unpaid household labour as an insurable risk, it is important to place this discussion within broader international, legal, gender and social policy frameworks. The recognition that unpaid domestic work has measurable economic value which is not only theoretically valid but also increasingly supported by empirical research and institutional practices across countries.

International precedents reinforce the insurability of unpaid labour loss, with Canada's well-established concept of 'homemaker loss', actuarially assessed for legal and insurance purposes, and the widespread practice in the United States of forensic economists routinely calculating the value of lost household services. Forensic economists routinely calculate the value of lost household services using standardised time-use surveys and replacement cost methods, which are admissible in court and used in settlement negotiations. These practices confirm that such services are quantifiable as well as essential to household functioning, fulfilling a foundational criterion of insurability, namely the ability to define and measure the loss (Folbre, 2024).

From a legal standpoint, several European countries indirectly acknowledge the economic value of unpaid labour through wrongful death claims and survivor benefits. For instance, under Polish civil law, Article 446 §3 of the Civil Code provides for compensation to close relatives for non-pecuniary damage resulting from the death of a family member. Judicial practice in Poland has also included compensation for the loss of services provided by the deceased, such as caregiving and household work, particularly when children are involved (Błaszczak-Przybycińska & Marszałek, 2019). These precedents suggest that even in the absence of formal employment, the contribution of domestic work is recognised as compensable and thus implicitly insurable.

Women continue to perform a disproportionate share of unpaid domestic and caregiving work across the world. According to OECD data (2023), women spend on average twice as much time as men on unpaid household tasks. This imbalance exacerbates income inequality and also exposes women to greater economic vulnerability in the event of household disruption. Insurance products that explicitly recognise and protect the economic value of unpaid labour can therefore serve as tools of gender equity by formalising compensation for work historically rendered invisible in economic and legal systems.

As presented in earlier chapters, governments in some countries, for example Germany and Sweden, have introduced pension credits for time spent caregiving, while others have expanded publicly funded long-term care systems. The inclusion of unpaid work in household satellite accounts, as advocated by the UN System of National Accounts and implemented in several OECD countries, further supports the view that such work holds economic value worthy of protection. Private insurance mechanisms can complement these public efforts by addressing immediate and individualised household risks that fall outside the scope of state welfare.

While standard life insurance compensates for the loss of income from market labour, it generally does not account for the loss of non-market labour. This omission leaves many families, particularly those with young children and/or disabled members, financially exposed to the costs of replacing essential services such as caregiving, cooking, and household maintenance. A dedicated household support insurance product provides a practical and cost-effective solution by addressing the short- to medium-term disruption caused by the loss of a household member performing unpaid work.

As was mentioned, this is a non-catastrophic, individual-level risk with moderate variance, typically affecting only one household at a time, and its occurrence can be priced using well-established mortality tables and time-use datasets. Moreover, product design features, including waiting periods, service caps, and exclusion clauses, can further reduce adverse selection and moral hazard. The concentration of support needs within the first three years post-loss enhances pricing predictability and product clarity.

The following part of the chapter contains reflections and extensions regarding the potential design and structure of the proposed household insurance. These considerations also place the earlier discussions from subsections 1.1 to 1.3 within this broader context.

The household functions both as a consumption and production unit in modern economies, with unpaid domestic work constituting a fundamental yet invisible component of socio-economic systems. As Suh and Folbre (2016) demonstrate, this work creates substantial economic value despite its exclusion from conventional market metrics. Similarly, Błaszczak-Przybycińska and Marszałek (2019) found that unpaid household work in Poland alone represents between 15% and 35% of GDP, depending on the valuation methodology employed. Despite this economic significance, such labour receives no formal protection in standard insurance products, creating a substantial vulnerability for households (Marszałek, 2025), and this protection gap becomes particularly acute when examining the gendered nature of unpaid work. According to Charmes (2019), women perform approximately 76.2% of total unpaid care work worldwide, devoting over 3.4 times more hours than men in regions like sub-Saharan Africa. The economic value of these services would require significant financial resources to replace them through market alternatives, yet the existing life insurance products treat death exclusively through the lens of market productivity, ignoring these broader household losses. This systematic exclusion not only undervalues women's contributions but also leaves households financially vulnerable when losing the primary provider of domestic services.

The proposed insurance concept builds on three established research streams that collectively provide a robust framework for valuing and protecting unpaid household work. First, the valuation of unpaid domestic work through satellite accounts and time-use data has been extensively developed by international organisations. Van de Ven et al. (2018) established methodologies for incorporating household production into national accounts, demonstrating that including unpaid labour could increase GDP estimates by up to

40% in some G7 countries. These frameworks provide the technical foundation for quantifying the economic value that would be lost upon the death of a household member.

Second, legal-economic compensation methods for personal injury and wrongful death cases offer practical precedents for valuing lost household services. Cushing and Rosenbaum (2012) and Tinari (1998) developed comprehensive frameworks for estimating the replacement cost of domestic services in litigation contexts. The Ogden Tables (2020) in the United Kingdom provide actuarial guidance for calculating dependency losses that include both earnings and household services, establishing legal recognition of these non-market contributions. These precedents demonstrate that courts and legal systems already acknowledge the compensable nature of unpaid household work.

Third, behavioural household finance provides insights into how households adapt to major disruptions and reorganise their domestic production. Marszałek (2025) stressed that household needs are dynamic, varying across life-cycle stages and responding to external shocks through various coping mechanisms. This understanding is crucial for designing insurance products that provide appropriate support during the adaptation period rather than attempting to permanently replace lost services.

International precedents exist in Nordic countries, where comprehensive welfare systems partially recognise unpaid domestic contributions. Policies supporting parental leave, home care allowances, and pension credits for caregiving demonstrate governmental acknowledgement of the value of unpaid labour. However, no comprehensive insurance product directly compensates for the loss of unpaid work due to death or incapacitation, leaving a significant gap in household protection systems.

The BHLI product encompasses two complementary coverage models designed to address different aspects of household disruption. The compensation model covers the provision of specific domestic services previously performed by the insured individual. When purchasing the policy, policyholders specify the scope of tasks they typically perform and estimate the hours required for these activities. This approach aims to directly replace lost household work with equivalent services, maintaining household functionality during the adjustment period.

The support model extends beyond basic service replacement to include additional crisis support services identified as necessary through expert assessment. This model notably includes psychological consultations, responding to survey findings where 87% of the participants indicated they would need personal psychological support and 92% expressed the need for such services for their children following a family crisis (Marszałek, 2025). The support model recognises that household disruption involves not only practical challenges but also emotional and psychological dimensions requiring professional intervention. The insurance design incorporates several key parameters derived from empirical research on household needs and preferences. The standard

coverage period is one year, with services available for selected periods within that timeframe. While typical service periods range from one to two weeks, survey data reveal that 30% of families report needing extended assistance for up to six months following the insured event. This variation suggests the need for flexible insurance packages offering different duration options to accommodate diverse household situations.

To address this gap, the bridge insurance concept provides a time-limited benefit stream over three years, helping households maintain stability during the most difficult period of adjustment. This support is especially critical in households with dependent children, where the sudden loss of caregiving or income-generating functions can lead to immediate disruptions in daily life. The value of these services is commonly benchmarked against the minimum wage for domestic assistance, although the actual replacement costs may vary by region and service quality. In many cases, particularly where young children are involved, the total market value of lost labour and care can be substantial.

Protection in the form of support for the family after the death of an adult household member would be offered as an additional option within a life insurance policy. The authors propose that this would be a term insurance policy (e.g. for 20 years), providing coverage for a specific period in the household life cycle, namely during the time when children are present in the household. The support would be available to the family for a period of, for example three years, gradually decreasing in subsequent years (e.g. at a rate of 100%, 75%, and 50%).

The proposed three-year duration of household support following the death of a family member is grounded in legal practice and also in empirical evidence. When the death results from the fault of a third party, such as in traffic accidents, medical negligence, workplace incidents or criminal acts, surviving family members may pursue civil claims for compensation; however, the adjudication of such claims is often protracted. An analysis of 1,875 verdicts issued by the Polish civil courts between 2017 and 2020 found that the average time between the death and final judgement was 1.8 years, with a median of 2.8 years, and in numerous cases extending far longer. This legal delay creates a significant financial vulnerability period during which the family must reorganise its functioning and replace lost services long before any compensation is received.

Moreover, actuarial models must also take into account the heightened mortality risk for surviving spouses. This vulnerability further supports the need for structured and extended assistance. In addition, Dębicka, Heilpern, and Marciniuk (2023) stressed the actuarial importance of mortality dependence in pricing household-level insurance products, underlining that the death of one spouse can significantly affect the survival probability of the other. In light of these findings, a structured, time-bound insurance model providing assistance over three years fills a critical compensation gap as well as acknowledges the complex, layered consequences of household member loss – both economic and emotional.

The benefit structure follows a declining schedule that reflects progressive household adaptation. Full benefits (100%) are provided in the first year post-loss, decreasing to 75% in the second year and 50% in the third year. This structure acknowledges that households gradually develop new routines and coping mechanisms while maintaining affordability through reduced long-term obligations. The declining benefit approach balances adequate initial support with sustainable premium levels, making the product accessible to a broader range of households.

The insurance contract is assumed to be concluded for a specific household, the composition of which is determined at the time of purchase of the insurance contract. This assumption reflects a closed child population approach, whereby children born after the start of the insurance period are not taken into account in the valuation of the contract. Potential avenues for refining the model, including extensions to account for an open child population, are discussed in Section 3.4, where recommendations for further development of the insurance contract are presented.

The insurance targets households with two adults and dependent children, activated upon the death of the first adult. This 'first-loss' principle recognises the mutual dependency within adult partnerships for maintaining household functionality. The death of either partner creates significant disruption requiring external support, regardless of whether the deceased was the primary income earner or the primary provider of domestic services.

Not all household activities fall within the insurable scope of this product. Focus data (a study conducted with 120 participants across ten focus groups in Poland in 2024) revealed significant reluctance among parents to delegate certain responsibilities, particularly those involving direct childcare. Only 23% of parents expressed willingness to outsource childcare activities, with a mere 2% indicating actual intent to use such services, while 83% rejected the idea outright. Activities such as caregiving, grooming, education, reading, and child transportation were widely viewed as non-transferable, reflecting deep-seated preferences about parental roles and responsibilities.

These preferences necessitate alternative solutions within the insurance framework; for instance – rather than attempting to replace home cooking with hired cooks – the insurance might provide catering services, which appear more acceptable to families. The product includes a curated list of covered services from which policyholders select those most relevant to their situation, ensuring that benefits align with actual household needs and preferences.

Standard insurance exclusions apply to maintain actuarial soundness and prevent adverse selection, which include exclusions for death resulting from war, pandemics, or other mass events that could trigger simultaneous claims beyond the insurer's capacity. A suicide clause with an appropriate waiting period is included to prevent moral hazard while respecting the legitimate needs of households facing mental health challenges.

The current model prioritises direct service provision over cash payouts to ensure that benefits are used for their intended purpose. This approach

guarantees that grieving families receive the necessary household support without the additional burden of identifying, vetting and managing service providers. Insurance companies can leverage economies of scale to negotiate favourable rates with service providers and ensure quality standards through ongoing monitoring and feedback systems.

However, this service-centric approach presents certain limitations that must be acknowledged. The survey responses indicated divided preferences regarding benefit delivery, with 36% of respondents preferring monetary compensation and 57% expressing amenability to either service provision or cash (Marszałek, 2025). Cash payouts would enable families to compensate relatives and/or friends who provide informal assistance during the crisis period, recognising that social networks often mobilise to support grieving households. The flexibility of cash benefits also accommodates varying household needs and preferences that may not align perfectly with standardised service packages.

The tension between ensuring appropriate benefit use and providing household flexibility represents a fundamental design challenge. One potential solution involves a hybrid approach offering a base level of guaranteed services supplemented by discretionary cash benefits for additional needs. This structure would maintain the insurance product's core purpose while acknowledging the diverse ways households organise support during crises.

Pricing this insurance product requires careful analysis of multiple parameters that influence both claim frequency and severity. Expected hours of support needed post-death vary significantly based on household composition, the age of children, and the deceased's role within the household. Time-use data from across the EU indicate that women in households with young children dedicate an average of 217 minutes daily to household tasks, while men contribute 92 minutes. These disparities must be reflected in premium calculations to ensure actuarial fairness. The minimum hourly wage for domestic assistance provides a baseline for calculating benefit values, although actual costs may vary based on local labour markets and service quality requirements. The total value of unpaid work over a six-month period can reach substantial sums, particularly in households with young children where the combination of childcare, household maintenance, and meal preparation represents a full-time occupation. Annual mortality probabilities must be adjusted not only for age and gender but also for the increased mortality risk experienced by surviving spouses in the years immediately following their partner's death.

Life-cycle adjustments are essential for maintaining appropriate coverage throughout the insurance term. The economic value of household services varies dramatically between a household with infant children and one with teenagers approaching independence. Similarly, the transition to retirement fundamentally alters both household income patterns and domestic labour requirements. Dynamic benefit calculations that respond to these life-cycle changes ensure that coverage remains relevant and valuable throughout the policy term.

Bridge insurance serves as a complement rather than a replacement for traditional life insurance products. While whole-life insurance addresses permanent income loss and long-term financial security, bridge insurance focuses on temporary dysfunction and adaptation support. This complementary relationship suggests opportunities for product bundling and cross-selling within comprehensive household protection packages.

The relatively short benefit period and specific focus on domestic services allow for lower premiums compared to traditional life insurance, making the product accessible to households that might not afford comprehensive whole-life coverage. Furthermore, bridge insurance serves an educational role by raising awareness of household vulnerability to non-income losses. Many consumers remain unaware of their dependency on unpaid household services until faced with their sudden absence. By quantifying and insuring these services, the product promotes greater appreciation for domestic labour's economic value.

The development of household support insurance extends beyond individual risk management to address systemic social and economic issues. Recognition of the economic value of unpaid work through insurance markets reinforces ongoing efforts to include household production in national accounts and economic policy discussions. This market-based validation complements governmental initiatives such as pension credits for caregivers and public investment in care infrastructure.

Gender equity represents a central concern given women's disproportionate responsibility for unpaid household work. Insurance products that explicitly value and protect these contributions help address the economic vulnerabilities created by traditional gender roles. By providing financial protection for unpaid labour, such insurance acknowledges that household work creates real economic value deserving of formal recognition and protection. This recognition is particularly important in ageing societies where care demands are increasing while family sizes shrink, creating growing pressure on informal care systems.

The insurance mechanism also contributes to household resilience by providing a structured framework for managing domestic disruption. Rather than relying solely on informal networks or depleting savings to hire replacement services, households can access predetermined support through insurance benefits. This systematic approach to household risk management promotes greater economic stability and may reduce the long-term impacts of household disruption on surviving family members.

While comprehensive in scope, certain losses remain beyond the reach of insurance coverage. The emotional bonds between household members, the comfort of familiar routines, and the security of established relationships cannot be replaced through market mechanisms. Social identity disruption, particularly for surviving spouses who must navigate changed social roles and relationships, represents a profound loss that insurance cannot address. The reduced capacity for self-care and mutual support within the household extends beyond practical tasks to encompass emotional and psychological dimensions that resist commodification.

These intangible losses are typically addressed through legal remedies such as awards for pain and suffering in wrongful death cases rather than through insurance mechanisms. The proposed bridge insurance acknowledges these limitations by focusing on practical support that may indirectly facilitate emotional healing by reducing stress and maintaining stability during the grieving process. By alleviating financial and practical pressures, insurance enables households to focus on emotional recovery and relationship rebuilding.

The proposed Bridging Household Life Insurance represents an innovative response to the multidimensional vulnerabilities households face upon losing a member. By integrating insights from economic valuation of unpaid work, legal compensation frameworks, and behavioural finance, this product addresses a significant protection gap in current insurance markets. The recognition and protection of unpaid household work through insurance mechanisms marks an important evolution in how societies value and support the full spectrum of economic contributions within households.

While the product cannot replace the irreplaceable human connections and emotional support lost when a household member dies, it provides crucial practical assistance during the most vulnerable period of adjustment. This support offers households something invaluable in the face of tragedy: time to grieve, adapt, and reorganise without the immediate pressure of maintaining all household functions. The development and implementation of such insurance products represent a business opportunity and a social innovation that promotes more resilient, equitable, and sustainable household economies. As societies continue to struggle with changing demographics, evolving gender roles and increasing care demands, innovative insurance solutions that recognise the full value of household contributions will become increasingly important for ensuring economic security and social well-being.

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## 2 Actuarial Modelling of the Insurance Contract

*Joanna Dębicka*

### 2.1 Multi-State Model

In this chapter, a contract is defined as a written agreement between parties that establishes mutual rights and obligations, resulting in specified cash flows. It is assumed that the contract is concluded between a married couple (or partners) and an insurance company.

The analysis concerns a contract issued at time 0 and terminating at future time  $n$  denoting the contract term (*the insurance period*). Variable  $t$  ( $t \geq 0$ ) refers to the time elapsed since the beginning of the contract. Having practical applications in view, the study focuses on a discrete-time framework, with particular attention given to annual time intervals. In this context, the contract duration is divided into full years. Time points are indexed by integers  $t = 0, 1, 2, \dots$  where  $t = 0$  denotes the contract inception and subsequent values represent the end of consecutive policy years. For the  $t$ -th policy year, cash flows paid in advance (e.g. period premiums) are assumed to occur at time  $t - 1$  (i.e. at the beginning of year  $t$ ), while cash flows paid from below (e.g. immediate annuity) are assumed to occur at time  $t$  (i.e. at the end of year  $t$ ).

*Multi-State Models* (MSMs) constitute a comprehensive stochastic modelling framework frequently employed in actuarial science for modelling insurance contracts that depend on the future lifetime of the insured (Booth et al., 2020; Dębicka, 2013; Dickson et al., 2019; Haberman & Pitacco, 1999; Norberg, 2008; Rotar, 2014). The MSM represents a person's life as a series of transitions between defined states, such as health, illness, or death. Each state reflects a specific condition, and at any given time the individual can be only in exactly one state. MSMs can also be used for small groups, for example a married couple (Dębicka et al., 2022, 2023, 2025; Denuit et al., 2001; Gerber, 1995).

In formal terms, MSM is a pair of two sets  $(S, T)$ . The set  $S = \{1, 2, \dots, N\}$  is a set space in which at any time the individual risk is in one of a finite number of states labelled  $1, 2, \dots, N$ . Set  $T$  contains direct transitions between states of the state space. The pair  $(i, j)$  denotes a direct transition from state  $i$  to state  $j$

( $i \neq j$ ;  $i, j \in S$ ). The pair  $(S, T)$  describes all possible individual risk events as far as its evolution is concerned (until the end of the contract).

Due to the fact that *Bridging Household Life Insurance* (BHLI) is a contract in which there are two insured persons (spouses or partners), it belongs to the so-called group insurance, an example of which is marriage insurance. In the remainder of this chapter, for the sake of simplicity, it is assumed that the insured individuals are spouses. However, it should be emphasised that the same assumptions and model structure can also be applied to partners in informal relationships.

Note that the purpose of marital insurance is usually to provide income security in the event of the death of a spouse and to help maintain the current standard of living for the widow or widower. Typically, this type of contract is open-ended (concluded as long as one of the spouses is alive, the so-called *last surviving status*) and the fixed annuity instalment is paid for the rest of the life of the remaining spouse. The MSM for such marriage insurance takes the form:

$$(S, T) = \left( \{1, 2, 3, 4\}, \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \right) \quad (2.1)$$

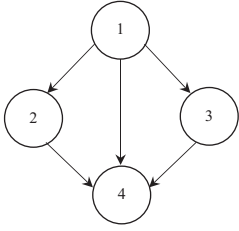
and its graphical presentation with an explanation of the meaning of each state is shown in Figure 2.1A.

The analysis of state transitions (i.e. the evolution of the contract states from inception to termination) is one of the fundamental factors affecting its valuation. In this study, a discrete-time stochastic process  $\{X(t) : t = 0, 1, 2, \dots, n\}$  to model the evolution of a modelled multi-state contract  $(S, T)$  is considered. Process  $\{X(t)\}$  takes values in finite state space  $S$ . Note that  $X(t) = i$  (for  $i \in S$ ) means that at time  $t$  (i.e. the time that has elapsed from the beginning of the contract), the insureds are affected by the life situation to which state  $i$  has been assigned. It is assumed that only one transition may occur per time unit, corresponding to a single random event. This modelling approach reflects practical applications, particularly in the context of contracts where the subject of insurance is human life (Booth et al., 2020; Bowers et al., 1986; Haberman & Pitacco, 2018).

An insurance contract encompasses not only the insured event (in this case, the death of the spouse), but also the manner in which the contract is implemented throughout its duration. Notably, the structure of the MSM is shaped by the types of cash flows (especially benefits) that arise upon the realisation of the contract. In the rest of this chapter, the cash flow realised at time  $t$  if  $X(t) = j$  is denoted by  $\phi_j(t)$  (Dębicka, 2012; Dębicka et al., 2025). In multi-state insurance models, an individual's presence in a given state or transition between states can generate financial consequences. The realisation of insurance contracts results in cash flow streams, which include cash flows such as periodic  $p_j(t)$  or fixed-time  $\pi_j(t)$  premiums, annuity due  $\ddot{b}_j(t)$

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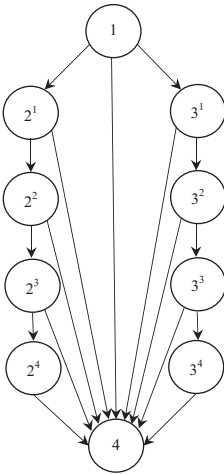
### A. MSM for marriage insurance



State space:

- 1 - both spouses are alive
- 2 - the wife is dead
- 3 - the husband is dead
- 4 - both spouses are dead

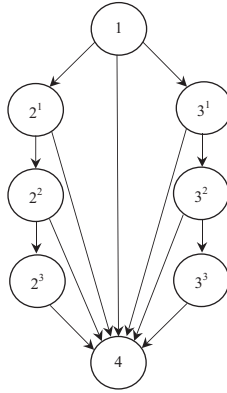
### B. MMSM for marriage insurance



State space:

- 1 - both spouses are alive
- $2^\eta$  - the wife died  $\eta$ -th years ago ( $\eta=1,2,3$ )
- $2^4$  - the wife died at least 4 years ago
- $3^\eta$  - the husband died  $\eta$ -th year ago ( $\eta=1,2,3$ )
- $3^4$  - the husband died at least 4 years ago
- 4 - both spouses are dead

### C. MMSM for marriage insurance after reduction



State space

- 1 - both spouses are alive
- $2^\eta$  - the wife died  $\eta$ -th year ago ( $\eta=1,2,3$ )
- $3^\eta$  - the husband died  $\eta$ -th year ago ( $\eta=1,2,3$ )
- 4 - both spouses are dead or one of the spouses died at least 4 years ago

Figure 2.1 Standard and modified multi-state models for marriage insurance.

Source: Author's own elaboration.

and immediate annuity  $b_j(t)$  benefits, lump sums  $d_j(t)$  and transition benefits  $c_{ij}(t)$  (for a discrete-time model, it means that  $X(t-1) = i \neq j$ ,  $X(k) = j$  and  $X(t+1) \neq j$ ). Each premium  $p_j(t)$  is classified as a cash flow of type  $p$ . Similarly,  $\pi_j(t)$ ,  $\ddot{b}_j(t)$ ,  $b_j(t)$ ,  $d_j(t)$  represent cash flows of types  $\pi, \ddot{b}, b, d$ , respectively. The transition benefit  $c_{ij}(t)$  is considered as type  $c_i$  cash flow, as

its value may depend on the process state immediately prior to the transition to state  $j$ . Therefore it can be concluded that  $\wp \in \{p, \pi, \bar{b}, b, d, c_1, \dots, c_N\}$ . Cash flows  $\pi, p, \bar{b}, b, d$  relate to process  $\{X(t)\}$  remaining in a given state, while  $c_s$  correspond to cash flows tied to transitions between states. The set direct transition  $T$  includes two types: those associated with cash flows and those without. According to the definition, that pair  $(i, j)$  is of the *cf* (cash flow) type if  $t$  exists for which  $c_{ij}(t) \neq 0$ . Transitions not associated with any cash flows of type  $c_i$ s are classified as *not-cf*.

In the case of classical  $n$ -year term marital life insurance, the primary type of benefit paid upon the death of a spouse is a death annuity paid from below. Specifically, the surviving spouse receives a constant annuity starting from the time of the partner's death and continuing until the end of the insurance period; namely, the husband receives a constant annuity after the death of his wife  $b_2$ , and similarly, the wife receives a constant annuity after the death of her husband  $b_3$ . Assuming a constant periodic premium  $p$  is paid while both spouses are alive during the first  $m$  year of the insurance period, the resulting cash flows from such a contract can be described as follows:

$$\wp_j(t) = \begin{cases} p & \text{for } j = 1 \text{ and } t = 0, 1, \dots, m-1 \\ b_2 & \text{for } j = 2 \text{ and } t = 1, 2, \dots, n \\ b_3 & \text{for } j = 3 \text{ and } t = 1, 2, \dots, n \\ 0 & \text{besides} \end{cases} \quad (2.2)$$

Similarly to classical marriage insurance, BHLI provides annuity-style benefits upon the death of a spouse; however, its primary objective is to protect against the consequences of losing the unpaid domestic work that a deceased spouse provided for the household. This is important during the first three years of widowhood when the mourning process has a particularly negative impact on the functioning of the household, especially in situations of joint child raising. While the purpose of the insurance is not to provide direct payments to the surviving insured but rather to arrange for paid care services through the insurer, the financing of these services is treated as an annuity benefit in terms of pricing the insurance contract.

BHLI is a fixed-term contract concluded for  $n$  years, in which the husband receives annuity  $(b_2)$  after the death of his wife for the first three years of widowhood, and similarly a wife after the loss of her husband  $(b_3)$ . The amounts of the annuity for a husband and wife may differ from each other  $(b_2 \neq b_3)$ . Moreover, if one of the spouses dies in the last year of the contract, then the period of payment of benefits may exceed the insurance period.

Initially let us assume that the amount of annuity does not depend on the number and age of the children forming a household with their parents.

This form of BHLI is hereafter referred to as *Scenario 0*, which serves as a reference case for further analysis and comparisons.

Since the annuity is payable only for a limited period – specifically during the first three years of widowhood – and certain benefits may be disbursed after the formal end of the insurance contract if the insured becomes eligible in the final year of coverage, the model (2.1) is insufficient and requires modification to reflect both time-limited and deferred payment mechanisms, hence the method proposed by Dębicka (2012) was applied. An important advantage of this approach is that the resulting model, known as *the Modified Multi-State Model* (MMSM), enables the use of matrix notation for the representation of actuarial values.

First, the methodology for constructing the MMSM is presented, followed by its application to the BHLI contract according to *Scenario 0*.

The MSM ( $S, T$ ) model can be modified if it meets the following assumptions:

Assumption B1.

For each  $i$ , all pairs  $(i, j) \in T$  are of the same type, *cf* or *not-cf*.

Assumption B2.

For each  $i$ , such that  $(i, j) \in T^{cf}$  and each  $t \in \{1, 2, \dots, n\}$ , we have  $c_{ij}(t) = c_j(t)$ .

If assumptions B1 and B2 are met, then MMSM can be constructed based on the four-step procedure.

STEP 1 (*the set of policy conditions*). The MSM modification procedure is based on the sets of parameters  $\Gamma^{(\wp, j)}$  introduced in (Haberman & Pitacco, 1999) defining the conditions for the implementation of individual types of cash flow  $\wp$  (benefit or premium) realised at the state  $j \in S$  resulting from the insurance contract.

For all cash flows whose exact moment of realisation is not known in advance, the set of policy conditions is formally represented by a set of five parameters, that is for each type of cash flow  $\wp \in \{p, b, \ddot{b}, c_1, \dots, c_N\}$  and state  $j \in S$ :

$$\Gamma^{(\wp, j)} = \left( n_1^{(\wp, j)}, n_2^{(\wp, j)}, f^{(\wp, j)}, s^{(\wp, j)}, r^{(\wp, j)} \right), \quad (2.3)$$

where  $\left( n_1^{(\wp, j)}, n_2^{(\wp, j)} \right)$  denotes the period of insurance liability,  $f^{(\wp, j)}$  is the deferral period preceding the first realisation of a given type of cash flow,  $s^{(\wp, j)}$  means the maximum period for the realisation of a given type of cash flow and  $r^{(\wp, j)}$  is the stopping time (from policy issue) of cash flow  $\wp$  realised at state  $j$ .

Whereas for all cash flows whose exact moment of realisation is predetermined (i.e. for  $\wp \in \{\pi, d\}$ ), the set of parameters is defined as

$$\Gamma^{(\wp, j)} = \left( t_1^{(\wp, j)}, t_2^{(\wp, j)}, \dots, t_k^{(\wp, j)} \right), \quad (2.4)$$

where  $t_1^{(\wp, j)} < t_2^{(\wp, j)} < \dots < t_k^{(\wp, j)} \leq n$  denotes predetermined moments in time at which a given type of cash flow occurs, provided that process  $\{X(t)\}$  is in state  $j$  at those moments.

It appears that the modification process depends solely on the sets of policy parameters associated with cash flows occurring at stochastic moments within the policy term.

STEP 2 (*cash flows realisation period*). This step is to determine the period of implementation of all cash flows  $n^*$  resulting from the terms of the contract

$$n^* = \max_{\wp, j} \left\{ n^{\Gamma^{(\wp, j)}} \right\}, \quad (2.5)$$

where  $n^{\Gamma^{(\wp, j)}} = \min \left\{ n_2^{(\wp, j)} + s^{(\wp, j)} - 1, r^{(\wp, j)} \right\}$ .

STEP 3 (*the state space reflecting insurance conditions*). Let  $S^{\Gamma^{(\wp, j)}}$  be a state space taking into account the set of insurance conditions  $\Gamma^{(\wp, j)}$

$$S^{\Gamma^{(\wp, j)}} = \left\{ 1, 2, \dots, j-1, j^1, j^2, \dots, j^{\gamma^{(\wp, j)}}, j+1, \dots, N \right\}, \quad (2.6)$$

where

$$\gamma^{(\wp, j)} = \begin{cases} \min \left\{ s^{(\wp, j)} + 1, r^{(\wp, j)} \right\} & \text{for } s^{(\wp, j)} \leq n_2^{(\wp, j)} \\ \min \left\{ s^{(\wp, j)}, r^{(\wp, j)} \right\} & \text{for } n_2^{(\wp, j)} < s^{(\wp, j)} < \infty \\ f^{(\wp, j)} & \text{for } \begin{cases} n_2^{(\wp, j)} < s^{(\wp, j)} = \infty \text{ and} \\ \left\{ n_2^{(\wp, j)} < f^{(\wp, j)} < r^{(\wp, j)} \right\} \text{ or} \\ f^{(\wp, j)} < n_2^{(\wp, j)} \leq r^{(\wp, j)} \text{ or} \\ f^{(\wp, j)} = n_2^{(\wp, j)} < r^{(\wp, j)} \end{cases} \\ 0 & \text{besides} \end{cases} \quad (2.7)$$

and  $j^{(\eta)}$  means that the insured risk is in state  $j$  and lasts in this state for the  $\eta$ -th unit of time ( $\eta = 1, 2, \dots, \gamma^{(\emptyset, j)} - 1$ ) and  $j^{\gamma^{(\emptyset, j)}}$  means that the insured risk is in state  $j$  and lasts in this state for at least  $\gamma^{(\emptyset, j)}$  time units.

Note that if the insured risk at time  $t = 0, 1, 2, \dots, n^*$  is in state  $j$ , then in principle all types of cash flows (premiums and benefits) can exist. Let for each state  $j \in S$

$$\gamma^{(\cdot, j)} = \max_{\emptyset} \gamma^{(\emptyset, j)}. \quad (2.8)$$

Then the new state space  $S^*$  resulting from fitting  $S$  to all sets of parameters  $\Gamma^{(\emptyset, j)}$  is defined as follows (compare (2.6) and (2.8)):

$$S^* = \left\{ 1^1, \dots, 1^{\gamma^{(\cdot, 1)}}, 2^1, \dots, 2^{\gamma^{(\cdot, 2)}}, \dots, j^1, \dots, j^{\gamma^{(\cdot, j)}}, \dots, N^1, \dots, N^{\gamma^{(\cdot, N)}} \right\} \quad (2.9)$$

and size  $N^*$  of state space  $S^*$  is defined in the following way

$$N^* = N + \sum_{j \in S} 1_{\{\gamma^{(\cdot, j)} \geq 2\}} \cdot (\gamma^{(\cdot, j)} - 1). \quad (2.10)$$

STEP 4 (*the set of direct transitions reflecting insurance conditions*). If in the set of direct transitions between states  $T$  there exists a direct transition  $(i, j)$ , then in the new set  $T^*$  there are transitions  $(i^1, j), \dots, (i^{\gamma^{(\cdot, i)}}, j)$  if state  $i$  has been separated in STEP 3. Additionally, if state  $j$  has been separated into more states (e.g.  $j^1, j^2, \dots, j^{\gamma^{(\cdot, j)}}$ ), then we obtain the following direct transitions  $(i^1, j^1), \dots, (i^{\gamma^{(\cdot, i)}}, j^1)$ . Note that if only one state  $j$  has been separated, then we obtain only one direct transition  $(i, j^1)$ .

To sum up, as a result of the four-step procedure, an MMSM is constructed. This model is defined by a pair of sets  $(S^*, T^*)$ , where set space  $S^*$  is determined in STEP 3 and the set of direct transitions  $T^*$  is specified in STEP 4.

The following part of this section examines the application of the described methodology for constructing the MMSM to the BHLI contract under *Scenario 0*.

### ***Scenario 0 (Modified Multi-State Model)***

Before modifying the model illustrated in Figure 2.1A, it is necessary to verify whether the cash flows arising from the realisation of the BHLI contract

satisfy assumptions B1 and B2. It should be noted that the marriage contract (as with the BHLI contract) does not include any lump-sum benefits of type  $c_i$ , but exclusively provides annuity-type benefits. Consequently, all the transitions in the model are of the not- $cf$  type. As a result, assumptions B1 and B2 are satisfied, and an MMSM can be constructed based on the four-step procedure.

STEP 1. In the case of the BHLI contract under consideration, there are two types of life annuity for which sets of policy conditions, defined by (2.3), are as follows:  $\Gamma^{(b,2)} = (0, n, 0, 3, \infty)$  and  $\Gamma^{(b,3)} = (0, n, 0, 3, \infty)$ . Another type of cash flow is the period insurance premiums ( $p$ ) paid when both spouses are alive (at state 1). If the insurance premium is paid periodically for the first  $m$  ( $m \leq n$ ) units of the insurance period, then  $\Gamma^{(p,1)} = (0, m, 0, \infty, m)$ .

STEP 2. For BHLI, we have that  $n^{\Gamma^{(b,2)}} = n^{\Gamma^{(b,3)}} = \min\{n+3-1, \infty\} = n+2$

and  $n^{\Gamma^{(p,1)}} = \min\{m+\infty-1, m\} = m-1$ . Thus finally, from (2.5), one obtains  $n^* = n+2$ .

STEP 3. For BHLI, based on (2.6), we have

$$\begin{aligned} - S^{\Gamma^{(b,2)}} &= \{1, 2^1, 2^2, 2^3, 2^4, 3, 4\} & \text{because } n_2^{(b,2)} &\leq s^{(b,2)} & \text{and} \\ &\gamma^{(b,2)} = \min\{3+1, \infty\} = 4, \\ - S^{\Gamma^{(b,3)}} &= \{1, 2, 3^1, 3^2, 3^3, 3^4, 4\} & \text{because } n_2^{(b,3)} &\leq s^{(b,3)} & \text{and} \\ &\gamma^{(b,3)} = \min\{3+1, \infty\} = 4, \\ - S^{\Gamma^{(p,1)}} &= \{1, 2, 3, 4\} & \text{because } n_2^{(p,1)} < s^{(p,1)} & \text{and } f^{(p,1)} < n_2^{(p,1)} = r^{(p,1)} & \text{we} \\ &\text{obtain } \gamma^{(p,1)} = 0+1 = 1. \end{aligned}$$

Thus, from (2.9) and (2.10), one directly obtains

$$S^* = \{1, 2^1, 2^2, 2^3, 2^4, 3^1, 3^2, 3^3, 3^4, 4\} \quad (2.11)$$

and  $N^* = 4 + (4-1) + (4-1) = 10$ .

STEP 4. For BHLI, we obtain

$$T^* = \left\{ \begin{array}{l} (1, 2^1), (1, 3^1), (1, 4), \\ (2^1, 2^2), (2^1, 4), (2^2, 2^3), (2^2, 4), (2^3, 2^4), (2^3, 4), (2^4, 4), \\ (3^1, 3^2), (3^1, 4), (3^2, 3^3), (3^2, 4), (3^3, 3^4), (3^3, 4), (3^4, 4) \end{array} \right\}. \quad (2.12)$$

As a result, the MMSM for *Scenario 0* is created, defined by a pair of sets according to (2.11) and (2.12) – see Figure 2.1B. The MMSM also allows for the case when the BHLI contract assumes decreasing annuities, that is  $b_{21} > b_{22} > b_{23}$  and  $b_{31} > b_{32} > b_{33}$ .

It should be noted that when the insured risk reaches state  $2^4$  (or analogously, state  $3^4$ ), it signifies that all benefits have been disbursed and the insurance coverage concluded, whereas states  $2^\eta$  and  $3^\eta$  (for  $\eta = 1, 2, 3$ ) are called a *reflex state*; namely, after one unit of time the insured risk leaves this state. Since state  $2^3$  (as well as state  $3^3$ ) is a transient state with zero duration, then if the husband survives the third year of widowhood, the insured risk transitions to state  $2^4$ , otherwise – in the event of his death – it moves to state 4 (similarly for state  $3^4$ ). As a result, the MMSM presented in Figure 2.1B may be simplified by merging states  $2^4$  and  $3^4$  with state 4, producing a reduced multi-state structure (see Figure 2.1C). Henceforth, the modified model for *Scenario 0* presented in Figure 2.1C is denoted as  $(S_{BHLI(0)}, T_{BHLI(0)})$ .

□

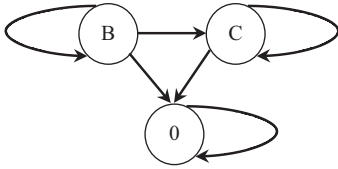
The concept of BHLI is based on the idea that the amount of annuity benefit should be flexibly (i.e. dynamically) adjusted to the household's needs arising from the work and efforts of each insured individual for the family at the time of death. Accordingly, the traditional assumption that the amount of the annuity is invariant with respect to the number and age of the children residing in the household should be abandoned.

Following the classification introduced in Section 1.2, and for the purposes of the MSM used to describe the BHLI contract, the following household types are distinguished:

- 0 – households with no minors (young children), comprising household types HH1, HH4, and HH5.
- B – households with at least one child, the youngest aged 0 to 6 years (hereafter referred to as 'households with a Baby'), corresponding to all HH2 households.
- C – households with at least one child, the youngest aged 7 to 17 years (hereafter referred to as 'households with a Child'), corresponding to all HH3 households.

Therefore, in accordance with the life-cycle framework, the household may transition between types in each period of the insurance contract's duration. However, such transitions must follow a predefined sequence, consistent with the assumption introduced in Section 1.4, whereby a deterministic household development cycle – not introducing additional children – is assumed (see Figure 2.2).

A. For household of type B



B. For household of type C.

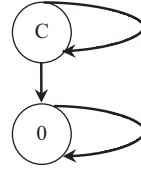


Figure 2.2 Household type transition diagram (closed child population).

Source: Author's own elaboration.

Given the assumption of a closed child population, the BHLI contracts that include child-related benefits may be offered to families with young children (referred to in this monograph as *Scenario B*; cf. Figure 2.2A) or to families with children aged between 7 and 17 years (referred to as *Scenario C*; cf. Figure 2.2B). These two scenarios differ in the age structure of the insured child population, which affects both the configuration of the state space and the amount of benefit payments.

### Scenario B (Modified Multi-State Model)

If, in accordance with the insurance contract, the amounts of cash flows depend on the household type, then each state in which the cash flows are realised should be subdivided into three states, 0, B, and C (cf. Figure 2.2A), representing the possible household structure at the time of cash flow realisation. Thus, one obtains for each state  $j \in \{1, 2^1, 2^2, 2^3, 3^1, 3^2, 3^3\}$  the states  $j_0, j_B, j_C$ , and the new state space for the BHLI contract  $S_{\text{BHLI}(B)}$  consists of 22 states.

Therefore, the transitions between the newly defined states must account for all possible developments in household composition (see Figure 2.2A). Then the new set of direct transitions for the BHLI contract according to *Scenario B*, i.e.  $T_{\text{BHLI}(B)}$  consist of 60 pairs. The final MMSM for *Scenario B*  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$  is presented in Figure 2.3.

The MSM presented in Figure 2.3 is comprehensive, as it allows not only for the adjustment of the benefit amount depending on the period of widowhood and the current composition of the household, but also for the differentiation of the insurance premium amount depending on the current structure of the household.

The initial state in the model presented in Figure 2.3 is state  $1_B$ , thus the model is dedicated to families with the youngest child up to six years of age, denoted as  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$ .

□

**Scenario C (Modified Multi-State Model)**

If the BHLI contract is completed by a family with a child aged 7–17 (cf. Figure 2.2B), then all states and transitions marked in red in the diagram in Figure 2.3 are not needed, and the model contains 15 states and 33 transitions between them. For notation purposes, this model is described by  $(S_{\text{BHLI}(C)}, T_{\text{BHLI}(C)})$ ; the initial state is  $1_C$ .

□

Moreover, if the BHLI regards a family without children or with adult children (according to *Scenario 0*), then all states and transitions marked in red and green are not needed in the model presented in Figure 2.3. The remaining model is identical to that shown in Figure 2.1C, namely  $(S_{\text{BHLI}(0)}, T_{\text{BHLI}(0)}) \equiv (S^*, T^*)$ .

Thus, the MSM presented in Figure 2.3 includes all the considered scenarios in the sense that

$$(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)}) \subset (S_{\text{BHLI}(C)}, T_{\text{BHLI}(C)}) \subset (S_{\text{BHLI}(0)}, T_{\text{BHLI}(0)}). \quad (2.13)$$

The change in the household structure during the insurance contract period affects the reduction of the MSM. The level of reduction depends on the initial composition of the household and the transitions in its structure, as seen in Figure 2.2A.

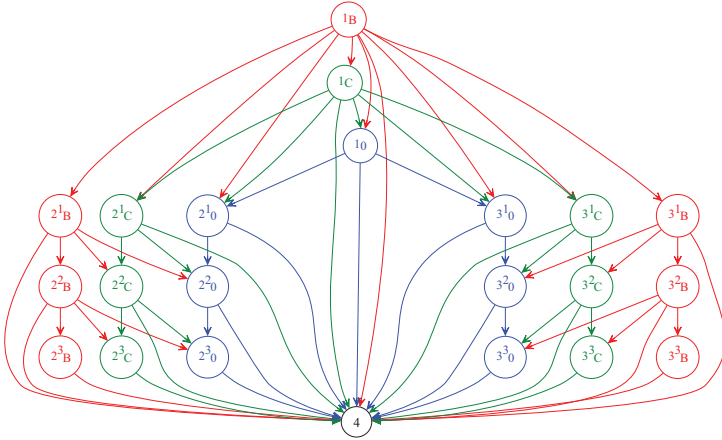


Figure 2.3 Modified multi-state model for the Bridging Household Life Insurance contract.

Source: Author's own elaboration. The diagram was generated using Graphviz software (AT&T Labs Research, 2023).

## 2.2 Model's Probabilistic Structure

The analysis of state transitions (i.e. the evolution of the contract states from inception to termination) is one of the fundamental factors affecting its valuation. Note that after changing  $(S, T)$  into the reduced MMSM  $(S^*, T^*)$  (c.f. MSM in Figure 2.1A into MMSM in Figure 2.1C), it is necessary to introduce a new discrete-time stochastic process  $\{X^*(t) : t = 0, 1, 2, \dots, n^*\}$  that describes the evolution of the insured risk. Process  $\{X^*(t)\}$  takes values in finite state space  $S^*$ , and it is assumed that only one transition may occur per time unit, corresponding to a single random event. Note that  $X^*(t) = i$  (for  $i \in S^*$ ) means that at time  $t$  those insured are affected by the life situation to which state  $i$  has been assigned.

In actuarial literature, the family of Markov chains (especially non-homogeneous) is considered a sufficiently flexible class of stochastic processes, allowing for the adjustment of model parameters to the specific conditions of the insurance under analysis (Hoem, 1969, 1988; Norberg, 2014; Waters, 1984; Wolthuis, 1994). In this context it is assumed that the process  $\{X^*(t) : t = 0, 1, 2, \dots, n^*\}$  follows the properties of a Markov chain (c.f. Billingsley, 1995). Moreover, the initial state is denoted by 1 (i.e.  $X^*(0) = 1$ ), and all the other states are assumed to be reachable from this starting point.

If the process is observed at discrete time points, the introduction of matrix notation constitutes a convenient and commonly applied approach. Consequently, let

$$\mathbf{Q}(t) = [q_{ij}(t)]_{i,j=1}^{N^*} \quad (2.14)$$

be the matrix transition, where  $q_{ij}(t) = P(X^*(t+1) = j | X^*(t) = i)$  is the transition probability of process  $\{X^*(t)\}$  going to state  $j$  under the condition that  $X^*(t) = i$ . It is important to note that each  $\mathbf{Q}(t)$  is a transition matrix; therefore, it satisfies the following conditions:

- for each  $i \in S^*$  the sum of the elements in every row equals one (i.e.  $\sum_{j=1}^{N^*} q_{ij}(t) = 1$ )
- $q_{ij}(t) = 0$  if there is no direct transition from state  $i$  to state  $j$  in the MSM (i.e.  $(i, j) \notin T^*$ ).

The probabilistic distributions of  $\{X^*(t)\}$  take the following form:

$$\mathbf{P}(t) = (p_1(t), p_2(t), p_3(t), \dots, p_{N^*}(t))^T \in \mathbb{R}^{N^*}, \quad (2.15)$$

where  $p_i(t) = P(X^*(t) = i)$  denotes the state probability at time  $t$ . In particular, the initial distribution  $\mathbf{P}(0) = (p_1(0), p_2(0), \dots, p_{N^*}(0))^T \in \mathbb{R}^{N^*}$ , assuming that  $X^*(0) = 1$ , has the form  $\mathbf{P}(0) = (1, 0, 0, \dots)^T \in \mathbb{R}^{N^*}$ .

Under the assumption that  $\{X^*(t)\}$  is a non-homogeneous Markov chain vector,  $\mathbf{P}(t)$  can be determined from the sequence of the matrix transition  $\{\mathbf{Q}(t)\}_{t=0}^{n^*-1}$  in line with the formula (c.f. Dębicka, 2012, 2013):

$$\mathbf{P}^T(t) = \mathbf{P}^T(0) \prod_{k=0}^{t-1} \mathbf{Q}(k). \quad (2.16)$$

In order to characterise the probabilistic structure of  $\{X^*(t)\}$  over the entire insurance period, i.e. at any time  $k = 0, 1, 2, \dots, n^*$ , the following matrix

$$\mathbf{D} = \begin{pmatrix} \mathbf{P}(0)^T \\ \mathbf{P}(1)^T \\ \vdots \\ \mathbf{P}(n^*)^T \end{pmatrix} \in \mathbb{R}^{(n^*+1) \times N^*} \quad (2.17)$$

is introduced.

Before presenting  $\{\mathbf{Q}(t)\}_{t=0}^{n^*-1}$ , which uniquely defines the matrix  $\mathbf{D}$  for the BHLI contract scenarios (according to (2.16)), some useful notation and assumptions are specified.

Let  $x$  and  $y$  be the ages at the policy issue of the woman (wife, female partner) and the man (husband, male partner), respectively. Moreover, let  $a_k(t) = a_k + t$  denote the age of the  $k$ -th minor child ( $m_k$ ) at time  $t$  during the term of the insurance contract, where it is assumed that  $a_k(0) = a_k$  and  $0 \leq a_1 \leq a_2 \leq \dots$ .

Let  $K^k(a)$  be a random variable representing the total number of future years of life remaining for person  $k$  aged  $a$ . In this monograph,  $k \in \{w, m, m_1, m_2, \dots\}$  indicates the specific individual in the household to whom variable  $K^k(a)$  refers ( $w, m$  stand for woman and man, respectively). The one-year probability of death at age  $a + t$  is denoted by

$$q_{a+t}^k = P(K^k(a+t) = 0) \quad (2.18)$$



where the matrix elements are determined directly from the life tables for women ( $p_{x+t}, q_{x+t}$ ) and men ( $p_{y+t}, q_{y+t}$ ).

In order to facilitate a subsequent analysis, the transition matrix was decomposed and expressed in block matrix form. This representation enables a more concise definition of the transition matrix for  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$  and  $(S_{\text{BHLI}(C)}, T_{\text{BHLI}(C)})$  models, where

$$\begin{aligned} \mathbf{Q}_0(t) &= \begin{pmatrix} p_{x+t}p_{y+t} & q_{x+t}p_{y+t} & 0 & 0 & p_{x+t}q_{y+t} & 0 & 0 & q_{x+t}q_{y+t} \\ 0 & 0 & p_{y+t} & 0 & 0 & 0 & 0 & q_{y+t} \\ 0 & 0 & 0 & p_{y+t} & 0 & 0 & 0 & q_{y+t} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & p_{x+t} & 0 & q_{x+t} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{x+t} & q_{x+t} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{A}(k) & \overline{\mathbf{A}}(k) \\ \mathbf{Z}_7^T & 1 \end{pmatrix}. \end{aligned} \quad (2.21)$$

Matrix  $\mathbf{A}(t) \in R^{7 \times 7}$  constitutes a part of the transition matrix  $\mathbf{Q}_0(t)$  and corresponds to states in which premiums and benefits are paid, specifically, those in which at most one spouse has died, and the duration of widowhood does not exceed three years. In contrast, vector  $\overline{\mathbf{A}}(t) \in R^{7 \times 1}$  represents the absorbing state that marks the termination of the insurance contract. This may be interpreted as a complement to the events described by matrix  $\mathbf{A}(t)$ , namely situations in which at least one spouse has been deceased for more than three years; moreover,  $\mathbf{Z}_7^T = (0, 0, 0, 0, 0, 0, 0) \in R^{1 \times 7}$  is *zero vector*.

□

### **Scenario B (Transition Probability Matrix)**

Clearly, in order to describe the transition matrix for  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$ , the survival and death probabilities related solely to the future lifetime of the insured spouses are insufficient. The probabilistic structure is influenced by two key factors: the primary one, which relates to the future lifetime of the insured spouses, and the external one, which accounts for the future lifetime

of the children and the number of children in the household. These external elements jointly define the stage of the household development cycle and must be incorporated into the model. Therefore, let

$B(t)$  – denotes the maximum possible number of children aged six years or less at the beginning of the  $t + 1$ -th policy year, under the assumption that children grow older but do not die.

$C(t)$  – denotes the maximum possible number of children over the age of six (but not yet adults at the beginning of the  $t + 1$ -th policy year), again assuming that children age but do not die.

Under assumption A1, probability  $q^B(t)$  of death of all children aged 0 to 6 in the  $t + 1$ -th year of the insurance contract equals

$$q^B(t) = \begin{cases} 1 & \text{for } B(t) = 0 \\ \prod_{k=1}^{B(t)} q_{a_k+t} & \text{for } B(t) > 0 \end{cases} \quad (2.22)$$

Similarly,  $q^C(t)$  denotes the probability that all children over the age of 6 (but not yet adults) die within the  $t + 1$ -th policy year, and equals

$$q^C(t) = \begin{cases} 1 & \text{for } C(t) = 0 \\ \prod_{k=B(t)+1}^{B(t)+C(t)} q_{a_k+t} & \text{for } C(t) > 0 \end{cases} \quad (2.23)$$

Note that  $B(t) = C(t) = 0$  indicates that a permanent change has occurred in the household structure and in the group of insured children. This situation arises when:

- a child reaches the age threshold (i.e.  $a_k + t = 6$  or  $a_k + t = 17$ ) and exits their original age group in year  $t + 1$ .
- The child dies before time  $t + 1$  and the household composition is updated accordingly in year  $t + 1$ .
- The child has already left the household in previous years ( $t = \{1, 2, \dots, t - 1\}$ )

In such cases the change is irreversible, and therefore  $q^B(t) = q^C(t) = 1$ .

Let  $H_B(t)$  denote the household type at time  $t$ , where  $H_B(t) = h$ ,  $h \in \{B, C, 0\}$  and  $t = 0, 1, 2, \dots, n^*$ . In each year of the insurance contract, the household may change its type; however, these changes must follow a specific cycle, as illustrated in Figure 2.2A – thus, they are random in nature. Therefore, it is obvious to assume that  $\{H_B(t); t = 0, 1, 2, \dots\}$  is a stochastic process,

taking values from a finite set space  $\{B, C, 0\}$ . Furthermore, it is assumed that this process is modelled by a non-homogeneous Markov chain. This approach is consistent with the literature, in which households are assumed to progress through successive stages of development according to a Markov chain (e.g. Du & Kamakura, 2006; Han et al., 2020) or general life-cycle models (e.g. Fella et al., 2017; Keyfiz & Caswell, 2005).

Accordingly, let matrix  $\mathbf{H}_B(t) = [h_{gk}(t)]_{g,k \in \{B,C,0\}} \in R^{3 \times 3}$  represent the transition probability matrix describing the change in household type in the  $(t+1)$ -th policy year, where

$$h_{gk}(t) = P(H_B(t+1) = k \mid H_B(t) = g). \quad (2.24)$$

This matrix is associated with the household development cycle and reflects the possible transitions in household composition occurring in the  $(t+1)$ -th year of the insurance contract, resulting from demographic changes related to the ageing and mortality of children. Taking into account the introduced notation and the order-preserving structure of household type transitions, matrix  $\mathbf{H}_B(t)$  takes the following form:

$$\mathbf{H}_B(t) = \begin{bmatrix} h_{BB}(t) & h_{BC}(t) & h_{B0}(t) \\ 0 & h_{CC}(t) & h_{C0}(t) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - q^B(t) & q^B(t) \cdot (1 - q^C(t)) & q^B(t) \cdot q^C(t) \\ 0 & 1 - q^C(t) & q^C(t) \\ 0 & 0 & 1 \end{bmatrix}. \quad (2.25)$$

In light of assumption A1, process  $\{H_B(t); t = 0, 1, 2, \dots\} \equiv \{H_B(t)\}$  serves as a *contextual factor* in determining the probabilistic structure of the  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$  model. While it influences the cash flows associated with the model, it does not directly affect the stochastic process  $\{X^*(t)\}$  itself. Given this distinction, the probabilistic structure of the model exhibits a dual stochastic nature, represented by the pair  $\{(X^*(t), H_B(t)); t = 0, 1, 2, \dots\}$ , and is described by a Markov chain with a contextual component – hereafter in this book referred to as the *Contextual Markov Chain*.

Given that, for a household with a small child up to six years of age, the longest possible realisation of process  $\{H_B(t)\}$  results from the youngest child reaching adulthood, the following order of states is assumed in transition matrix  $\mathbf{Q}_B(t)$  of the  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$  model

$$S_{\text{BHLI}(B)} = \{1_B, 2_B^1, 2_B^2, 2_B^3, 3_B^1, 3_B^2, 3_B^3, 1_C, 2_C^1, 2_C^2, 2_C^3, 3_C^1, 3_C^2, 3_C^3, 1_0, 2_0^1, 2_0^2, 2_0^3, 3_0^1, 3_0^2, 3_0^3, 4\}. \quad (2.26)$$

Matrix  $\mathbf{Q}_B(t)$  is a block matrix given by the following formula:

$$\mathbf{Q}_B(t) = \begin{pmatrix} \mathbf{Q}_{BB}(t) & \mathbf{Q}_{BC}(t) & \mathbf{Q}_{B0}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{Q}_{CB}(t) & \mathbf{Q}_{CC}(t) & \mathbf{Q}_{C0}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{Q}_{0B}(t) & \mathbf{Q}_{0C}(t) & \mathbf{Q}_{00}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{Z}_7^T & \mathbf{Z}_7^T & \mathbf{Z}_7^T & 1 \end{pmatrix} \in R^{22 \times 22}, \quad (2.27)$$

where  $\mathbf{Q}_{gk}(t) = h_{gk}(t) \cdot \mathbf{A}(t)$  and  $g, k \in \{B, C, 0\}$ . Taking into account (2.25), the matrix  $\mathbf{Q}_B(t)$  can be written as:

$$\begin{aligned} \mathbf{Q}_B(t) &= \begin{pmatrix} h_{BB}(t) \cdot \mathbf{A}(t) & h_{BC}(t) \cdot \mathbf{A}(t) & h_{B0}(t) \cdot \mathbf{A}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{0} & h_{CC}(t) \cdot \mathbf{A}(t) & h_{C0}(t) \cdot \mathbf{A}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{0} & \mathbf{0} & 1 \cdot \mathbf{A}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{Z}_7^T & \mathbf{Z}_7^T & \mathbf{Z}_7^T & 1 \end{pmatrix} \\ &= \begin{pmatrix} (1 - q^B(t)) \cdot \mathbf{A}(t) & q^B(t) \cdot (1 - q^C(t)) \cdot \mathbf{A}(t) & q^B(t) \cdot q^C(t) \cdot \mathbf{A}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{0} & (1 - q^C(t)) \cdot \mathbf{A}(t) & q^C(t) \cdot \mathbf{A}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{0} & \mathbf{0} & \mathbf{A}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{Z}_7^T & \mathbf{Z}_7^T & \mathbf{Z}_7^T & 1 \end{pmatrix}, \end{aligned} \quad (2.28)$$

where the matrix  $\mathbf{0} = [0]_{i,j=1}^7$  contains only zero elements and is called a *zero matrix*.

□

### Scenario C (Transition Probability Matrix)

Note that  $\{H_C(t); t = 0, 1, 2, \dots\}$  is a stochastic process, taking values from the finite set space  $\{C, 0\}$ . Process  $\{H_C(t)\}$  serves as a contextual factor in determining the probabilistic structure of the  $(S_{\text{BHLI}(C)}, T_{\text{BHLI}(C)})$  model (cf. Figure 2.2B). In a similar way to *Scenario B*, it can be shown that

$$\mathbf{Q}_C(t) = \begin{pmatrix} (1 - q^C(t)) \cdot \mathbf{A}(t) & q^C(t) \cdot \mathbf{A}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{0} & \mathbf{A}(t) & \bar{\mathbf{A}}(t) \\ \mathbf{Z}_7^T & \mathbf{Z}_7^T & 1 \end{pmatrix} \in R^{15 \times 15}. \quad (2.29)$$

□

Since relation (2.10) holds, the elements of the matrix (2.28) were colour-coded to correspond to the states shown in the graphical representation of model  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$  in Figure 2.3, that is

$$\mathbf{Q}_B(t) = \begin{pmatrix} (1-q^B(t)) \cdot \mathbf{A}(t) & q^B(t) \cdot (1-q^C(t)) \cdot \mathbf{A}(t) & q^B(t) \cdot q^C(t) \cdot \mathbf{A}(t) & \overline{\mathbf{A}}(t) \\ \mathbf{0} & (1-q^C(t)) \cdot \mathbf{A}(t) & q^C(t) \cdot \mathbf{A}(t) & \overline{\mathbf{A}}(t) \\ \mathbf{0} & \mathbf{0} & \mathbf{A}(t) & \overline{\mathbf{A}}(t) \\ \mathbf{Z}_7^T & \mathbf{Z}_7^T & \mathbf{Z}_7^T & 1 \end{pmatrix} \quad (2.30)$$

In the case of model  $(S_{\text{BHLI}(C)}, T_{\text{BHLI}(C)})$ , the transition matrix consists of the elements of matrix  $\mathbf{Q}_B(t)$  highlighted in green and blue (cf. (2.23)), while for model  $(S_{\text{BHLI}(0)}, T_{\text{BHLI}(0)})$ , it includes only the blue elements and is the same as the matrix (2.21).

It should be noted that all elements of the matrix (2.30) can be determined based on one-year death probabilities  $q_t$ , where  $t$  denotes the age of a household member at a given point during the insurance contract – provided in the standard life tables.

Within the framework of this study, notation  $\{XH_B(t); t = 0, 1, 2, \dots\} = \{(X^*(t), H_B(t)); t = 0, 1, 2, \dots\}$  was implemented to denote the process describing the evaluation of insured risk in model  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$ . Analogously for model  $(S_{\text{BHLI}(C)}, T_{\text{BHLI}(C)})$  let  $\{XH_C(t); t = 0, 1, 2, \dots\} = \{(X^*(t), H_C(t)); t = 0, 1, 2, \dots\}$  where  $\{H_C(t); t = 0, 1, 2, \dots\}$  is a stochastic process taking values from finite set space  $\{C, 0\}$ . For  $(S_{\text{BHLI}(0)}, T_{\text{BHLI}(0)})$ , it was assumed that process  $\{X_0(t)\}$  describes the evaluation of the insured risk.

## 2.3 Actuarial Value of Cash Flows

In life insurance mathematics, the valuation of a contract relies fundamentally on the proper quantification of all future financial cash flows. These include both premium and benefit payments, which are realised over time and contingent on the evolution of the insured risk. The actuarial value of cash flows provides a unified framework for converting these future uncertain payments into present values, using both the assumed interest rate and the probability of future states of the insured risks. This section introduces the formal definition

of actuarial value and presents methods for evaluating it within the MSM structure applied for BHLI contracts.

Given the complexity and dimensionality of the MSM of the BHLI contract, it is advantageous to employ the matrix notation of actuarial quantities, particularly with regard to the premiums as the main aim of Section 2.4. This approach enables a more compact and transparent representation of both cash flows and discounting mechanisms, which is especially valuable in MSMs. Thus, in this section discount factors and cash flows were also formulated using matrix notation.

In insurance mathematics, the *discounting function* serves to determine the present value of the future payments. Denote by  $v(k, t)$  the discount factor for a payment of one unit received at time  $t$ , valued at time  $k$  (for  $0 \leq k \leq t$ ). In particular, function  $v(0, t)$  is used to express future insurance cash flow in terms of its present value at the start of the contract.

Assuming a discrete-time framework over contract period  $t = 0, 1, 2, \dots, n^*$  defines the discount vector

$$\mathbf{V} = (v_0, v_1, \dots, v_{n^*})^T \in R^{\binom{n^*+1}{1} \times 1}, \quad (2.31)$$

where  $v_t = v(0, t)$  represents the value at time 0 of one unit of currency payable at time  $t$ .

In general, discount factors are derived from the interest rate applicable over a given time period. Under continuous compounding and a time-varying instantaneous interest rate  $r(t)$ , the discount factor from time 0 to  $t$  is given by:

$$v_t = e^{-\int_0^t r(s) ds}. \quad (2.32)$$

While interest rate modelling may involve stochastic dynamics (see, e.g. Dębicka et al., 2022; Gibson et al., 2010; Jajuga, 2006; Mehalla, 2021), this monograph considers the case of constant interest rate  $r > 0$ . In such a setting, discount factors for time periods  $[0, t]$  (given by (2.32)), are simplified to

$$v_t = v(0, t) = (1 + r)^{-t} = \left( \frac{1}{1 + r} \right)^t = v^t, \quad (2.33)$$

which ensures a deterministic valuation framework. A constant interest rate model, independent of external influences, is commonly used when changes in interest rates are negligible or when the primary interest lies in analysing other sources of risk, such as demographic or biometric variability (Bowers et al., 1986). Accordingly, this monograph applies a constant interest rate assumption to reduce model complexity and focuses analysis of the dynamic

changes in household composition that play a central role in the valuation of multi-life insurance contracts to which BHLI belongs.

The next step concerns the cash flows arising from the realisation of the BHLI contract, serving as the basis for subsequent actuarial contract valuation. Each cash flow is characterised by its nature (premium or benefit), frequency (single or periodic), and timing of payment (in advance or from below); hence, the authors consider a single premium (type  $\pi$  cash flow) paid at the beginning of the insurance contract and a period premium paid in the first  $m$  year of the insurance contract ( $m \leq n_2$ ) which are cash flows paid in advance. Thus, immediate annuities (type  $b$  cash flow) are periodic cash flows paid from below. Note that all  $\pi, p, b$  types of the cash flows are connected with the remaining process  $\{XH_B(t)\}$  (or  $\{XH_C(t)\}, \{X_0(t)\}$  respectively) in the considered state, which means that all transitions from the set of direct transition  $T_{BHLI(B)}$  ( $T_{BHLI(C)}, T_{BHLI(0)}$  respectively) are *non-cf*. Therefore, assumptions B1 and B2 are satisfied, and it is possible to record cash flows resulting from the performance of a contract in matrix form.

For the model  $(S^*, T^*)$  with process  $\{X^*(t)\}$ , let  $C^{\Gamma^{(\varphi, j)}} = \left[ \varphi_i^{\Gamma^{(\varphi, j)}}(t) \right]_{\substack{i=1,2,\dots,N^* \\ t=0,1,\dots,n^*}} \in R^{(n^*+1) \times N^*}$  denote the matrix of cash flows  $\varphi$  realised in accordance with set of parameters  $\Gamma^{(\varphi, j)}$  (cf. (2.3)), where  $\varphi_i^{\Gamma^{(\varphi, j)}}(t)$  is a cash flow of type  $\varphi$  realised at time  $t$  when  $X^*(t) = i$ , with  $i \in S^*$  and  $j \in S$ . This implies that for a particular MMSM  $(S^*, T^*)$ , one can define as many matrices  $C^{\Gamma^{(\varphi, j)}}$  as there are sets  $\Gamma^{(\varphi, j)}$  representing conditions of realisation of different types of cash flows for the MSM  $(S, T)$ .

At any given moment  $t$  during the term of the insurance contract, any of the types of cash flows  $\varphi$  may be realised. Let  $cf_j^*(t)$  (cash flows arising from MMSM) denote the total cash flow realised at time  $t$  conditional on the process  $\{X^*(t)\}$  being in state  $j$ , i.e.  $X^*(t) = j$

$$cf_j^*(t) = \sum_{\varphi \in \{\pi, p, b, \ddot{b}, d, c\}} \varphi_j(t). \quad (2.34)$$

Under (2.34), the *cash flow matrix*  $C = (cf_i^*(k))_{\substack{i=1,2,\dots,N^* \\ k=0,1,\dots,n^*}}$  defined for insurance

contract modelled by  $(S^*, T^*)$  can be expressed as follows:

$$C = \sum_{\varphi \in \{\pi, p, b, \ddot{b}, d, c\}} \sum_{j \in S^*} C^{\Gamma^{(\varphi, j)}}. \quad (2.35)$$

The identification of the non-zero elements of the cash flow matrix  $\mathbf{C}^{\Gamma^{(\varphi,j)}}$  depends on the type of cash flow  $\varphi$  and, more specifically, on its characteristics – namely the frequency and timing of payments. According to (Dębicka, 2012), for  $(S^*, T^*)$ , the following properties hold:

- 1 If  $\varphi$  is a benefit paid from below (i.e.  $\varphi \in \{b, c\}$ ) is such that  $\Gamma^{(\varphi,j)} \neq (0, 0, 0, 0, 0)$ , then all elements of cash flow matrix  $\mathbf{C}^{\Gamma^{(\varphi,j)}}$  outside the region bounded by the following elements

$$\begin{aligned} w_1(j) &= \varphi_{j(f^{(\varphi,j)}+1)}^{\Gamma^{(\varphi,j)}} \left( \min \left\{ n_1^{(\varphi,j)} + f^{(\varphi,j)} + 1, n^* \right\} \right) \\ w_2(j) &= \varphi_{j(f^{(\varphi,j)}+1)}^{\Gamma^{(\varphi,j)}} \left( \min \left\{ n_2^{(\varphi,j)} + f^{(\varphi,j)}, n^* \right\} \right) \\ w_3(j) &= \varphi_{j(\min\{\gamma^{(\varphi,j)}, s^{(\varphi,j)}\})}^{\Gamma^{(\varphi,j)}} \left( \min \left\{ n_1^{(\varphi,j)} + \min\{\gamma^{(\varphi,j)}, s^{(\varphi,j)}\}, n^* \right\} \right) \\ w_4(j) &= \varphi_{j(\min\{\gamma^{(\varphi,j)}, s^{(\varphi,j)}\})}^{\Gamma^{(\varphi,j)}} \left( \min \left\{ n_2^{(\varphi,j)} - 1 + \min\{\gamma^{(\varphi,j)}, s^{(\varphi,j)}\}, n^* \right\} \right) \end{aligned} \quad (2.36)$$

are equal to zero.

- 2 If  $\varphi$  is a periodical cash flow paid in advance (i.e.  $\varphi \in \{\ddot{b}, p\}$ ) is such that  $\Gamma^{(\varphi,j)} \neq (0, 0, 0, 0, 0)$ , then all elements of cash flow matrix  $\mathbf{C}^{\Gamma^{(\varphi,j)}}$  outside the region bounded by these elements

$$\begin{aligned} w_1(j) &= \varphi_{j(f^{(\varphi,j)}+1)}^{\Gamma^{(\varphi,j)}} \left( \min \left\{ n_1^{(\varphi,j)} + f^{(\varphi,j)} + 1, n^* \right\} - 1 \right) \\ w_2(j) &= \varphi_{j(f^{(\varphi,j)}+1)}^{\Gamma^{(\varphi,j)}} \left( \min \left\{ n_2^{(\varphi,j)} + f^{(\varphi,j)}, n^* \right\} - 1 \right) \\ w_3(j) &= \varphi_{j(\min\{\gamma^{(\varphi,j)}, s^{(\varphi,j)}\})}^{\Gamma^{(\varphi,j)}} \left( \min \left\{ n_1^{(\varphi,j)} + \min\{\gamma^{(\varphi,j)}, s^{(\varphi,j)}\}, n^* \right\} - 1 \right) \\ w_4(j) &= \varphi_{j(\min\{\gamma^{(\varphi,j)}, s^{(\varphi,j)}\})}^{\Gamma^{(\varphi,j)}} \left( \min \left\{ n_2^{(\varphi,j)} - 1 + \min\{\gamma^{(\varphi,j)}, s^{(\varphi,j)}\}, n^* \right\} - 1 \right) \end{aligned} \quad (2.37)$$

are equal to zero.

3 If  $\wp$  is a single cash flow (i.e.  $\wp \in \{\pi, d\}$ ) is such that  $\Gamma^{(\wp, j)} \neq (-)$ ,

and if  $\gamma^{(\cdot, j)} > 1$ , then  $\Gamma^{(\wp, j)} = \Gamma^{(\wp, j^1)} = \dots = \Gamma^{(\wp, j^{\gamma^{(\cdot, j)}})}$  and for each  $i \in \{j^1, j^2, \dots, j^{\gamma^{(\wp, j)}}\}$  at time  $t \in \Gamma^{(\wp, j)}$ , it holds that  $\wp_i^{\Gamma^{(\wp, j)}}(t) \neq 0$ .

For MMSM model  $(S^*, T^*)$  presented in Figure 2.1C, there are two annuity benefits realised according to sets of policy conditions:  $\Gamma^{(b, 2)} = (0, n, 0, 3, \infty)$  and  $\Gamma^{(b, 3)} = (0, n, 0, 3, \infty)$  and the period insurance premiums paid for the first  $m$  ( $m \leq n$ ) units of the insurance period, i.e.  $\Gamma^{(p, 1)} = (0, m, 0, \infty, m)$ . Then, by applying the equation (2.36):

– for matrix  $\mathbf{C}^{\Gamma^{(b, 2)}}$

$$\begin{aligned} w_1(2) &= b_{21}(1) \\ w_2(2) &= b_{21}(n) \\ w_3(2) &= b_{23}(3) \\ w_4(2) &= b_{23}(n+2) \end{aligned} \quad , \quad (2.38)$$

– for matrix  $\mathbf{C}^{\Gamma^{(b, 3)}}$

$$\begin{aligned} w_1(2) &= b_{31}(1) \\ w_2(2) &= b_{31}(n) \\ w_3(2) &= b_{33}(3) \\ w_4(2) &= b_{33}(n+2) \end{aligned} \quad . \quad (2.39)$$

Moreover, by applying (2.37) for matrix  $\mathbf{C}^{\Gamma^{(p, 1)}}$  one obtains

$$\begin{aligned} w_1(1) = w_3(1) &= p_{11}(0) = p_1(0) \\ w_2(1) = w_4(1) &= p_{11}(m-1) = p_1(m-1) \end{aligned} \quad . \quad (2.40)$$

Based on (2.35) and (2.38) to (2.40), the cash flow matrix can be expressed in the following form:

$$\begin{aligned}
 \mathbf{C} &= \mathbf{C}^{I(p,1)} + \mathbf{C}^{I(b,2)} + \mathbf{C}^{I(b,3)} \\
 &= \begin{pmatrix}
 w_1(1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \bullet & w_1(2) & 0 & 0 & w_1(3) & 0 & 0 & 0 \\
 \bullet & \bullet & \bullet & 0 & \bullet & \bullet & 0 & 0 \\
 \bullet & \bullet & \bullet & w_3(2) & \bullet & \bullet & w_3(3) & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 w_2(1) & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 \\
 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 \\
 0 & w_2(2) & \bullet & \bullet & w_2(3) & \bullet & \bullet & 0 \\
 0 & 0 & \bullet & \bullet & 0 & \bullet & \bullet & 0 \\
 0 & 0 & 0 & w_4(2) & 0 & 0 & w_4(3) & 0
 \end{pmatrix} \in R^{(n+3) \times 8} \quad (2.41)
 \end{aligned}$$

where  $\bullet$  indicates non-zero elements.

It should be noted that the implementation of a life insurance contract generates two opposing streams of cash flows: the premium stream, representing payments made by the insured to the insurer, and the benefit stream, representing payments made by the insurer to the insured or their beneficiaries. From the perspective of the *insurer's total loss fund*, premiums are treated as outflows, as they reduce the fund's total value (are negative), whereas benefits are considered inflows, as they increase its value (are positive).

Therefore, in order to properly value the BHLI contract, it is necessary to explicitly define matrix  $\mathbf{C}$  in a manner that reflects both the detailed terms of the contract and the direction of cash flows. The elements of the cash flow matrix in the case of the insurer's total loss fund will now be defined for each of the three BHLI settlement scenarios under consideration.

### Scenario 0 (Specification of Cash Flows under Decreasing Survivor Benefits)

Assume that the insured pays a constant annual premium  $p$  throughout the entire insurance period of  $n$  years (hence,  $m = n$ ). Upon the death of one of the spouses, the surviving partner is entitled to an annual annuity benefit. Let  $b_2^0$  denote the survivor benefit received by the man after the death of his wife, and  $b_3^0$  the benefit received by the woman after the death of her husband.

According to the policy conditions, these annuity benefits are non-increasing over time. Let  $\alpha_1, \alpha_2, \alpha_3$  be *reduction coefficients* with  $0 < \alpha_i \leq 1$

and  $\alpha_1 \leq \alpha_2 \leq \alpha_3$  that regulate the level of annuity instalment in the 1st, 2nd, and 3rd years of obtaining the benefit. Consequently, the amount of the survivor benefit in a given year depends on both the time of the spouse's death and the duration since that event.

Given these assumptions, the cash flows resulting from the realisation of the BHLI contract can be specified as follows:

$$\varphi_j(t) = \begin{cases} p & \text{for } (j = 1 \wedge t = 0, 1, \dots, n-1) \\ \alpha_i b_2^0 & \text{for } (j = 2^i \wedge t = i, i+1, \dots, n+i-1) \text{ and } i \in \{1, 2, 3\} \\ \alpha_i b_3^0 & \text{for } (j = 3^i \wedge t = i, i+1, \dots, n+i-1) \text{ and } i \in \{1, 2, 3\} \\ 0 & \text{besides} \end{cases} \quad (2.42)$$

The cash flows presented by (2.42) are used to describe elements of the cash flow matrix (2.41) representing the insurer's total loss fund. As a result, one obtains the following form of the cash flow matrix:

$$\mathbf{C}_{(0)} = \begin{pmatrix} -p & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -p & \alpha_1 b_2^0 & 0 & 0 & \alpha_1 b_3^0 & 0 & 0 & 0 \\ -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & 0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & 0 & 0 \\ -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 & 0 \\ 0 & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 & 0 \\ 0 & 0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & 0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 & 0 \\ 0 & 0 & 0 & \alpha_3 b_2^0 & 0 & 0 & \alpha_3 b_3^0 & 0 \end{pmatrix} \in R^{(n+3) \times 8}. \quad (2.43)$$

However, for a contract with a single premium paid in advance at the beginning of the insurance contract, the form of the matrix (2.43) is analogous, with the difference that the first column is as follows  $\left( \pi \quad 0 \quad \dots \quad 0 \right)^T$  – cf. (2.4) and property 3 described above.

□

### Scenario B (Specification of Cash Flows under Decreasing Survivor Benefits)

It should be noted that in the construction of  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$  model, each of the original states  $i = 1, 2, 3$  (cf. Figure 2.1A) has been decomposed into three states denoted as  $i_h$  where  $h = \{B, C, 0\}$ . Therefore, when specifying the cash flow matrix in *Scenario B*, the first step is to define the sets of cash flow parameters corresponding to these states, namely  $\Gamma^{(p,1B)} = (0, m, 0, \infty, m)$ ,  $\Gamma^{(p,1C)} = \Gamma^{(p,1_0)} = (1, m, 0, \infty, m)$ ,  $\Gamma^{(b,2B)} = \Gamma^{(b,2C)} = \Gamma^{(b,2_0)} = (0, n, 0, 3, \infty)$ ,  $\Gamma^{(b,3B)} = \Gamma^{(b,3C)} = \Gamma^{(b,3_0)} = (0, n, 0, 3, \infty)$ . The sets of parameters describing the realisation of benefits are identical to those for the model presented in Figure 2.1A, namely  $\Gamma^{(b,2B)} = \Gamma^{(b,2C)} = \Gamma^{(b,2_0)} = \Gamma^{(b,2)}$  and  $\Gamma^{(b,3B)} = \Gamma^{(b,3C)} = \Gamma^{(b,3_0)} = \Gamma^{(b,3)}$ . Since in the model  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$  the initial state is  $1_B$ , then  $\Gamma^{(p,1B)} = \Gamma^{(p,1)}$  and premiums paid in states  $1_C$  and  $1_0$  can only be realised from moment 1 ( $n_1 = 1$ ) due to the fact that  $(1_B, 1_C)$ ,  $(1_B, 1_0)$  are direct transitions in set  $T_{\text{BHLI}(B)}$ .

Once these sets have been identified, the procedure for determining the cash flow matrix follows the structure previously introduced in *Scenario 0*, with the modification that cash flow matrix  $C_B$  representing the insurer's total loss fund is now defined over the extended state space as

$$C = \sum_{h \in \{B, C, 0\}} \left( -C^{\Gamma^{(p,1_h)}} + C^{\Gamma^{(b,2_h)}} + C^{\Gamma^{(b,3_h)}} \right) \in R^{(n+3) \times 22}. \quad (2.44)$$

Similarly to *Scenario 0*, it is assumed that the insured individual pays a constant annual premium  $p$  throughout the entire insurance period while both spouses are alive, regardless of the household type. Upon the death of one spouse, the surviving partner becomes entitled to an annuity benefit, with its amount depending on the family situation (type of household). Let  $b_2^h$  denote the benefit received by the husband following the death of his wife, and  $b_3^h$  the benefit received by the wife following the death of her husband, given that  $H(t) = h$  (for  $h \in \{B, C, 0\}$ ) at the time of death. Thus, assuming that the benefits are non-increasing, the corresponding cash flows are defined as follows:

$$\wp_j(t) = \begin{cases} p & \text{for } (j = 1 \wedge t = 0, 1, \dots, n-1) & \text{and } (j = 8, 15 \wedge t = 1, 2, \dots, n-1) \\ \alpha_i b_2^h & \text{for } (j = 2_h^i \wedge t = i, i+1, \dots, n+i-1) & \text{and } i \in \{1, 2, 3\} \text{ and } h \in \{B, C, 0\} \\ \alpha_i b_3^h & \text{for } (j = 3_h^i \wedge t = i, i+1, \dots, n+i-1) & \text{and } i \in \{1, 2, 3\} \text{ and } h \in \{B, C, 0\} \\ 0 & \text{besides} \end{cases} \quad (2.45)$$

As a result, the following form of the cash flow matrix is obtained:

$$C_{(B)} = \begin{pmatrix} -p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -p & \alpha_1 b_2^B & 0 & 0 & \alpha_1 b_3^B & 0 & 0 & -p & \alpha_1 b_2^C & 0 & 0 \\ -p & \alpha_1 b_2^B & \alpha_2 b_2^B & 0 & \alpha_1 b_3^B & \alpha_2 b_3^B & 0 & -p & \alpha_1 b_2^C & \alpha_2 b_2^C & 0 \\ -p & \alpha_1 b_2^B & \alpha_2 b_2^B & \alpha_3 b_2^B & \alpha_1 b_3^B & \alpha_2 b_3^B & \alpha_3 b_3^B & -p & \alpha_1 b_2^C & \alpha_2 b_2^C & \alpha_3 b_2^C \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -p & \alpha_1 b_2^B & \alpha_2 b_2^B & \alpha_3 b_2^B & \alpha_1 b_3^B & \alpha_2 b_3^B & \alpha_3 b_3^B & -p & \alpha_1 b_2^C & \alpha_2 b_2^C & \alpha_3 b_2^C \\ 0 & \alpha_1 b_2^B & \alpha_2 b_2^B & \alpha_3 b_2^B & \alpha_1 b_3^B & \alpha_2 b_3^B & \alpha_3 b_3^B & 0 & \alpha_1 b_2^C & \alpha_2 b_2^C & \alpha_3 b_2^C \\ 0 & 0 & \alpha_2 b_2^B & \alpha_3 b_2^B & 0 & \alpha_2 b_3^B & \alpha_3 b_3^B & 0 & 0 & \alpha_3 b_2^C & \alpha_3 b_2^C \\ 0 & 0 & 0 & \alpha_3 b_2^B & 0 & 0 & \alpha_3 b_3^B & 0 & 0 & 0 & \alpha_3 b_2^C \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_1 b_3^C & 0 & 0 & -p & \alpha_1 b_2^0 & 0 & 0 & \alpha_1 b_3^0 & 0 & 0 & 0 \\ \alpha_1 b_3^C & \alpha_2 b_3^C & 0 & -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & 0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & 0 & 0 \\ \alpha_1 b_3^C & \alpha_2 b_3^C & \alpha_3 b_3^C & -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_1 b_3^C & \alpha_2 b_3^C & \alpha_3 b_3^C & -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 & 0 \\ \alpha_1 b_3^C & \alpha_2 b_3^C & \alpha_3 b_3^C & 0 & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 & 0 \\ 0 & \alpha_2 b_3^C & \alpha_3 b_3^C & 0 & 0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & 0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 & 0 \\ 0 & 0 & \alpha_3 b_3^C & 0 & 0 & 0 & \alpha_3 b_2^0 & 0 & 0 & \alpha_3 b_3^0 & 0 \end{pmatrix} \in R^{(n+3) \times 22}.$$

(2.46)

□

### Scenario C (Specification of Cash Flows under Decreasing Survivor Benefits)

Under the same assumptions denoting benefits and premiums as in *Scenario B*, and given the smaller number of possible states that process  $\{H(t)\}$  can assume (see Figure 2.2B), as well as the fact that the initial state is  $1_C$ , the cash flows in Scenario C take the following form:

$$\wp_j(t) = \begin{cases} p & \text{for } (j = 1 \wedge t = 0, 1, \dots, n-1) & \text{and } (j = 8 \wedge t = 1, 2, \dots, n-1) \\ \alpha_i b_2^h & \text{for } (j = 2_h^i \wedge t = i, i+1, \dots, n+i-1) & \text{and } i \in \{1, 2, 3\} \text{ and } h \in \{C, 0\} \\ \alpha_i b_3^h & \text{for } (j = 3_h^i \wedge t = i, i+1, \dots, n+i-1) & \text{and } i \in \{1, 2, 3\} \text{ and } h \in \{C, 0\} \\ 0 & \text{besides} \end{cases}.$$

(2.47)

The equation (2.47) leads to the following form of the cash flow matrix:

$$C_{(C)} = \begin{pmatrix} -p & 0 & 0 & 0 & 0 & 0 & 0 \\ -p & \alpha_1 b_2^C & 0 & 0 & \alpha_1 b_3^C & 0 & 0 \\ -p & \alpha_1 b_2^C & \alpha_2 b_2^C & 0 & \alpha_1 b_3^C & \alpha_2 b_3^C & 0 \\ -p & \alpha_1 b_2^C & \alpha_2 b_2^C & \alpha_3 b_2^C & \alpha_1 b_3^C & \alpha_2 b_3^C & \alpha_3 b_3^C \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -p & \alpha_1 b_2^C & \alpha_2 b_2^C & \alpha_3 b_2^C & \alpha_1 b_3^C & \alpha_2 b_3^C & \alpha_3 b_3^C \\ 0 & \alpha_1 b_2^C & \alpha_2 b_2^C & \alpha_3 b_2^C & \alpha_1 b_3^C & \alpha_2 b_3^C & \alpha_3 b_3^C \\ 0 & 0 & \alpha_2 b_2^C & \alpha_3 b_2^C & 0 & \alpha_2 b_3^C & \alpha_3 b_3^C \\ 0 & 0 & 0 & \alpha_3 b_2^C & 0 & 0 & \alpha_3 b_3^C \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -p & \alpha_1 b_2^0 & 0 & 0 & \alpha_1 b_3^0 & 0 & 0 \\ -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & 0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & 0 \\ -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -p & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 \\ 0 & \alpha_1 b_2^0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & \alpha_1 b_3^0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 \\ 0 & 0 & \alpha_2 b_2^0 & \alpha_3 b_2^0 & 0 & \alpha_2 b_3^0 & \alpha_3 b_3^0 \\ 0 & 0 & 0 & \alpha_3 b_2^0 & 0 & 0 & \alpha_3 b_3^0 \end{pmatrix} \in R^{(n+3) \times 15}. \quad (2.48)$$

□

A compact and consistent representation of all cash flow payments under the contract was possible due to obtaining cash flow matrices in this section. This matrix representation is the foundation for the valuation of the contract, using matrix-based actuarial techniques developed for MSMs (Dębicka, 2012, 2013; Dębicka et al., 2025).

Before proceeding to the valuation of the insurance contract in Section 2.4, the actual (present) value and the actuarial value of the cash flows must be defined within the framework of the MMSM. In such models, all types of cash flows are associated with the occupation of states by the stochastic process that represents the evolution of the insured risk.

For MMSM model  $(S^*, T^*)$  with process  $\{X^*(t)\}$ , let  $Y_k^{\wp, j}(t)$  denote the *discounted value* at time  $k$  of the cash flow  $\wp$  realised at time  $t$  when  $X^*(t) = j$

$$Y_k^{\wp, j}(t) = \begin{cases} v^{t-k} 1_{\{X^*(t)=j\}} \wp_j(t) & \text{for } 0 \leq k < t \\ 1_{\{X^*(t)=j\}} \wp_j(k) & \text{for } 0 \leq k = t \end{cases}, \quad (2.49)$$

where  $1_A$  denotes the indicator of set  $A$ .

Clearly  $Y_k^{\wp, j}(t)$  is a random variable whose distribution depends on the distribution of the process  $\{X^*(t)\}$ . Thus it is useful to introduce the actuarial value at time  $k$  of cash flow  $\wp$  realised at time  $t$ , when  $X^*(t) = j$  and are  $X^*(k) = i$  defined as

$$E\left(Y_k^{\wp, j}(t) \mid X^*(k) = i\right) = \begin{cases} v^{t-k} q_{ij}(k, t) \wp_j(t) & \text{for } 0 \leq k < t \\ \wp_j(t) & \text{for } 0 \leq k = t \text{ and } i = j \\ 0 & \text{for } 0 \leq k = t \text{ and } i \neq j \end{cases} \quad (2.50)$$

In particular, at the beginning of the insurance contract process,  $\{X^*(t)\}$  is at state 1 (i.e.  $X^*(0) = 1$ ). Thus, the *actuarial value* at time 0 of cash flow  $\wp$  realised at time  $t$ , when  $X^*(t) = j$  and  $X^*(0) = 1$ , takes the following form:

$$E\left(Y_0^{\wp, j}(t) \mid X^*(0) = 1\right) = \begin{cases} v^t p_j(t) \wp_j(t) & \text{for } t > 0 \\ \wp_j(t) & \text{for } t = 0 \text{ and } j = 1 \\ 0 & \text{for } t = 0 \text{ and } j \neq 1 \end{cases} \quad (2.51)$$

The sum of all payments of a given type realised in a given state  $j$  throughout the entire insurance period, is as follows:

$$E\left(Y_0^{\wp, j}(0, n^*) \mid X^*(0) = 1\right) = \begin{cases} \sum_{t=0}^{n^*-1} v^t p_j(t) \wp_j(t) & \text{for } \wp \text{ paid in advance} \\ \sum_{t=1}^{n^*} v^t p_j(t) \wp_j(t) & \text{for } \wp \text{ paid from below} \end{cases} \quad (2.52)$$

In particular, for cash flows with periodic frequency equal to one paid in advance (namely unit payments paid in advance e.g.  $\wp \in \{p, \ddot{b}\}$ ) the following notation is used:

$$\ddot{a}_{1j}(0, n^*) = \sum_{t=0}^{n^*-1} v^t p_j(t). \quad (2.53)$$

This section concludes with the total value of all payments of a given type for the scenarios under consideration.

### Scenario 0 (Actuarial Value of Cash Flows)

$$\begin{aligned} & E\left(\Upsilon_0^{\wp, j}(0, n^*) \mid X_0^*(0) = 1\right) = \\ & = \begin{cases} \sum_{t=0}^{n^*-1} v^t p_1(t)p & \text{for } \wp = p \text{ and } j = 1 \\ \sum_{t=i}^{n^*+i-1} v^t p_{2i}(t)\alpha_i b_2 & \text{for } \wp = b \text{ and } j = 2^i \wedge i \in \{1, 2, 3\} \\ \sum_{t=i}^{n^*+i-1} v^t p_{3i}(t)\alpha_i b_3 & \text{for } \wp = b \text{ and } j = 3^i \wedge i \in \{1, 2, 3\} \end{cases}. \end{aligned} \quad (2.54)$$

□

### Scenario B (Actuarial Value of Cash Flows)

$$\begin{aligned} & E\left(\Upsilon_0^{\wp, j}(0, n^*) \mid XH_B(0) = 1_B\right) = \\ & = \begin{cases} \sum_{t=0}^{n^*-1} v^t p_{1_B}(t)p & \text{for } \wp = p \text{ and } j = 1_B \\ \sum_{t=1}^{n^*-1} v^t p_j(t)p & \text{for } \wp = p \text{ and } j \in \{1_C, 1_0\} \\ \sum_{t=i}^{n^*+i-1} v^t p_{2_h^i}(t)\alpha_i b_2^h & \text{for } \wp = b \text{ and } j = 2_h^i \wedge i \in \{1, 2, 3\} \wedge h \in \{B, C, 0\} \\ \sum_{t=i}^{n^*+i-1} v^t p_{3_h^i}(t)\alpha_i b_3^h & \text{for } \wp = b \text{ and } j = 3_h^i \wedge i \in \{1, 2, 3\} \wedge h \in \{B, C, 0\} \end{cases}. \end{aligned} \quad (2.55)$$

□

**Scenario C (Actuarial Value of Cash Flows)**

$$\begin{aligned}
& E\left(\gamma_0^{\wp,j}(0, n^*) \mid XH_C(0) = 1_C\right) = \\
& = \begin{cases} \sum_{t=0}^{n^*-1} v^t p_{1_C}(t) p & \text{for } \wp = p \text{ and } j = 1_C \\ \sum_{t=1}^{n^*-1} v^t p_{1_0}(t) p & \text{for } \wp = p \text{ and } j = 1_0 \\ \sum_{t=i}^{n^*+i-1} v^t p_{2_h^i}(t) \alpha_i b_2^h & \text{for } \wp = b \text{ and } j = 2_h^i \wedge i \in \{1, 2, 3\} \wedge h \in \{C, 0\} \\ \sum_{t=i}^{n^*+i-1} v^t p_{3_h^i}(t) \alpha_i b_3^h & \text{for } \wp = b \text{ and } j = 3_h^i \wedge i \in \{1, 2, 3\} \wedge h \in \{C, 0\} \end{cases}
\end{aligned}
\tag{2.56}$$

□

In all three scenarios, if the contracts are implemented with a single premium payable in advance at the beginning of the insurance period, then the actuarial value of such a cash flow  $\wp = \pi$  is

$$\begin{aligned}
& E\left(\gamma_0^{\wp,j}(0, n^*) \mid X_0^*(0) = 1\right) = E\left(\gamma_0^{\wp,j}(0, n^*) \mid XH_B(0) = 1_B\right) = \\
& = E\left(\gamma_0^{\wp,j}(0, n^*) \mid XH_C(0) = 1_C\right) = \pi.
\end{aligned}
\tag{2.57}$$

**2.4 Net Premium Calculation**

This section is devoted to the introduction and formulation of matrix-based expressions used for premium calculation within the multiple-state model framework.

As outlined in Section 2.3, a life insurance contract involves two opposing cash flow streams: premiums, treated as outflows from the insurer's total loss fund, and benefits, considered inflows to this fund. Let  $Z$  denote *the insurer's total loss*, defined as the difference between the discounted value of all benefit payments and the present value of all premiums paid by the insured over the contract term. Thus, the insurer's total loss may be expressed as:

$$Z = \sum_{\wp \in \{b, \bar{b}, d, c\}} \sum_{j \in S^*} \sum_{t=0}^{n^*} \gamma_0^{\wp,j}(t) - \sum_{\wp \in \{\pi, p\}} \sum_{j \in S^*} \sum_{t=0}^{n^*-1} \gamma_0^{\wp,j}(t). \tag{2.58}$$

A premium is referred to as a net premium if the *equivalence principle* is satisfied, namely when the expected value of the total loss is equal to zero,

that is  $E(Z) = 0$  (cf. Bowers et al., 1986; Dickson et al., 2019; Gerber, 1995). Therefore, under the multi-state insurance model, the equivalence principle takes the form:

$$\sum_{\emptyset \in \{b, \bar{b}, d, c\}} \sum_{j \in S} E\left(r_0^{\emptyset, j}(0, n^*) \middle| X^*(0) = 1\right) = \sum_{\emptyset \in \{\pi, p\}} \sum_{j \in S} E\left(r_0^{\emptyset, j}(0, n^*) \middle| X^*(0) = 1\right). \quad (2.59)$$

The *net single premium*  $\pi$  payable at the time of concluding the insurance contract (when  $X(0) = 1$ ) can be determined directly from the formula (2.59) as

$$\pi = \pi_1(0) = \sum_{\emptyset \in \{b, \bar{b}, d, c\}} \sum_{j \in S^*} E\left(r_0^{\emptyset, j}(0, n^*) \middle| X^*(0) = 1\right). \quad (2.60)$$

Whereas, the *net period premium* with a fixed amount  $p$ , payable for the first  $m$  units of the insurance contract if  $X(t) = 1$  for  $t = 0, 1, \dots, m-1$  is obtained from (2.59) using (2.53):

$$p = \frac{\sum_{\emptyset \in \{b, \bar{b}, d, c\}} \sum_{j \in S^*} E\left(r_0^{\emptyset, j}(0, n^*) \middle| X^*(0) = 1\right)}{\sum_{t=0}^{m-1} v^t p_1(t)} = \frac{\pi}{\ddot{a}_{11}(0, m)}. \quad (2.61)$$

Since MMSM are typically complex, such as in the case of the BHLI model, with their probabilistic structure and the present values determined by the interest rate, it is convenient to represent them in matrix form. Consequently, expressing the net premiums as defined in equations (2.41) and (2.42), using a matrix formulation, appears natural. This approach significantly simplifies their computation and facilitates further analytical developments.

Based on (Dębicka, 2013), for  $(S^*, T^*)$ , the equivalence principle can be rewritten in the following form:

$$E(Z) = \mathbf{V}^T \text{Diag}(\mathbf{CD}^T) \mathbf{S} = 0, \quad (2.62)$$

where  $\mathbf{S} = (1, 1, \dots, 1)^T \in R^{N^*}$  and for any  $\mathbf{E} = (e_{ij})_{i,j=1}^{n^*+1}$  matrix  $\text{Diag}(\mathbf{E})$  is defined as

$$\text{Diag}(\mathbf{E}) = \begin{pmatrix} e_{11} & 0 & \dots & 0 \\ 0 & e_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & e_{n^*+1, n^*+1} \end{pmatrix}. \quad (2.63)$$

Cash flow  $cf_i(k)$  is the sum of inflows and outflows to the insurer's total loss fund (cf. (2.34)). Then  $\mathbf{C} = \mathbf{C}_{\text{in}} + \mathbf{C}_{\text{out}}$  where  $\mathbf{C}_{\text{in}}$  consists only of an income to this fund (benefits of type  $b, \bar{b}, d, c$ ) and  $\mathbf{C}_{\text{out}}$  consists only of an outgo from this fund (premiums of type  $\pi, p$ ).

Based on (2.62), equation (2.59) can be reformulated as follows:

$$\mathbf{V}^T \text{Diag}(-\mathbf{C}_{\text{out}} \mathbf{D}^T) \mathbf{S} = \mathbf{V}^T \text{Diag}(\mathbf{C}_{\text{in}} \mathbf{D}^T) \mathbf{S}. \quad (2.64)$$

Applying (2.64) in (2.53) along with (2.60) and (2.61), one obtains matrix formulas for the following actuarial values, respectively:

- *n-year temporary life annuity due* (annuity instalment equal to one unit)

$$\ddot{a}_{11}(0, n^*) = \mathbf{V}^T (\mathbf{I} - \mathbf{I}_{n^*+1}^* \mathbf{I}_{n^*+1}^T) \mathbf{D} \mathbf{J}_1, \quad (2.65)$$

- *net single premium  $\pi$*  (paid when  $X^*(0) = 1$ )

$$\pi = \pi_1(0) = \mathbf{V}^T \text{Diag}(\mathbf{C}_{\text{in}} \mathbf{D}^T) \mathbf{S}, \quad (2.66)$$

- *net period premium* with fixed amount  $p$ , payable for the first  $m$  units of the insurance contract if  $X^*(t) = 1$  for  $t = 0, 1, \dots, m-1$

$$p = \frac{\pi}{\ddot{a}_{11}(0, m)} = \frac{\mathbf{V}^T \text{Diag}(\mathbf{C}_{\text{in}} \mathbf{D}^T) \mathbf{S}}{\mathbf{V}^T (\mathbf{I} - \sum_{t=m+1}^{n^*+1} \mathbf{I}_t \mathbf{I}_t^T) \mathbf{D} \mathbf{J}_1}, \quad (2.67)$$

where

$$\mathbf{I}_t = \left( 0, \dots, 0, 1, 0, \dots, 0 \right)^T \in R^{n^*+1} \text{ for } t = 1, 2, \dots, n^* + 1, \quad (2.68)$$

$$\mathbf{J}_i = \left( 0, \dots, 0, 1, 0, \dots, 0 \right)^T \in R^{N^*} \text{ for } i = 1, 2, \dots, N^*, \quad (2.69)$$

and  $\mathbf{I}$  is the identity matrix of size  $(n^* + 1) \times (n^* + 1)$ .

In certain cases a periodic premium may be required if process  $\{X^*(t)\}$  occupies multiple states (not only state 1). Let  $S^p \subset S^*$  denote the set of such states, where  $X^*(t) = i$  and  $i \in S_p$  for  $t = 0, 1, \dots, m-1$  implies that a periodic premium  $p_{S^p}$  is paid. A general formula for determining such a premium was derived in (Dębicka & Zmyślona, 2018)

$$p_{S^p} = \frac{\mathbf{V}^T \text{Diag}(\mathbf{C}_{\text{in}} \mathbf{D}^T) \mathbf{S}}{\sum_{i \in S^p} \mathbf{V}^T (\sum_{t=0}^{m-1} \mathbf{I}_t \mathbf{I}_t^T) \mathbf{D} \mathbf{J}_i} = \frac{\mathbf{V}^T \text{Diag}(\mathbf{C}_{\text{in}} \mathbf{D}^T) \mathbf{S}}{\sum_{i \in S^p} \ddot{a}_{1(i)}(0, m-1)}, \quad (2.70)$$

where  $\ddot{a}_{1(i)}(t_1, t_2)$  denotes the actuarial value of the stream of unit benefits arising from the life annuity-due contract payable in period  $[t_1, t_2)$  if  $X^*(t) = i$  for  $t \in [t_1, t_2)$ . The actuarial value is calculated at the beginning of insurance period ( $t = 0$ ). It is tacitly assumed that  $X^*(t) = i$ .

In practical applications, parameter  $m$  in equation (2.70) should be selected to ensure that at least one premium can be generated in each state within subspace  $S^p$ . Therefore, determining the lower limit of the number of contributions, that is  $m$ , is related to determining the shortest path between state 1 and state  $i$ . For this purpose, the graph optimisation technique can be used because the multiple-state model can be represented as a directed graph, where the states correspond to nodes and direct transitions to edges. The length of a path is defined by the number of direct transitions it includes. The shortest path is the one with the minimum number of such transitions. Importantly, the shortest path – if it exists – must consist of distinct states, with no repetitions. This type of path can be effectively found using Dijkstra's algorithm (Dijkstra, 1959), a widely recognised method in graph-based modelling and optimisation. Its application within the context of MSMs was demonstrated in (Dębicka & Zmysłona, 2018).

The matrix representation of actuarial values, as provided in equations (2.65) to (2.67) and in (2.70), not only facilitates the computation of net premiums and life annuities but also highlights the structural decomposition of the insurer's total loss  $Z$ . Specifically, matrix  $\mathbf{D}$  depends exclusively on the probabilistic structure of process  $\{X^*(t)\}$ , while matrix  $\mathbf{V}$  reflects solely the impact of the interest rate. In turn, matrix  $\mathbf{C}$  captures the nature of the cash flows and thereby characterises the type of insurance contract under consideration. This decomposition allows for a clear separation of the model's key components: the evolution of the insured risk, the financial environment, and the contract structure.

In the context of the implementation of the BHLI contract, two streams of cash flows are created: a stream of premiums ( $p$  or  $\pi$  depending on the detailed insurance conditions) paid by the insured to the insurance company, and a stream of immediate annuity instalments (type  $b$  cash flow) paid by the insurer to the surviving spouse.

### Scenario B (Insurance premiums)

Within  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$ , the subspace of states  $S^p$  is identified as  $\{1_B, 1_C, 1_0\}$ . In this case, the shortest path from state  $1_B$  to  $1_C$  consists of a single direct transition  $(1_B, 1_C)$  and a similar structure applies to the transition from state  $1_B$  and  $1_0$ . Since each of these transitions corresponds to a one-step path, it follows that in order to ensure that at least one premium is generated in each of the relevant states, parameter  $m$  must satisfy condition  $2 \leq m \leq n$ . Based on (2.70), once this condition is fulfilled, the premium payable in multiple states can be determined using the following formula:

$$p_{S^p} = \frac{\mathbf{V}^T \text{Diag}(\mathbf{C}_{\text{in}(B)} \mathbf{D}_{(B)}^T) \mathbf{S}}{\ddot{a}_{1_B(1_B)}(0, m-1) + \ddot{a}_{1_B(1_C)}(0, m-1) + \ddot{a}_{1_B(1_0)}(0, m-1)}, \quad (2.71)$$

where

$$\mathbf{C}_{\text{in}(B)} = \sum_{h \in \{B, C, 0\}} \left( \mathbf{C}^{\Gamma(b, 2_h)} + \mathbf{C}^{\Gamma(b, 3_h)} \right) \in R^{(n+3) \times 22}, \quad (2.72)$$

and  $\mathbf{D}_{(B)}^T$  is described for process  $\{X_{HB}(t)\}$  and  $\{\mathbf{Q}_B(t)\}_{t=0}^{n^*-1}$  defined by (2.30).

For  $m = 1$ , single premium  $\pi = \pi_{1_B}(0)$  is calculated based on (2.66) with the cash flow matrix given by (2.72).

□

### Scenario C (Insurance premiums)

Within  $(S_{\text{BHLI}(C)}, T_{\text{BHLI}(C)})$ , the subspace of states  $S^P$  is specified as  $\{1_C, 1_0\}$ . In this case, the shortest path from state  $1_C$  to  $1_0$  consists of a single direct transition  $(1_C, 1_0)$  and similarly to Scenario B, parameter  $m$  must satisfy condition  $2 \leq m \leq n$  and

$$p_{SP} = \frac{\mathbf{V}^T \text{Diag}(\mathbf{C}_{\text{in}(C)} \mathbf{D}_{(C)}^T) \mathbf{S}}{\ddot{a}_{1_C(1_C)}(0, m-1) + \ddot{a}_{1_C(1_0)}(0, m-1)}, \quad (2.73)$$

where

$$\mathbf{C}_{\text{in}(C)} = \sum_{h \in \{C, 0\}} \left( \mathbf{C}^{\Gamma(b, 2_h)} + \mathbf{C}^{\Gamma(b, 3_h)} \right) \in R^{(n+3) \times 15}, \quad (2.74)$$

and  $\mathbf{D}_{(C)}^T$  is described for process  $\{X_{HC}(t)\}$  and  $\{\mathbf{Q}_C(t)\}_{t=0}^{n^*-1}$  defined by (2.29).

For  $m = 1$ , analogously to Scenario B,  $\pi = \pi_{1_C}(0)$  is calculated based on (2.66) with the cash flow matrix given by (2.74).

□

### Scenario 0 (Insurance premiums)

In the case of *Scenario 0*, the premium is paid only when  $X(t) = 1$  for  $1 \leq m \leq n$ . To calculate the period premium with fixed amount  $p$ , the straightforward formula (2.67) is applied:

$$p = \frac{\mathbf{V}^T \text{Diag}(\mathbf{C}_{\text{in}(0)} \mathbf{D}_{(0)}^T) \mathbf{S}}{\ddot{a}_{11}(0, m)}, \quad (2.75)$$

where the cash flow matrix is defined as follows:

$$\mathbf{C}_{\text{in}(0)} = \mathbf{C}^{\Gamma(b,2)} + \mathbf{C}^{\Gamma(b,3)} \in R^{(n+3) \times 8}, \quad (2.76)$$

and  $\mathbf{D}_{(0)}^T$  is described for process  $\{X_0^*(t)\}$  and  $\{\mathbf{Q}_0(t)\}_{t=0}^{n^*-1}$  defined by (2.21), whereas the single premium is derived by formula (2.66), with the cash flow matrix specified by (2.76). □

In Section 3.1 the formulas introduced in this chapter are applied to price BHLI contracts across the selected European Union countries and to examine the impact of the COVID-19 pandemic on the actuarial values associated with such contracts.

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# 3 Case Studies and Assessment of Practical Implementation Feasibility

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## 3.1 Premiums in EU Countries – Comparative Analysis

The aim of this section was to conduct a comparative analysis of net premiums calculated under the *Bridging Household Life Insurance* (BHLI) model across selected EU countries. The analysis was based on the actuarial framework developed in Chapter 2 and focused on how premium amounts vary depending on household composition, particularly the number of adult members, the presence and age of children, and the age structure of spouses.

Special attention was given to the impact of the COVID-19 pandemic on premium levels, as the health and economic disruptions it caused have significantly affected household dynamics and caregiving needs.

By examining premium levels across multiple household types and national contexts, this analysis aimed to highlight the adaptability of the proposed insurance contract across different markets. The results also serve as an empirical basis for evaluating whether the proposed mechanism offers adequate protection and fair treatment of diverse family structures.

To perform a relevant cross-country comparison of net insurance premiums, the analysis was structured into four interconnected stages.

The first stage involved identifying the *demographic structure of households*. This required defining a representative household type based on publicly available Eurostat data, which captures key life-course transition points (particularly the average age at which individuals leave the parental home and the age at which they typically have children) for both women and men in each country. In addition to determining the ages of adult household members (e.g. spouses or partners), this stage also takes into consideration the number of dependent children and their age profiles. The resulting household composition reflects the socio-demographic characteristics typical of each country. The analysis focused on single-family households classified primarily according to the age of adult partners and the number of dependent children.

The next stage focused on the *life table-based transition framework*, which determines the probabilistic structure of the multi-state model. This involved estimating the transition probabilities between states within a multi-state

model determined for each household type. These probabilities were derived from national life tables, which reflect country-specific mortality and survival patterns.

The third stage addresses the *financial assumptions* underlying the valuation of the insurance contract, which includes specifying the interest rate used in the present value calculations, as well as the benefit amounts paid under the contract. These assumptions were aligned in such a way as to ensure comparability across the analysed countries.

Finally, the analysis proceeded to the *premium valuation and cross-country comparison*. This involved an actuarial estimation of both single and periodic net premiums for each household type across the selected EU countries. The calculations were based on the demographic profiles and financial assumptions established in the previous stages, providing a consistent framework for evaluating the relative cost of insurance coverage across different national contexts. Importantly, the adopted assumptions also enabled an assessment of the impact of the COVID-19 pandemic on the level of net premiums.

The remainder of this section describes each step of the comparative analysis in detail.

### **3.1.1 Demographic Structure of Households**

In selecting household representatives for the comparative analysis, two main factors were taken into account. First, given that BHLI is primarily intended for families with caregiving responsibilities (particularly those with young children under the age of six, who require intensive care and nurturing), young families were chosen as the analytical focus. Second, as the analysis aims to encompass various types of household configurations, the selected family needed to have demographic potential to raise at least three children.

To identify a household type that meets these criteria while remaining representative of the socio-demographic conditions in a given country, the mean age of women at the birth of a fourth or higher-order child was used as the starting point. The corresponding age of the male partner and the ages of the children were determined in accordance with the procedures provided in Table 3.1.

The country-specific age profiles of model household members shown in Table 3.2 are based on demographic indicators taken from the 2023 Eurostat database (introduced in Table 1.2). Due to the use of life tables in the next stage of the analysis, the ages of family members were rounded to full years.

With regard to age, households were classified as either *young* (both partners aged 34 or younger) or *mature* (partners aged 35–59). For the purposes of the analysis, a *mixed-age household* was also distinguished (where one partner, typically the female, is under 35 and the other is – 35 or older). These categories reflect fertility potential and life-cycle stage and were aligned with national demographic patterns. Accordingly, countries in the sample were grouped based on the predominant age structure observed in representative

Table 3.1 Procedures for determining the age of members of the model household

| <i>Age of household members</i> | <i>Age assignment procedure</i>                                                                                                                |
|---------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| Woman (wife/partner)            | Mean age of women at birth of fourth and higher order child                                                                                    |
| Man (husband/partner)           | Age of the woman plus the difference between the average ages of leaving the parental household between the men and the women                  |
| Oldest child (the first one)    | The difference between the mean age of women at the birth of a fourth and higher order child and the mean age at the birth of the first child  |
| Middle child (the second one)   | The difference between the mean age of women at the birth of a fourth and higher order child and the mean age at the birth of the second child |
| Youngest child (the third one)  | The difference between the mean age of women at the birth of a fourth and higher order child and the mean age at the birth of the third child  |

*Source:* Authors' own elaboration.

family units. In this framework, the model households are categorised as follows (cf. Table 3.2):

- Young households (e.g. Bulgaria, Malta, Slovakia),
- Mixed-age households (e.g. Cyprus, Czechia, Germany, Greece, Croatia, Hungary, Italy, Poland, Romania, Slovenia),
- Mature households (e.g. Belgium, Bulgaria, Denmark, Estonia, Finland, France, Ireland, Lithuania, Luxembourg, Latvia, Netherlands, Portugal, Spain, and Sweden).

In terms of family composition, the analysis distinguished between couples without children and couples with dependent children, further subdivided by the number of children (one, two, or three). The ages of the children were detailed in Table 3.2. Therefore, the analysis encompassed:

- *Childless Household*,
- *One-Child Household* (with a child at the age specified in the 'Age of the first child' column),
- *Two-Child Household* (with children at the ages specified in the columns 'Age of the first child' and 'Age of the second child'),
- *Three-Child Household*.

*Childless households* are included in the analysis concerning the young age of the partners. The presence of an adult child is uncommon, except in cases of adoption or fostering. Within this category, two specific subtypes stand out as particularly important for analysis. The first is the *Dual-Earner Childless*

Table 3.2 Age of members of the model household in EU countries

| <i>Country</i> | <i>Code</i> | <i>Women's age</i> | <i>Men's age</i> | <i>Age of the first child</i> | <i>Age of the second child</i> | <i>Age of the third child</i> |
|----------------|-------------|--------------------|------------------|-------------------------------|--------------------------------|-------------------------------|
| European Union | EU27        | 34                 | 36               | 4                             | 2                              | 1                             |
| Austria        | AT          | 35                 | 37               | 5                             | 3                              | 2                             |
| Belgium        | BE          | 35                 | 36               | 5                             | 3                              | 2                             |
| Bulgaria       | BG          | 30                 | 34               | 3                             | 1                              | 1                             |
| Croatia        | HR          | 33                 | 36               | 4                             | 2                              | 1                             |
| Cyprus         | CY          | 34                 | 35               | 4                             | 1                              | 0                             |
| Czechia        | CZ          | 34                 | 37               | 5                             | 2                              | 0                             |
| Denmark        | DK          | 36                 | 37               | 6                             | 4                              | 2                             |
| Estonia        | EE          | 36                 | 38               | 7                             | 4                              | 2                             |
| Finland        | FI          | 35                 | 36               | 5                             | 3                              | 2                             |
| France         | FR          | 35                 | 36               | 6                             | 3                              | 2                             |
| Germany        | DE          | 34                 | 36               | 5                             | 2                              | 1                             |
| Greece         | EL          | 33                 | 37               | 2                             | 0                              | 0                             |
| Hungary        | HU          | 32                 | 35               | 3                             | 1                              | 0                             |
| Ireland        | IE          | 36                 | 37               | 4                             | 2                              | 1                             |
| Italy          | IT          | 34                 | 36               | 2                             | 0                              | 0                             |
| Latvia         | LV          | 35                 | 36               | 7                             | 4                              | 2                             |
| Lithuania      | LT          | 35                 | 37               | 7                             | 4                              | 2                             |
| Luxembourg     | LU          | 35                 | 36               | 4                             | 2                              | 1                             |
| Malta          | MT          | 34                 | 33               | 4                             | 1                              | 0                             |
| Netherlands    | NL          | 35                 | 36               | 5                             | 3                              | 2                             |
| Poland         | PL          | 34                 | 36               | 6                             | 3                              | 2                             |
| Portugal       | PT          | 35                 | 36               | 5                             | 2                              | 1                             |
| Romania        | RO          | 31                 | 36               | 4                             | 2                              | 2                             |
| Slovakia       | SK          | 31                 | 34               | 3                             | 1                              | 0                             |
| Slovenia       | SI          | 34                 | 37               | 5                             | 3                              | 2                             |
| Spain          | ES          | 35                 | 37               | 4                             | 2                              | 1                             |
| Sweden         | SE          | 36                 | 36               | 6                             | 4                              | 2                             |

*Source:* Authors' own elaboration based on (Eurostat Data Browser, 2023).

*Couple*, typically found in countries with low fertility rates and high levels of female labour force participation. The second is the *Young High-Fertility Household*, which refers to couples who, based on demographic trends for women aged 25–34, are statistically likely to have three or more children in the future.

To compare the impact of life expectancy differences across countries on the insurance premium level, a mixed-age household was selected for analysis, consisting of a 34-year-old woman and a 36-year-old man. Premiums were calculated for both childless couples and couples with dependent children. In this case, the household was assumed to include children aged four, two, and one. The selection of this household type was based on demographic data from countries that have been members of the European Union since 2020,

reflecting representative age and fertility structures within the EU context (cf. Table 3.2). In summary, the analyses included the following household types:

- Childless Household,
- One-Child Household (with a four-year-old child),
- Two-Child Household (with children aged four and two years),
- Three-Child Household (with children aged four, two, and one years).

Among households of this demographic profile, a particularly relevant example is the *Young Two-Child Family Household*, typically composed of partners in their early thirties with two young children aged approximately two and four. This household structure reflects a stable phase of family development and corresponds to the average fertility patterns observed across many European countries. It thus serves as a benchmark for assessing the affordability and adequacy of the proposed insurance contract under typical early-family conditions within the EU.

### 3.1.2 Life Table – Based Transition Framework

The numerical examples presented in this section were based on total (unisex) national life tables, which are independent of the gender of the insured individuals. This approach is required by European Union legislation which prohibits the differentiation of insurance premiums and benefits based on gender. In particular, the judgement of the Court of Justice of the European Union in the *Test-Achats case* (C-236/09) invalidated Article 5(2) of Directive 2004/113/EC, establishing that all elements used in premium calculations (including the selection of life tables) must be gender-neutral (Court of Justice of the European Union, 2011). This prohibition also extends to indirect forms of discrimination, such as incorporating assessment of children's lifetimes as family members of the insured into the probabilistic structure of the model. If the age and corresponding mortality probabilities of a child influence the premium amount, applying gender-differentiated mortality rates would violate the principle of equal treatment as outlined in the Charter of Fundamental Rights of the European Union (European Parliament, Council of the European Union, 2012). Thus, in all cases where premium levels depend on demographic characteristics of individuals, regardless of whether they are the insured parties, total (unisex) life tables must be used (Council of the European Union, 2004).

To introduce the probabilistic structure of the model, life tables for the year 2023 were selected, as they represent the most recent data published by Eurostat for EU member states. To analyse the impact of the COVID-19 pandemic on net premiums, life tables for the years 2019 and 2021 were also incorporated. Table 3.3 shows the life expectancy at birth for individuals born in each of the reference years, as reported by Eurostat.

In Table 3.3, years in which the Eurostat life tables reported zero mortality probabilities for specific age groups are marked in grey. Due to these data anomalies, the affected countries were excluded from the net premium calculations. In the comparative analysis of BHLI contract valuations based on the 2023 life tables, the following countries were excluded: Cyprus, Latvia, Lithuania, Luxembourg, Malta, and Slovenia. In addition, for the specific analysis of the impact of the COVID-19 pandemic on premium amounts, Estonia and Ireland were also excluded.

It is assumed that the premiums for all analysed insurance contracts were calculated for a 20-year *insurance period* ( $n = 20$ ). In the comparative analysis of national model households, the chosen time horizon was determined by the age of the youngest child in three-child households. The goal was to

Table 3.3 Life expectancy at a given exact age (total) in EU countries (2023).

| Country     | Code | 2019 | 2021 | 2023 |
|-------------|------|------|------|------|
| Austria     | AT   | 82.0 | 81.3 | 81.9 |
| Belgium     | BE   | 82.1 | 81.9 | 82.5 |
| Bulgaria    | BG   | 75.1 | 71.2 | 75.8 |
| Croatia     | HR   | 78.6 | 76.6 | 78.6 |
| Cyprus      | CY   | 82.3 | 81.3 | 82.9 |
| Czechia     | CZ   | 79.3 | 77.2 | 79.9 |
| Denmark     | DK   | 81.5 | 81.5 | 81.8 |
| Estonia     | EE   | 79.0 | 77.2 | 79.1 |
| Finland     | FI   | 82.1 | 81.9 | 81.6 |
| France      | FR   | 83.0 | 82.4 | 83.0 |
| Germany     | DE   | 81.3 | 80.8 | 81.1 |
| Greece      | EL   | 81.7 | 80.2 | 81.8 |
| Hungary     | HU   | 76.5 | 74.1 | 76.7 |
| Ireland     | IE   | 82.8 | 82.3 | 82.9 |
| Italy       | IT   | 83.6 | 82.7 | 83.5 |
| Latvia      | LV   | 75.7 | 73.1 | 75.6 |
| Lithuania   | LT   | 76.5 | 74.2 | 77.6 |
| Luxembourg  | LU   | 82.7 | 82.7 | 83.4 |
| Malta       | MT   | 82.7 | 82.3 | 83.4 |
| Netherlands | NL   | 82.2 | 81.4 | 81.9 |
| Poland      | PL   | 78.0 | 75.4 | 78.4 |
| Portugal    | PT   | 81.9 | 81.5 | 82.5 |
| Romania     | RO   | 75.6 | 72.8 | 76.4 |
| Slovakia    | SK   | 77.8 | 74.6 | 78.2 |
| Slovenia    | SI   | 81.6 | 80.7 | 82.0 |
| Spain       | ES   | 84.0 | 83.3 | 84.0 |
| Sweden      | SE   | 83.2 | 83.1 | 83.4 |

Source: Authors' own elaboration based on (Eurostat Data Browser, 2023).

ensure that in all countries the insurance period lasts at least until the youngest child reaches legal adulthood.

The 20-year insurance term was also used in comparative analyses of households that reflect typical demographic structure within the European Union. In the households with dependent children, the eldest child is four years old. If one parent were to pass away in the final year of the contract, the child would be 24 years old (e.g. possibly completing their higher education at that time). In accordance with the terms of the BHLI contract, benefits would continue for an additional two years, until the eldest child reaches the age of 26. This aligns with the average age of leaving the parental home, that is 27 years for men and 25 years for women (see Table 1.2).

The choice of the insurance period directly influences the length of the transition matrix sequence  $\{\mathbf{Q}(t)\}_{t=0}^{21}$  needed to determine the probabilistic structure of the multi-state model. In the case of a childless household (*Scenario 0*), the elements of matrix  $\mathbf{Q}_0(t)$  are determined directly from the death probabilities for insured spouses (partners) listed in the life table, cf. (2.21). When a household with small children (*Scenario B*) is analysed, it is necessary first to determine matrix  $\mathbf{H}_B(t)$  which represents the changes in household composition during the insurance period, as described in formula (2.25). Once matrix  $\mathbf{H}_B(t)$  is established, the transition matrix  $\mathbf{Q}_B(t)$  can be derived using formula (2.28). Matrix  $\mathbf{H}_B(t)$  can also be determined based on the life tables. Tables 3.4–3.6 present elements  $q^B(t)$  and  $q^C(t)$  (according to

Table 3.4 Evaluation of child mortality probabilities used to derive elements of transition matrix  $\mathbf{H}_B(t)$  for a 20-year BHLI contract – one-child household (aged four)

| $t$ | $a_1(t)$ | $B(t)$ | $C(t)$ | $q^B(t)$  | $q^C(t)$   | Household type |
|-----|----------|--------|--------|-----------|------------|----------------|
| 0   | 4        | 1      | 0      | $q_4$     | 1          | $H_B(t)=B$     |
| 1   | 5        | 1      | 0      | $q_{4+1}$ | 1          | $B(t) > 0$     |
| 2   | 6        | 1      | 0      | $q_{4+2}$ | 1          | $C(t) \geq 0$  |
| 3   | 7        | 0      | 1      | 1         | $q_{4+3}$  | $H_B(t)=C$     |
| ... | ...      | ...    | ...    | ...       | ...        | $B(t) = 0$     |
| 13  | 17       | 0      | 1      | 1         | $q_{4+13}$ | $C(t) > 0$     |
| 14  | 18       | 0      | 0      | 1         | 1          | $H_B(t)=0$     |
| ... | ...      | ...    | ...    | ...       | ...        | $B(t) = 0$     |
| 22  | 26       | 0      | 0      | 1         | 1          | $C(t) = 0$     |

Source: Authors' own elaboration.

formulas (2.22) and (2.23), respectively) are calculated based on the child's one-year probability of death for 20-year insurance contracts specified for different types of households with children aged four, two, and one.

Note that the cells in Tables 3.4–3.6 were colour-coded to correspond to the states' colours used in the graphical presentation of model  $(S_{\text{BHLI}(B)}, T_{\text{BHLI}(B)})$  shown in Figure 2.3. Each colour represents the household type in a given year of the insurance contract: red for households with a small child ( $B$ ), green for those with a young child aged 7–17 ( $C$ ), and blue for households without dependent young children (0). It is evident that changes in household type are closely correlated with the age progression of the youngest child.

The elements of matrices  $\{\mathbf{H}_C(t)\}_{t=0}^{21}$  were determined in a similar way for households with young children, the youngest being not less than seven years old.

### 3.1.3 Financial Assumptions

The financial assumptions for the BHLI contract concern both the amounts of interest rate and the benefits. They were designed to ensure realism and cross-country comparability of premiums for long-term insurance policies.

*Table 3.5* Evaluation of child mortality probabilities used to derive elements of transition matrix  $\mathbf{H}_B(t)$  for a 20-year BHLI contract – two-child household (aged four and two)

| $t$ | $a_1(t)$ | $a_2(t)$ | $B(t)$ | $C(t)$ | $q^B(t)$           | $q^C(t)$             | Household type                              |
|-----|----------|----------|--------|--------|--------------------|----------------------|---------------------------------------------|
| 0   | 2        | 4        | 2      | 0      | $q_4, q_2$         | 1                    | $H_B(t) = B$<br>$B(t) > 0$<br>$C(t) \geq 0$ |
| 1   | 3        | 5        | 2      | 0      | $q_{4+1}, q_{2+1}$ | 1                    |                                             |
| 2   | 4        | 6        | 2      | 0      | $q_{4+2}, q_{2+2}$ | 1                    |                                             |
| 3   | 5        | 7        | 1      | 1      | $q_{2+3}$          | $q_{4+3}$            |                                             |
| 4   | 6        | 8        | 1      | 1      | $q_{2+4}$          | $q_{4+4}$            |                                             |
| 5   | 7        | 9        | 0      | 2      | 1                  | $q_{4+5}, q_{2+5}$   | $H_B(t) = C$<br>$B(t) = 0$<br>$C(t) > 0$    |
| ... | ...      | ...      | ...    | ...    | ...                | ...                  |                                             |
| 13  | 15       | 17       | 0      | 2      | 1                  | $q_{4+13}, q_{2+13}$ |                                             |
| 14  | 16       | 18       | 0      | 1      | 1                  | $q_{2+14}$           |                                             |
| 15  | 17       | 19       | 0      | 1      | 1                  | $q_{2+15}$           |                                             |
| 16  | 18       | 20       | 0      | 0      | 1                  | 1                    | $H_B(t) = 0$<br>$B(t) = 0$<br>$C(t) = 0$    |
| ... | ...      | ...      | ...    | ...    | ...                | ...                  |                                             |
| 22  | 24       | 26       | 0      | 0      | 1                  | 1                    |                                             |

Source: Authors' own elaboration.

Table 3.6 Evaluation of child mortality probabilities used to derive elements of transition matrix  $\mathbf{H}_B(t)$  for a 20-year BHLI contract – three-child household (aged four, two and one)

| $t$ | $a_1(t)$ | $a_2(t)$ | $a_3(t)$ | $B(t)$ | $C(t)$ | $q^B(t)$                    | $q^C(t)$                       | Household type                              |
|-----|----------|----------|----------|--------|--------|-----------------------------|--------------------------------|---------------------------------------------|
| 0   | 1        | 2        | 4        | 3      | 0      | $q_4, q_2, q_1$             | 1                              | $H_B(t) = B$<br>$B(t) > 0$<br>$C(t) \geq 0$ |
| 1   | 2        | 3        | 5        | 3      | 0      | $q_{4+1}, q_{2+1}, q_{1+1}$ | 1                              |                                             |
| 2   | 3        | 4        | 6        | 3      | 0      | $q_{4+2}, q_{2+2}, q_{1+2}$ | 1                              |                                             |
| 3   | 4        | 5        | 7        | 2      | 1      | $q_{2+3}, q_{1+3}$          | $q_{4+3}$                      |                                             |
| 4   | 5        | 6        | 8        | 2      | 1      | $q_{2+4}, q_{1+4}$          | $q_{4+4}$                      |                                             |
| 5   | 6        | 7        | 9        | 1      | 2      | $q_{1+5}$                   | $q_{4+5}, q_{2+5}$             |                                             |
| 6   | 7        | 8        | 10       | 0      | 3      | 1                           | $q_{4+6}, q_{2+6}, q_{1+6}$    | $H_B(t) = C$<br>$B(t) = 0$<br>$C(t) > 0$    |
| ... | ...      | ...      | ...      | ...    | ...    | ...                         | ...                            |                                             |
| 13  | 14       | 15       | 17       | 0      | 3      | 1                           | $q_{4+13}, q_{2+13}, q_{1+13}$ |                                             |
| 14  | 15       | 16       | 18       | 0      | 2      | 1                           | $q_{2+14}, q_{1+14}$           |                                             |
| 15  | 16       | 17       | 19       | 0      | 2      | 1                           | $q_{2+15}, q_{1+15}$           |                                             |
| 16  | 17       | 18       | 20       | 0      | 1      | 1                           | $q_{1+16}$                     |                                             |
| 17  | 18       | 19       | 21       | 0      | 0      | 1                           | 1                              | $H_B(t) = 0$<br>$B(t) = 0$<br>$C(t) = 0$    |
| ... | ...      | ...      | ...      | ...    | ...    | ...                         | ...                            |                                             |
| 22  | 23       | 24       | 26       | 0      | 0      | 1                           | 1                              |                                             |

Source: Authors' own elaboration.

A benchmark interest rate of 1% per year was assumed in numerical examples. On the one hand, this reflects the persistent low interest rate environment observed across global financial markets over the past decades (Del Negro et al., 2019; Holsboer, 2000; Reyna et al., 2022), whilst on the other, life insurance is inherently interest-rate sensitive, as both the valuation of assets and liabilities are correlated with changes in market rates (Eling & Holder, 2013a, 2013b). In a low interest rate environment, policyholders are less likely to lapse; this extends the duration of insurers' liabilities, which may outpace the duration of their assets, thereby worsening negative duration gaps (Antolin et al., 2011; Berends et al., 2013). If the difference between the duration of liabilities and assets is negative, it works to the insurer's disadvantage, for example when an insurance company offers 20-year life insurance and invests the funds in ten-year bonds. After ten years it must reinvest, but during this time interest rates may fall. New investments will yield lower returns, and obligations to customers will remain unchanged. In the European context, yields on ten-year government bonds have repeatedly fallen below regulatory technical interest rates (Kablau & Wedow, 2012), prompting concern over the long-term solvency of life insurers (Berdin & Gründl, 2015). This situation makes insurers more sensitive to changes in interest rates, particularly when their contracts provide long-term benefits. For this reason, many European regulators have restricted technical interest rates used in life insurance pricing; for example Germany's maximum actuarial interest rate was set at 1% for new contracts in 2025. A comprehensive review of how life insurers have adjusted to the challenges of continuously low interest rates can be found in (Suwanmalai & Zaby, 2022).

Hence, the assumption of a 1% interest rate is consistent with regulatory guidance and a cautious actuarial approach, ensuring a stable basis for long-term premium valuation. Therefore the discount vector given by (2.31) contains discount factors of the form  $v_t = (1 + 0.01)^{-t} = 0.99099^t$ .

In addition, to facilitate international comparison of premium levels across EU countries, benefits were expressed in unitary terms, assuming a standardised benefit amount of 1 monetary unit per period.

The benefit amounts were determined based on the time each spouse or partner dedicates to household responsibilities. Since both young and mature households were included in the analysis, Table 1.3 specifically examines the time that men and women devote to family-related tasks within household types II, III, and IV.

In the mentioned household types, men in households without dependent children (type IV) devote the least amount of time to domestic activities, and consequently this time was used as the reference value for a unit of benefit. Therefore, in *Scenario 0* of the insurance model of BHLI, the benefit allocated to the wife following the husband's death was set at  $b_3^0(t) = 1$ . In this household type, the wife spends approximately 79% more time on household tasks than the husband ( $103/184 = 1.79$ ); hence, the corresponding benefit for the husband in the event of the wife's death was assumed to be  $b_2^0(t) = 1.79$ .

In households with children aged seven or older (type III), men spend approximately 26% more time on domestic tasks compared to households without dependent children ( $130/103 = 1.26$ ). Accordingly, in *Scenario C* of the insurance model, the benefit was set at  $b_3^C(t) = b_3^0(t) \cdot 1.26 = 1.26$  for the wife following the husband's death. For women in this household type, the increase in time devoted to housework is equal to 58% (i.e.  $291/184 = 1.58$ ) relative to type IV households. Therefore, based on the benefit amount established in *Scenario 0*, the benefit for the husband in the event of the wife's death was set at  $b_2^C(t) = b_2^0(t) \cdot 1.58 = 2.83$ . Note that in a type III household, the woman spends 124% more time on household chores than the man (because  $291/130 = 2.24$ ), and this relationship is maintained in the amount of benefits because  $b_2^C(t) : b_3^C(t) = 2.24$ .

Benefits for spouses with dependent children (up to six years old) were determined in a similar way by comparing the working hours for a type II and type III household, thus  $b_3^B(t) = b_3^C(t) \cdot (179/130) = 1.74$  and  $b_2^B(t) = b_2^C(t) \cdot (399/179) = 3.87$ . Similarly to type III households, in this case (corresponding to *Scenario B*), the ratio of benefits between spouses  $b_2^B(t) : b_3^B(t) = 2.23$  corresponds to the ratio of work they devote to the household, that is  $399/179 = 2.23$ .

In accordance with the contract assumptions set out in Section 1.4, it was assumed that the reduction coefficients are:  $\alpha_1 = 1, \alpha_2 = 0.75, \alpha_3 = 0.5$ . Regarding the reduction coefficients, the benefit structure for a BHLI contract in accordance with *Scenario B* is illustrated in Figure 3.1. The graph also includes other scenarios, specifically *Scenario C* (with bars for  $H(t) = C$  and  $H(t) = 0$ ) and *Scenario 0* (with bars for  $H(t) = 0$ ).

Note that in the first year following the death of one of the spouses, the benefits under the BHLI contract range from 1 to 1.79 in *Scenario 0*, from 1

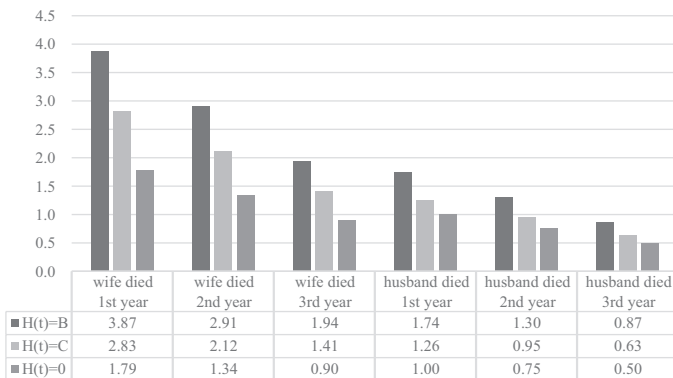


Figure 3.1 Benefit structure for the BHLI contracts.

Source: Authors' own elaboration.

to 2.83 in *Scenario C*, and from 1 to 3.87 in *Scenario B*. In subsequent years, regardless of the scenario, the benefit amount is reduced by 25% in the second year and by 50% in the third year.

The specified annuity instalment values and reduction coefficients enable the determination of the cash flow matrix given by formulas (2.72), (2.74) and (2.76) for *Scenario B*, *Scenario C*, and *Scenario 0* of BHLI contracts, respectively.

Assumptions about the interest rate and the unitary base benefit enabled the analysis to concentrate on the demographic aspects of premium differences, rather than on monetary or currency-specific variations.

### 3.1.4 *Premium Valuation and Cross-Country Comparison*

In the following analysis, attention was focused on annual premiums paid throughout the entire term of the BHLI contract, that is at time points  $t = 0, 1, \dots, 19$ . The annual premiums were calculated based on matrix formula (2.68), whereas the corresponding single premiums were derived using matrix formula (2.67).

In Table 3.7 the highest premiums for each household type are highlighted in dark grey, while the lowest in light grey. In all the household types, annual premiums would be lowest in Sweden for the 20-year BHLI policy. Among childless, one-child, and two-child households, those in Romania would attract the highest premiums, while for childless and three-child households, Estonia would carry the highest premiums. In comparison to Sweden, premium levels in these countries are significantly higher, namely by 164% for childless and one-child households and by 167% for households with two or three children.

Notably, the premium gap between Romanian and Swedish households resulted from two factors: age structure and national life tables. Although Swedish households comprise older women (by approximately five years) and older children (the oldest and middle are two years older), their premiums remain lower than those for Romanian families, which clearly indicates significantly higher mortality in Romania compared to Sweden (cf. Table 3.3).

In contrast, the age profiles of model households in Sweden and Estonia are very similar. Higher premiums for households with two or more children in Estonia only reflect significant differences in mortality patterns between the two countries.

It is also worth highlighting the relative premium increases across household types: on average, annual premiums for a one-child household are 22% higher than for a childless household, premiums for two-child households increase by 8% over one-child households, whilst premiums for three-child households are 3% more than for two-child households. The variations in net premium levels among different household types are directly influenced by differences in the age structure of children. However, the decreasing rate of

Table 3.7 Annual net premiums for national model households (based on National Life Tables 2023)

| Country code | Age at entry of |     |              |              |                | Type of household |                        |                               |             |
|--------------|-----------------|-----|--------------|--------------|----------------|-------------------|------------------------|-------------------------------|-------------|
|              | Woman           | Man | Oldest child | Middle child | Youngest child | Childless         | One-child (the oldest) | Two-child (oldest and middle) | Three-child |
| AT           | 35              | 37  | 5            | 3            | 2              | 0.01326           | 0.01588                | 0.01695                       | 0.01756     |
| BE           | 35              | 37  | 5            | 3            | 2              | 0.01435           | 0.01732                | 0.01849                       | 0.01916     |
| BG           | 30              | 34  | 3            | 1            | 1              | 0.02346           | 0.03004                | 0.03253                       | 0.03253     |
| CZ           | 34              | 37  | 5            | 2            | 0              | 0.01747           | 0.02090                | 0.02307                       | 0.02497     |
| DE           | 34              | 36  | 5            | 2            | 1              | 0.01414           | 0.01687                | 0.01864                       | 0.01937     |
| DK           | 36              | 37  | 6            | 4            | 2              | 0.01320           | 0.01523                | 0.01613                       | 0.01725     |
| EE           | 36              | 38  | 7            | 4            | 2              | 0.02743           | 0.03158                | 0.03426                       | 0.03675     |
| EL           | 33              | 37  | 2            | 0            | 0              | 0.01405           | 0.01852                | 0.02011                       | 0.02012     |
| ES           | 35              | 37  | 4            | 2            | 1              | 0.01152           | 0.01402                | 0.01501                       | 0.01560     |
| FI           | 35              | 37  | 5            | 3            | 2              | 0.01490           | 0.01789                | 0.01904                       | 0.01979     |
| FR           | 35              | 37  | 6            | 3            | 2              | 0.01616           | 0.01887                | 0.02068                       | 0.02145     |
| HR           | 33              | 37  | 4            | 2            | 1              | 0.01937           | 0.02376                | 0.02547                       | 0.02651     |
| HU           | 32              | 35  | 3            | 1            | 0              | 0.01979           | 0.02496                | 0.02695                       | 0.02811     |
| IE           | 36              | 37  | 4            | 2            | 1              | 0.01280           | 0.01561                | 0.01675                       | 0.01743     |
| IT           | 34              | 36  | 2            | 0            | 0              | 0.01051           | 0.01387                | 0.01501                       | 0.01501     |
| NL           | 35              | 37  | 5            | 3            | 2              | 0.01247           | 0.01485                | 0.01584                       | 0.01643     |
| PL           | 34              | 37  | 6            | 3            | 2              | 0.02457           | 0.02885                | 0.03168                       | 0.03287     |
| PT           | 35              | 37  | 5            | 2            | 1              | 0.01605           | 0.01902                | 0.02105                       | 0.02187     |
| RO           | 31              | 36  | 4            | 2            | 2              | 0.02587           | 0.03194                | 0.03432                       | 0.03433     |
| SE           | 36              | 37  | 6            | 4            | 2              | 0.01040           | 0.01208                | 0.01283                       | 0.01375     |
| SK           | 31              | 34  | 3            | 1            | 0              | 0.01762           | 0.02256                | 0.02436                       | 0.02537     |

Source: Authors' own elaboration.

premium increase as the number of children in the family increases indicates that the impact of having additional children on risk exposure is diminishing since certain household work must be done, regardless of whether a family has two or three young children.

In Figure 3.2, the premiums from Table 3.7 are ordered by the age of the woman in the household – from youngest to oldest.

In all the countries, there is a clear distinction between premiums for households without children and those with dependent children, with the latter consistently facing higher premium levels. However, no consistent relationship was observed between the woman's age and the amount of the premium,

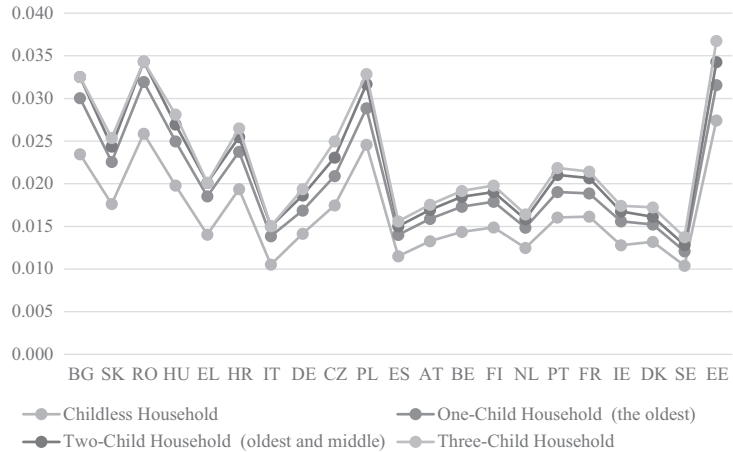


Figure 3.2 Annual net premiums for national model households ordered by woman's age (based on National Life Tables 2023).

Source: Authors' own elaboration.

which suggests that differences in premium levels are primarily driven by variations in national mortality patterns rather than by age structure alone.

Next, a comparative analysis was conducted using identical household structures across different countries to isolate and evaluate the impact of country-specific life tables on premium valuations. The analysis focused on a mixed-age household, which includes a 34-year-old woman and a 36-year-old man. Premiums were calculated for both childless couples and those with dependent children, with a maximum of three children aged four, two, and one. This configuration reflects typical demographic patterns observed in EU households, ensuring comparability across different countries.

The increases in premiums among household types show a pattern of reduced growth. While the transition from a childless household to a one-child household resulted in a substantial average premium increase of 23.3%, the subsequent increases were notably smaller – 7.2% from one to two children, and only 3.9% from two to three children.

In Table 3.8, the highest premiums for each household type are highlighted in dark grey, while the lowest are in light grey. Again, across all household types, annual premiums were lowest in Sweden for the 20-year BHLI contract, whereas the highest premiums in 2023 were observed in Bulgaria, which corresponds to its ranking as the country with the shortest life expectancy among the countries under consideration. The differences between the lowest and highest premiums across these countries are huge: 219% for childless

Table 3.8 Annual net premiums for typical EU households (based on National Life Tables 2023)

| Country code       | Type of household |                  |                     |                          |
|--------------------|-------------------|------------------|---------------------|--------------------------|
|                    | Childless         | One-child<br>(4) | Two-child<br>(4, 2) | Three-child<br>(4, 2, 1) |
| AT                 | 0.01218           | 0.01506          | 0.01616             | 0.01678                  |
| BE                 | 0.01321           | 0.01646          | 0.01766             | 0.01833                  |
| BG                 | 0.02958           | 0.03647          | 0.03910             | 0.04061                  |
| CZ                 | 0.01644           | 0.02032          | 0.02174             | 0.02257                  |
| DE                 | 0.01414           | 0.01739          | 0.01864             | 0.01937                  |
| DK                 | 0.01164           | 0.01431          | 0.01535             | 0.01595                  |
| EE                 | 0.02353           | 0.02945          | 0.03151             | 0.03275                  |
| EL                 | 0.01351           | 0.01658          | 0.01773             | 0.01842                  |
| ES                 | 0.01041           | 0.01270          | 0.01360             | 0.01414                  |
| FI                 | 0.01367           | 0.01694          | 0.01814             | 0.01888                  |
| FR                 | 0.01488           | 0.01841          | 0.01974             | 0.02052                  |
| HR                 | 0.01856           | 0.02267          | 0.02424             | 0.02522                  |
| HU                 | 0.02281           | 0.02767          | 0.02963             | 0.03080                  |
| IE                 | 0.01136           | 0.01389          | 0.01488             | 0.01548                  |
| IT                 | 0.01051           | 0.01293          | 0.01387             | 0.01442                  |
| NL                 | 0.01141           | 0.01400          | 0.01501             | 0.01561                  |
| PL                 | 0.02314           | 0.02883          | 0.03094             | 0.03215                  |
| PT                 | 0.01466           | 0.01796          | 0.01928             | 0.02005                  |
| RO                 | 0.02796           | 0.03431          | 0.03680             | 0.03826                  |
| SE                 | 0.00927           | 0.01149          | 0.01233             | 0.01281                  |
| SK                 | 0.02148           | 0.02656          | 0.02848             | 0.02961                  |
| Mean               | 0.0164            | 0.0202           | 0.0217              | 0.0225                   |
| Standard deviation | 0.0059            | 0.0073           | 0.0078              | 0.0081                   |

Source: Authors' own elaboration.

households and 217% for households with dependent children. The last two rows of Table 3.8 show the mean values and standard deviations of annual premiums. Based on these values, a one-sigma range (referred to as the typical range of premiums) was defined for each household type (cf. Figure 3.3). Note that premiums in Hungary, Poland, Estonia, Romania, and Bulgaria fall into the atypically high category, while those in Sweden, Spain, and Italy are considered atypically low.

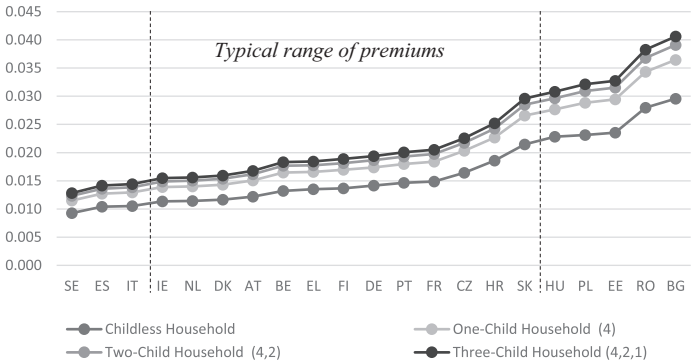


Figure 3.3 Annual net premiums for typical EU households ordered by premiums (based on National Life Tables 2023).

Source: Authors' own elaboration.

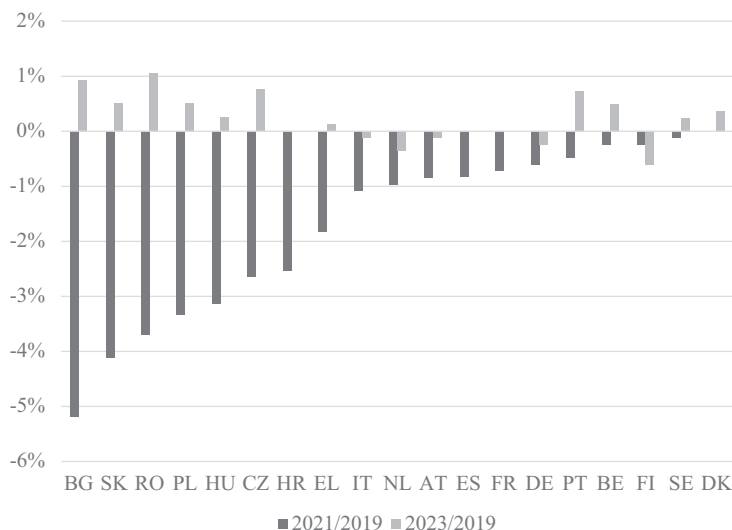
These findings are closely linked to life expectancy across different countries. In nations with higher life expectancy such as Sweden, Spain, and Italy, the cost of long-term insurance is generally lower. In contrast, in countries with shorter life expectancy, insurance providers require higher premiums to account for the increased risk of mortality.

The COVID-19 pandemic had a deep impact on life expectancy trends, and, consequently, on the insurers' risk assessments and pricing strategies. Sharp and uneven declines in life expectancy across countries during the pandemic forced insurers to adjust premiums to reflect the heightened and regionally variable mortality risks.

Figure 3.4 presents the percentage relative differences in life expectancy between the years 2021 and 2023, compared to 2019. Countries are shown in descending order according to the magnitude of the decline in life expectancy observed in 2021 relative to 2019.

In 2021 a significant decline in life expectancy was observed, particularly in Central and Eastern European countries such as Bulgaria, Slovakia, Romania, and Poland, with decreases reaching up to 5% compared to 2019. This clearly indicates the impact of the COVID-19 pandemic, which had its greatest effect in that year (especially the second and third waves), leading to excessive mortality.

A clear regional pattern emerges, namely the pandemic's effects on life expectancy were more severe in Central and Eastern Europe than in Western and Northern Europe. In countries with a significant decline in life expectancy (Bulgaria, Slovakia, Romania, and Poland), the problem of excessive mortality resulted from a combination of factors: poor quality or overloaded



*Figure 3.4* Relative differences in life expectancy in 2021 and 2023 compared to 2019 in EU countries.

*Source:* Authors' own elaboration based on (Eurostat Data Browser, 2023).

healthcare systems, delayed government action, and low public trust in institutions, whilst in Bulgaria and Romania a further factor was their low vaccination rates. Despite the introduction of strict lockdowns in Poland and Bulgaria, this did not translate into a reduction in mortality. Western and Northern European countries – including Denmark, Sweden, Finland, and Belgium – experienced either minor declines (less than 0.2%) or maintained relatively stable life expectancy levels. Each of these countries took a different approach to managing the pandemic (Denmark – short, cautious restrictions, rapid vaccinations; Sweden – no lockdowns; Finland – effective prevention and public trust; Belgium – very high acceptance of vaccinations).

By 2023, most countries showed signs of recovery, with life expectancy increasing compared to 2021 and, in many cases, approaching pre-pandemic levels. This suggests a partial normalisation and resilience of health systems and population outcomes post-crisis.

The increase in mortality in 2021 resulted in an increase in insurance contributions. Table 3.9 shows the annual premiums calculated for typical EU households (i.e. mixed-age household, which include a 34-year-old woman and a 36-year-old man with a maximum of three children aged four, two, and one) in 2019 and 2021.

*Table 3.9 Annual net premiums for typical EU households (based on National Life Tables 2019 and 2021)*

| <i>Country code</i> | <i>2019</i>              |                          |                             |                                  | <i>2021</i>              |                          |                             |                                  |
|---------------------|--------------------------|--------------------------|-----------------------------|----------------------------------|--------------------------|--------------------------|-----------------------------|----------------------------------|
|                     | <i>Type of household</i> |                          |                             |                                  | <i>Type of household</i> |                          |                             |                                  |
|                     | <i>Childless</i>         | <i>One-child<br/>(4)</i> | <i>Two-child<br/>(4, 2)</i> | <i>Three-child<br/>(4, 2, 1)</i> | <i>Childless</i>         | <i>One-child<br/>(4)</i> | <i>Two-child<br/>(4, 2)</i> | <i>Three-child<br/>(4, 2, 1)</i> |
| AT                  | 0.01222                  | 0.01505                  | 0.01610                     | 0.01673                          | 0.01312                  | 0.01620                  | 0.01736                     | 0.01806                          |
| BE                  | 0.01375                  | 0.01694                  | 0.01816                     | 0.01886                          | 0.01405                  | 0.01737                  | 0.01863                     | 0.01935                          |
| BG                  | 0.03008                  | 0.03706                  | 0.03980                     | 0.04134                          | 0.04281                  | 0.05277                  | 0.05659                     | 0.05881                          |
| CZ                  | 0.01709                  | 0.02086                  | 0.02239                     | 0.02328                          | 0.02076                  | 0.02545                  | 0.02729                     | 0.02837                          |
| DE                  | 0.01396                  | 0.01707                  | 0.01830                     | 0.01902                          | 0.01489                  | 0.01826                  | 0.01959                     | 0.02036                          |
| DK                  | 0.01234                  | 0.01503                  | 0.01613                     | 0.01679                          | 0.01182                  | 0.01444                  | 0.01549                     | 0.01608                          |
| EL                  | 0.01436                  | 0.01749                  | 0.01873                     | 0.01951                          | 0.01683                  | 0.02063                  | 0.02211                     | 0.02297                          |
| ES                  | 0.01124                  | 0.01355                  | 0.01453                     | 0.01512                          | 0.01141                  | 0.01386                  | 0.01484                     | 0.01543                          |
| FI                  | 0.01379                  | 0.01721                  | 0.01848                     | 0.01921                          | 0.01359                  | 0.01677                  | 0.01794                     | 0.01866                          |
| FR                  | 0.01546                  | 0.01906                  | 0.02046                     | 0.02127                          | 0.01565                  | 0.01928                  | 0.02068                     | 0.02149                          |
| HR                  | 0.01839                  | 0.02252                  | 0.02413                     | 0.02508                          | 0.02246                  | 0.02738                  | 0.02935                     | 0.03045                          |
| HU                  | 0.02508                  | 0.03012                  | 0.03229                     | 0.03361                          | 0.03095                  | 0.03742                  | 0.04005                     | 0.04164                          |
| IE                  | 0.01037                  | 0.01273                  | 0.01364                     | 0.01417                          | 0.01117                  | 0.01368                  | 0.01467                     | 0.01525                          |
| NL                  | 0.01133                  | 0.01387                  | 0.01485                     | 0.01544                          | 0.01221                  | 0.01496                  | 0.01605                     | 0.01669                          |
| PL                  | 0.02450                  | 0.03029                  | 0.03253                     | 0.03381                          | 0.02950                  | 0.03659                  | 0.03923                     | 0.04077                          |
| PT                  | 0.01539                  | 0.01875                  | 0.02008                     | 0.02089                          | 0.01591                  | 0.01944                  | 0.02086                     | 0.02170                          |
| RO                  | 0.03042                  | 0.03720                  | 0.03994                     | 0.04152                          | 0.03680                  | 0.04515                  | 0.04840                     | 0.05031                          |
| SE                  | 0.00959                  | 0.01189                  | 0.01272                     | 0.01320                          | 0.00977                  | 0.01208                  | 0.01296                     | 0.01349                          |
| SK                  | 0.02240                  | 0.02742                  | 0.02941                     | 0.03061                          | 0.02826                  | 0.03459                  | 0.03714                     | 0.03863                          |

*Source:* Authors' own elaboration.

Figures 3.5 and 3.6, based on data from Tables 3.3 and 3.9, show how annual premiums in 2021 and 2023 varied from those recorded in 2019, illustrating the impact of the pandemic.

The rise in premiums in 2021 was most strongly linked to the intensity of the COVID-19 pandemic's impact on each country, as evidenced by decreases in life expectancy and higher rates of excess mortality. In Nordic and Western European countries, it would have been possible to maintain BHLI premium levels or reduce them, despite the pandemic (as in Denmark and Finland). Conversely, many Central and Eastern European countries would have had to implement significant premium increases regardless of the size of households;

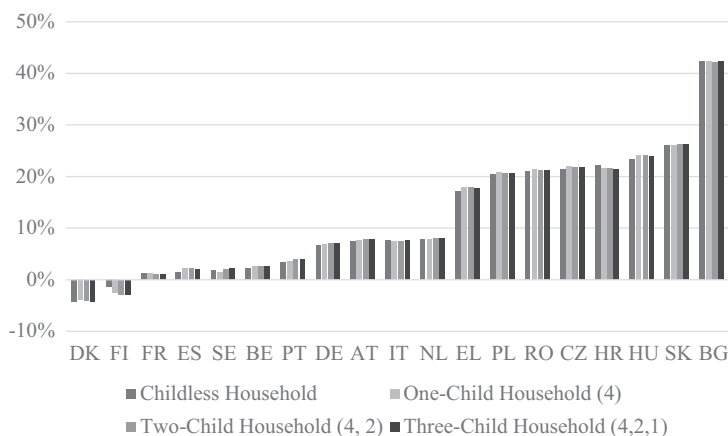


Figure 3.5 Relative differences in annual premiums in 2021 compared to 2019 in EU countries.

Source: Authors' own elaboration.

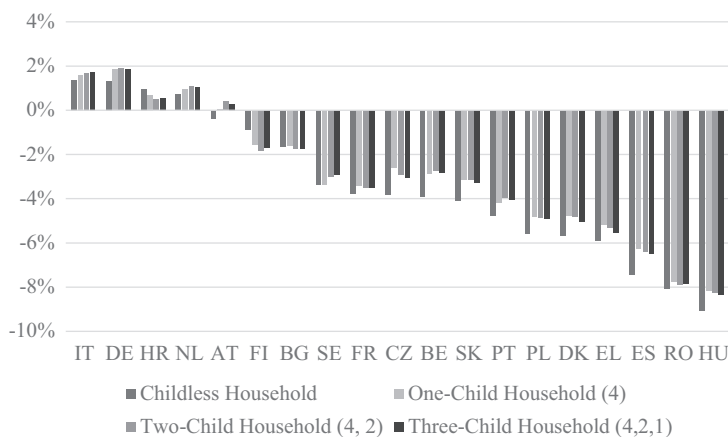


Figure 3.6 Relative differences in annual premiums in 2023 compared to 2019 in EU countries.

Source: Authors' own elaboration.

Bulgaria, in particular, would have experienced the largest increase in net premiums, estimated at 42%.

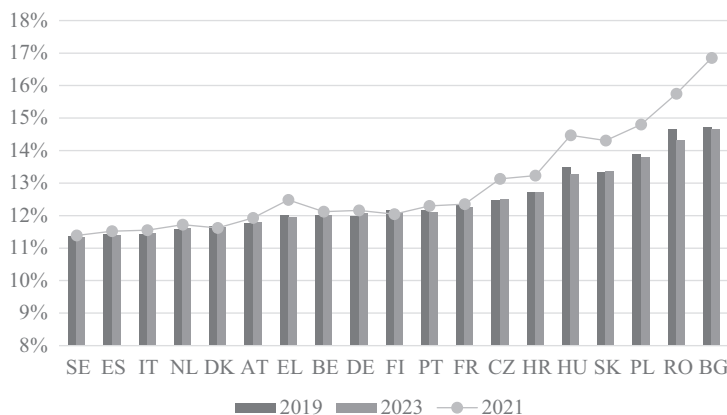
Based on Figures 3.5 and 3.6, it was possible to assess whether annual premiums might have returned to pre-COVID-19 levels. Prior to the pandemic, life expectancy in most EU countries was following an upward trend,

implying that the natural course would likely have led to a gradual decline in premiums rather than stagnation or increase. It should also be noted that even a ‘zero difference’ between 2023 and 2019 premiums does not necessarily indicate a true return to pre-pandemic trajectories.

Countries where premiums might have returned to or at least approached pre-pandemic levels by 2023 comprised Italy, Germany, Croatia, the Netherlands, and Austria. In these cases, insurance systems could have managed to offset the pandemic-induced increase in BHLI premiums, largely due to the stabilisation of life expectancy. For each household type, a typical range of changes (between –6% and 0%) was identified, and the relative differences observed in these countries could be considered exceptionally high.

In contrast, in the remaining countries, premiums appear to have remained above pre-pandemic levels, despite an observable downward trend. Hungary, Romania, and Spain stand out, where the relative differences in premiums would be considered unusually low, suggesting a delayed recovery of demographic indicators.

The COVID-19 pandemic not only influenced the amount of insurance contributions but also changed the relationship between periodic and single premium payments. The total value of annual premiums exceeds that of a single premium, which means that paying in instalments leads to higher overall costs for households. In 2019 and 2023, the relative differences between total annual premiums and single premiums ranged from 11% to 15% across the countries analysed; however, in 2021, this difference increased to 17%, potentially reflecting pandemic-related uncertainty.



*Figure 3.7* The relative difference between the total value of annual premiums and the single premium for the three-child household (4, 2, 1).

*Source:* Authors' own elaboration.

Figure 3.7 illustrates the relative difference between the sum of annual premiums paid over the policy term and the single premium for a representative household type, namely the three-child household (4, 2, 1) in ascending order for the year 2023; for other types of households the relationships were nearly identical.

The lowest relative differences (approximately 11–12%) were observed in Western and Northern European countries such as Sweden, Italy, Spain, the Netherlands, Denmark, Austria, and Greece, generally in those countries where life expectancy exceeds 80 years. In contrast, significantly higher differences were recorded in Central and Eastern European countries, particularly Bulgaria, Romania, Poland, Slovakia, and Hungary, where life expectancy is the lowest and does not exceed 80 years (cf. Table 3.3).

Data for 2021 show a noticeable increase in relative differences across all countries, particularly in Central and Eastern Europe. These differences result solely from the increase in mortality during that period, but in reality they could have been even greater. The increase in the gap between the total annual premium and the single premium in 2021 may reflect the higher level of uncertainty and increased risk perceived by insurers during the pandemic. In response to these factors, in gross premiums (i.e. administrative and acquisition costs and safety loading), institutions might have adjusted their pricing models by increasing the cost of annual premiums as a way to compensate for elevated financial risk.

By 2023, the differences decreased and returned to levels close to those observed in 2019 (analogous to Figure 3.6), suggesting a partial normalisation of the post-pandemic insurance market.

### **3.2 Advantages and Barriers**

One of the most promising starting points for developing a household assistance insurance product is the existing familiarity of European consumers with assistance-based insurance models. Unlike indemnity insurance, which primarily functions through the reimbursement of financial losses, assistance insurance centres on the provision of direct, immediate, service-based support. This model, centred on organised help during moments of disruption, is well established in many areas of life, and European consumers are not only accustomed to its logic but also increasingly expect such responsiveness from insurers. Widespread familiarity with assistance structures represents a significant conceptual advantage – it reduces barriers to product innovation and increases the likelihood that new offerings will be met with trust, comprehension, and engagement.

Across Europe, assistance insurance is widely regarded as a practical and efficient tool for managing everyday risks and emergencies. In the automotive sector, for instance, assistance services such as roadside towing, on-the-spot vehicle repairs, and the provision of a replacement car have become standard

components of car insurance policies. These benefits are highly valued for their immediacy and convenience, particularly in cross-border travel contexts where access to local support may be limited.

This logic of practical, real-time support also extends to travel insurance. Assistance benefits in this context typically include emergency medical consultations (increasingly via telemedicine), hotel accommodation in the event of delayed flights, co-ordination of medical repatriation, and legal support when required abroad. As international mobility increases and travellers face more complex logistical and legal challenges, the demand for comprehensive travel assistance is growing. In the area of home insurance, assistance insurance plays a crucial role in ensuring continuity and stability in household operations. When a pipe bursts, a lock jams, or the electricity supply fails, assistance policies enable policyholders to access a network of certified plumbers, locksmiths, and electricians promptly, without having to manage the logistics or bear the cost of the repair.

Based on the results of the 12th National Assistance Survey (ARC Rynek, 2024), assistance insurance products are clearly embedded in Polish consumer awareness. As many as 42% of Poles report holding an assistance policy, and nearly half of them have actually used the services offered. Roadside assistance, towing, and on-site vehicle repair remain by far the most frequently used services, with 79% of respondents reporting that they have used such services. A high satisfaction rate of 90% indicates that assistance is widely perceived as a practical and effective risk management tool. Consumers primarily associate assistance with help, support and a sense of security, attributes that strengthen their relationship with the insurer and increase the overall value of the insurance package. Notably, customers acquire these policies both directly from insurance companies and through agents, indicating broad market accessibility. The survey revealed that assistance products are no longer just niche offerings, but they have become an integral component of modern insurance portfolios. Their practical nature and positive public perception provide a strong foundation for the development of more complex support solutions, including innovative family-oriented assistance models aimed at addressing household needs in times of crisis.

The widespread familiarity with and the acceptance of insurance assistance across Europe provide a strong foundation for extending this model to new domains of household vulnerability. Traditionally associated with services such as roadside repairs or emergency travel assistance, assistance insurance has evolved into a trusted mechanism for delivering immediate, situation-specific support. Its popularity reflects not only consumer demand for practical risk management tools but also a growing preference for service-based rather than monetary solutions to everyday disruptions. This cultural embeddedness creates a favourable environment for adapting the assistance model to areas that remain underprotected, particularly the loss of unpaid domestic labour following the death or incapacitation of a household member.

Building on this familiarity, the proposed household support insurance adopts the established logic of assistance and applies it to the essential, yet often overlooked, sphere of non-market domestic work. Rather than introducing an entirely new product concept, it recontextualises an already familiar structure. Instead of providing technical repair services, this insurance would facilitate access to professionals who can support core household functions, such as meal preparation, cleaning, daily routines and, most importantly, the organisation of childcare and educational logistics. These services are essential to household continuity, particularly in families with young children or dependent members. By offering structured, in-kind support during a period of intense disruption, the product mirrors the operational features of existing assistance models while directly addressing a critical gap in currently offered insurance policies. Its design reflects practical household needs as well as behavioural patterns – leveraging the trust and clarity associated with assistance products to introduce a targeted, family-focused solution.

Another opportunity for implementing the proposed insurance product lies in the observable trend that Europeans are increasingly outsourcing household tasks to third parties. This transformation reflects a broader process of ‘servicisation’, a structural shift in both the economy and society in which an increasing share of value creation and employment derives from the services sector. In the context of domestic life, this means that households are progressively turning to external providers for routine tasks such as cleaning, caregiving, and other domestic support functions. Evidence from across Europe suggests that the outsourcing of household labour is no longer a marginal phenomenon. According to the OECD (2021), approximately 1.3% of workers in OECD countries are formally employed in household service roles, including domestic helpers, caregivers, and cleaners – a figure that underrepresents the true scale of the sector due to widespread informal employment. Many European countries have adopted fiscal incentives, for example tax deductions and/or social service vouchers to encourage the formalisation of domestic work, further normalising the practice and making it more accessible to a broad range of households.

In 2021, the domestic work sector in EU-27 countries was estimated to generate approximately €42.5 billion in value added, amounting to about 0.3% of EU gross domestic product (GDP) (DOMINA and Leone Moressa Foundation, 2023). Similarly, Zwysen (2024) confirmed that between 2001 and 2020, OECD countries experienced consistent growth in the outsourcing of household tasks, particularly among dual-income families and ageing populations. In Poland, although only around 15,900 domestic workers are formally registered, the market for professional home care and cleaning services continues to expand, supported by businesses advocating public subsidies or insurance products to increase access and affordability (Ziemacka, 2024). Globally, the private household services market is expected to grow at

a compound annual rate of 7–7.5%, driven by higher household incomes, ageing demographics and time constraints among working individuals. In urban areas of Central and Eastern Europe, including Poland, this growth is especially evident among self-employed providers delivering care and domestic support (Verified Market Reports, 2024).

Given these trends, it is reasonable to conclude that European households have developed a strong familiarity together with a growing reliance on service provision in the domestic sphere. This cultural readiness enhances the viability of an insurance model that provides structured assistance for domestic work following the loss or incapacitation of a household member. This alignment with established preferences significantly increases the likelihood of market acceptance and the successful implementation of the proposed insurance.

The growing demand for domestic services in Europe, fuelled by demographic shifts, changing family structures, and increased labour market participation of women, has been accompanied by a marked rise in labour supply, particularly due to the influx of Ukrainian refugees following the Russian invasion in 2022. This dual dynamic has significantly reshaped the landscape of care and household services in countries like Poland, where migration has become a key driver of the sector's expansion. In the Polish labour market, Ukrainian nationals (predominantly women) have filled gaps in low-skilled, labour-intensive occupations. According to the Polish Economic Institute, 65% of the refugees from Ukraine had entered the workforce by 2024, the highest employment rate among all OECD countries. This rapid integration has been facilitated by economic necessity and also by the relative openness of low-skilled sectors, including logistics, hospitality, and above all, domestic work. Migrants are increasingly employed in roles such as cleaners, informal caregivers, and household aides – occupations that are in growing demand across ageing European societies (more in: Filipek & Polkowska, 2020; Krivonos, 2025; Plomien & Schwartz, 2024).

This participation, however, often comes at the cost of professional downgrading. Despite relatively high levels of education among the refugee population, structural barriers, particularly the non-recognition of Ukrainian professional qualifications, have prevented many from accessing employment aligned with their skills. As emphasised by both the OECD and PIE, 46% of Ukrainian migrants in Poland work below their level of education, with this phenomenon disproportionately affecting women (Ziemacka, 2024). The long and bureaucratic process of diploma nostrification, combined with limited access to language courses and bridging programmes, has reinforced this occupational mismatch.

While the domestic services sector across Europe has expanded significantly, driven by both demand-side factors (i.e. longer working hours, changing gender roles and rising dual-income households) and supply-side factors (e.g. migrant labor), its growth is uneven across types of services. A notable

asymmetry persists: while tasks such as cleaning, cooking, and grocery shopping are readily outsourced by middle- and upper-income households, childcare remains the least likely task to be delegated to paid help. This divergence illustrates not only economic but also deep-rooted cultural and emotional boundaries around what constitutes appropriate outsourcing. According to van der Lippe, Frey, and Tsvetkova (2013), individuals are more likely to outsource household tasks they consider burdensome or unenjoyable. Cleaning and cooking are perceived as low-skill, repetitive chores, whereas childcare is associated with emotional intimacy, trust, and relational continuity, making it resistant to market-based delegation. This reluctance to outsource child-rearing duties exists even among highly educated and time-constrained individuals, suggesting that economic rationality alone does not shape household decisions.

Further insight was offered by du Toit and Heineken (2021), who interviewed South African households employing domestic workers. They found that employers are generally comfortable hiring cleaners through agencies or informal networks yet are unwilling to use such channels to find childcare providers instead of relying on family members and/or long-standing personal connections. This choice is rooted in concerns over child safety, moral responsibility, and the perception of good parenting, which remain deeply embedded in normative gender roles and familial expectations. The asymmetry is further explored, analysing motivations behind outsourcing across different types of households. The study also found that while single professionals and dual-income families are 'heavy users' of outsourced cleaning services, they remain sceptical of commodifying care work, particularly for young children. Families often seek a strong emotional fit and cultural alignment in nannies or caregivers, in contrast to the largely transactional logic governing cleaning or food preparation.

Cross-national evidence supports these findings. In European time-use data analysed by Hook (2006), women in dual-earner households continue to perform a disproportionate share of caregiving tasks even when other forms of domestic labour are outsourced. This suggests that care work is not only underpaid and undervalued in the market, but also symbolically associated with gender identity and maternal responsibility. Outsourcing may be perceived as a moral failure or a loss of parental control, particularly in cultures where intensive parenting is highly valued. Even when households outsource childcare, the arrangements tend to be informal, trust-based, and shaped by ethnic or kinship networks, as shown in studies of migrant care work in Western Europe (e.g. Horváth, 2008). In Poland and other Central and Eastern European countries, migrant women (especially from Ukraine) have increasingly filled gaps in domestic labour supply, but their employment in childcare remains limited compared to cleaning and caring for the elderly. This reflects both consumer-side caution and regulatory ambiguity surrounding childcare services.

To better understand household preferences regarding the delegation of domestic labour in the aftermath of a partner's death, a focus group study was conducted in winter 2024 in Poland. The study involved 140 participants to simulate real-life decision-making in the context of sudden widowhood or solo parenthood. The participants were asked to consider which household tasks they would be willing to delegate to third-party assistants selected and organised by an insurance provider. The findings revealed a clear stratification of tasks by their perceived delegability. The respondents showed the highest willingness to cede minor renovations and repairs (58%), followed by cleaning (31%), gardening (30%), and shopping (26%). Tasks such as childcare (22%) and meal preparation (21%) were considerably less likely to be entrusted to external service providers. This pattern remained broadly consistent across the age groups, although the 21–30 age cohort, a key demographic from the standpoint of long-term insurance product development, exhibited somewhat higher openness to outsourcing additional care responsibilities such as care for the elderly (21%), pet care (17%), and logistical childcare tasks like school pickups (17%).

These findings underscore a broader psychological and cultural distinction between 'technical' household tasks (such as repairs, cleaning, and shopping) and those that are 'relational' or 'emotional', including childcare, eldercare, and cooking. This distinction echoes patterns identified in earlier literature (e.g. Hook, 2006; Van der Lippe et al., 2013). Technical tasks are generally perceived as low-risk and emotionally neutral, whereas care work remains deeply embedded in notions of intimacy, trust, and moral responsibility. These symbolic meanings attached to different forms of domestic labour significantly complicate efforts to fully externalise such responsibilities through third-party service provision – even in circumstances of acute household disruption.

Importantly, the reluctance to delegate relational tasks such as childcare and eldercare raises practical design questions for insurance models based on assistance services rather than financial indemnities. In the current design of many assistance-based insurance policies, the insurer not only covers the cost of services but also organises and contracts providers directly. However, the focus group data suggests that such a model may be misaligned with beneficiary preferences when it comes to emotionally sensitive tasks. For example, the respondents indicated that while they may accept help with cleaning or gardening from a contracted third-party assistant, they would prefer childcare and care for the elderly to be provided by a trusted family member, even if that requires informal arrangements. This raises the possibility of designing hybrid insurance models where cash indemnities could be issued to allow families to compensate trusted caregivers within their own networks, including grandparents, adult siblings, and close friends. While this approach preserves user autonomy and aligns with cultural preferences, it introduces complexity and

potential risks of misuse or under-provision of care. Moreover, it contradicts the operational logic of many assistance-based insurance schemes, which are designed for efficiency and quality control through centrally managed services. Delegating care to the beneficiary's private network, while arguably more acceptable to the insured, may conflict with the insurer's capacity to ensure quality, continuity, and accountability. Thus, the findings highlight a core tension in the design of post-bereavement insurance services, namely the need to provide rapid and comprehensive support while recognising that not all domestic tasks are equally commodifiable. Emotional tasks and care work, even though time-intensive and economically valuable, resist standardisation and outsourcing. If insurance products are to be meaningful and effective in supporting households after a member's death, they must be sensitive not only to cost structures and labour supply but also to cultural notions of intimacy, autonomy, and moral duty embedded in everyday family life.

Given the varying levels of social acceptance regarding the outsourcing of specific domestic and caregiving tasks to external providers, it appears justified to consider a flexible model based on the individual preferences of policyholders. One potential solution could involve the introduction of a checklist, completed at the time of contract conclusion, in which the insured person would indicate which tasks they would be willing to delegate in the event of an insured loss. Such an approach would allow for the accommodation of cultural differences, individual psychological barriers, and varying degrees of trust in the institutions providing services on behalf of the insurer.

The authors contend that a key factor in enhancing public acceptance of innovative insurance instruments designed to support household functioning, such as BHLI, will be the transparency of service delivery processes and the quality of subcontracted providers. Should insurers engage reliable, well-reviewed contractors for domestic and caregiving services, and if the product functions on the market in a manner that builds trust and generates positive user feedback, a gradual rise in societal acceptance of such protection mechanisms may be anticipated. Over time, this could also contribute to a broader redefinition of social norms concerning the division of domestic responsibilities and the ways in which households adapt to the sudden reallocation of tasks following the death of a family member.

### **3.3 Application in Other Areas**

The ability to assess the value of non-market or unpaid labour performed for the benefit of households has broad applicability beyond the field of insurance, extending to areas such as social policy, insurance economics, and demographic analysis.

### **3.3.1 Social and Public Policy**

The ability to estimate the value of unpaid household work, including caring for children and the elderly, household chores, cooking and daily life management, holds significant importance for public institutions and also for the private sector. This is particularly relevant to the design of modern social policies (Edwards et al., 2020; Jones et al., 2023), the design of family and retirement benefits (Johnson, 2009), pension systems, and the development of innovative insurance products, including private long-term care (LTC) insurance (Jędrzychowska et al., 2025).

Incorporating the valuation of unpaid work enables the fairer design of benefit systems, whereby caregiving (especially long-term care) can be recognised as a socially meaningful activity, qualifying individuals for additional benefits or pension credits. As noted by Łyskawa and Bielawska (2024), LTC-inclusive benefit schemes can significantly reduce old-age poverty, particularly among women who dominate in long-term caregiving roles. Moreover, Łyskawa stressed that widowers often struggle more with household tasks after losing a partner – underlining the need for support in these domains. The designers of private LTC insurance draw on these same considerations, aiming to support households facing care responsibilities, whether informal or professional, for dependent family members. Thus, the authors' research on unpaid labour valuation provides a solid foundation for modelling such products, informing the determination of coverage amounts, in-kind benefits, payout thresholds, and premium calculation.

Moreover, valuing domestic work supports forecasting the social and economic impacts of demographic change, namely ageing populations, declining birth rates, and the rise in single-person households (Przybyła et al., 2020; Utz et al., 2004). In traditional care models, intergenerational and familial networks provided crucial support; however, with increasing migration, family dispersal, and evolving social structures (Groeger, 2024), these informal support systems are breaking down. The absence of intergenerational care transfers, especially in single-adult or childless households, creates a clear need for external caregiving services, often associated with real and sometimes unpredictable costs.

In this context, the valuation of unpaid work enables more accurate financial planning for later-life care needs and helps normalise the use of paid care services (Antonopoulos & Hirway, 2010; 'Unpaid Care', 2020). This is relevant for public budget forecasting as well as for personal insurance and investment decisions. Particularly vulnerable are older adults, especially women, living alone after the death of a spouse, who will increasingly depend on institutional and private support (Wang & Chonody, 2013). Research shows that older adults often feel shame and reluctance to ask for help with household or daily tasks, which is linked to a strong sense of independence and fear of losing autonomy (Sadio et al., 2025). Studies demonstrate that

seniors frequently avoid seeking assistance in order not to become a burden to family members or caregivers, which can lead to social isolation and a decline in quality of life. This phenomenon is reinforced by cultural norms emphasising the importance of self-reliance and dignity in later life (Horgan et al., 2024). The proposed insurance could help overcome this mental barrier, making it easier for older adults to accept necessary assistance without feelings of shame or loss of independence.

### **3.3.2 Demography and Family Policy**

A closely related field to the previously discussed area of science and economics, which stands to benefit from the authors' research, is social policy and demography. Particularly important is forecasting the demand for care services depending on the availability of unpaid domestic work (Coltrane, 2000), as well as analysing the impacts of existing social policies on the structure and productivity of households. Programmes such as Poland's 800+ child benefit (earlier 500+) or comprehensive institutional care systems in other European countries significantly influence family decisions regarding daily life organisation, the division of household duties and the return to work (Roman et al., 2025).

Modelling programmes that support young mothers' return to professional activity is crucial because caring for young children consumes the largest share of time devoted to domestic work. Enabling the mothers to return to the labour market often involves informal care arrangements, such as paying grandparents or other family members for childcare. Effective planning and evaluation of such programmes require precise valuation of the unpaid care work provided by informal caregivers, allowing for a realistic estimation of the actual costs involved. Examples of such support systems already exist in Europe (Franzoi et al., 2024; *Why Parenting Matters for Children in the 21st Century*, 2020), German family benefits combine financial support with flexible care options, while Scandinavian models offer a broad spectrum of care services tailored to families' needs (e.g. Sweden, Denmark). The authors' model incorporates changes in household types, enabling accurate cost estimations and assessment of policy impacts on family well-being.

An example of national legal regulations supporting families with young children in Poland is the Act of 4 February 2011 on the care of children up to three years of age (Journal of Laws 2023, item 204), which governs the functioning of nurseries, children's clubs, and daily caregivers. The purpose of this act is to facilitate parents' return to professional work by ensuring access to professional and institutional care for the youngest children. Additionally, significant support is provided by the Family Care Capital Act of November 17, 2021 (Journal of Laws 2023, item 883, which establishes a financial benefit enabling parents to cover costs related to childcare, including the employment of informal caregivers. These legal acts exemplify state

efforts aimed at improving the availability of care along with supporting the professional activation of young parents. This corresponds with the need for modelling the costs and effectiveness of programmes supporting the return to work discussed in this study.

It is also worth mentioning that in personal finance, children are often viewed as an investment in the future; however, awareness of the economic value of unpaid care work performed for children remains limited. Financially literate households should be able to estimate the full costs of child-rearing, including parental unpaid labour, to incorporate these costs into budgeting and long-term financial planning. Accounting for the value of unpaid work also has ethical and social implications, highlighting the significance of parenting and caregiving labour, which, although invisible in direct expenditure, generates measurable costs and social benefits (Dhungana, 2020). The literature stresses that the proper valuation of unpaid care work can improve family support policies and assist in designing effective social programmes.

### **3.3.3 *Economics and National Accounts***

In recent years, the concept of extended GDP measures has gained prominence by including not only traditional indicators of production and consumption but also the value of unpaid work performed within households. Domestic chores, childcare and caring for the elderly, and other forms of informal work play a crucial role in societal functioning, yet they remain excluded from standard macroeconomic accounts. A novel approach involves the application of Markov chains and actuarial accounting in modelling and estimating the value of this unpaid work. These methods allow for capturing demographic dynamics, diversity in household types, and transitions between formal and informal activities, enabling more precise and long-term economic forecasts. Consequently, satellite accounts that augment national accounts with unpaid work components become more realistic and useful in analysing social and economic policies.

Integrating unpaid labour into satellite accounts facilitates an accurate valuation of the real productivity of households (Barro et al., 2025; Definition and Measurement of Work and Unpaid Care Work, 2019). Traditional economic statistics fail to reflect the contribution of domestic work to the overall economy, leading to an underestimation of actual welfare and production levels. This valuation is important not only from a macroeconomic perspective but also in social policy, as it enhances understanding of how changes in household structures, such as an increase in single-person households or single-earner families, affect total production and demand for care services.

Another critical area benefiting from including unpaid labour in economic analyses is the study of inequalities and the distribution of domestic duties. A persistent phenomenon is the unequal division of household chores between genders, contributing to the gender gap in working hours, income, and

pension security. Women, who frequently undertake the majority of caregiving and domestic tasks, face reduced opportunities for career development and capital accumulation for retirement (Vageeshan & Kamalakar, 2025). Valuing this labour highlights its economic significance, potentially shaping fairer family and social policies (Kitts, 2006). Consideration and accurate calculation of women's contributions to family functioning are also important for the design of potential compensation programmes aimed at offsetting career breaks, part-time work, and similar issues due to their involvement in household labour and childcare, and often the care for elderly parents (Caregiving and Caregivers, 2023; Raley & Bianchi, 2006).

In the context of increasing migration and social tensions regarding refugees and migrants in European Union countries, understanding and valuing the actual contribution of work performed by these groups is critical. Informal labour, particularly in caregiving and domestic services, is often provided by migrants, who thereby support the functioning of host country economies (Brell et al., 2020; Naseh et al., 2024). The precise assessment of their economic input can contribute to a greater social acceptance of refugee and migrant labour, which is vital for macroeconomic stability and security amid demographic and social challenges.

### **3.3.4      *Law and Jurisprudence***

Contemporary challenges related to the valuation of household work and personal losses present significant opportunities to support judicial systems. As indicated in the first chapter, court compensations in cases of personal injury, including accidents where the injured party performed domestic work, especially women (Compensation for Damages Suffered by Women Performing Unpaid Domestic Works, 2023; Tinari, 2004), constitute an area where the proposed methodology can find a practical application. The accurate estimation of the value of unpaid work enables the more precise calculation of compensations and stresses the necessity of accounting for the loss of the deceased person's contribution to household functioning (Ragnarsdóttir et al., 2023). Similarly, this approach can be applied to estimating losses in civil proceedings concerning medical negligence or malpractice (e.g. Sukumar, 2023). Such cases often involve permanent impairment or loss of the ability to perform daily household duties, which have tangible financial and social consequences for the entire family.

In divorce proceedings, specific circumstances should be reflected in judicial decisions. In many families, women often forego or reduce their professional careers in favour of domestic work and child-rearing, while men continue their professional development (Ragnarsdóttir et al., 2023). Therefore, to ensure a fair division of contributions to family functioning, the courts should consider the division of assets and pension entitlements as well as the payment of capital compensating the value of domestic work performed by

the woman relative to the husband's involvement. Such a practice would help to provide equal opportunities and genuinely recognise unpaid work that has so far remained invisible in traditional property calculations.

The authors believe that the results of the research and the developed methodology for valuing household work and personal losses can significantly support the actions of the judiciary. The precise and comprehensive valuation of unpaid work will enable fairer and more adequate resolutions in legal proceedings, contributing to better recognition of the actual contributions of injured parties to household and family functioning. Hence, this study can become a valuable tool for legal practitioners, assisting in formulating judgements and compensations that reflect the actual losses and needs of the parties involved.

### **3.4 Recommendations for Further Study and Research**

The concept of the BHLI contract, described in Section 1.4, is a proposal for financial protection for households during periods of disruption following the death of a spouse. The actuarial model of this contract presented in Chapter 2 provides a starting point for further analysis, where both theoretical and practical issues remain open. In this section, the authors point out possible directions for further research, including the development of mathematical modelling methods, a deeper understanding of actuarial aspects, and the socioeconomic benefits associated with the potential implementation of the BHLI contract in insurance practice.

#### **3.4.1 *Structure of the BHLI Contract: Joint vs. Separate Contracts for the Parents?***

It is worth considering in-depth analyses of joint annuity insurance contracts for spouses (commonly referred to as spousal annuities or joint-life annuities), which offer an alternative to two separate individual contracts. This approach can offer economic as well as operational advantages within the framework of BHLI contracts.

From the insurer's perspective, a single joint contract can reduce administrative complexity and costs by eliminating the need to issue and manage two policies. This model can also enable offering more competitive insurance terms, including lower premiums for couples opting for joint coverage.

For policyholders, a joint contract simplifies the policy administration by minimising the need to monitor two parallel insurance agreements and facilitating household budget planning. Moreover, in certain jurisdictions (such as the United States, Canada, and some EU member states), joint annuities may benefit from preferential tax treatment, including reduced taxation of annuity income or survivor benefits.

Psychological and behavioural aspects also warrant consideration. A joint insurance policy may more accurately reflect the financial interdependence of household members, particularly in the context of shared expenditures and joint financial obligations. The decision to purchase a joint BHLI policy may demonstrate a mutual understanding of the risk of losing a partner and emphasise the equal contribution of spouses to the family. This can positively impact relationship cohesion and a sense of shared financial responsibility.

Accordingly, further research is recommended on the applicability and effectiveness of joint BHLI coverage (and more broadly, joint disability insurance arrangements), taking into account both economic efficiency and socio-psychological factors. Comparative cross-country analyses and empirical studies, particularly those exploring consumer preferences and attitudes toward joint insurance in the context of household budgeting, may provide valuable insights for future contract development and public policy.

### 3.4.2 *Refining the Actuarial Model: Relaxing the Independence Assumption and Incorporating the Health Status of Family Members*

In this monograph, the BHLI insurance contract was analysed under the assumption that the lifetimes of individual family members, including spouses and their children, are treated as independent random variables. In real-life settings, the lifetimes and health statuses of individuals living in the same household are often interrelated. A growing body of empirical research highlights significant correlations, particularly between the health of spouses. For example, not only the death of a partner but also the presence of a chronic illness can adversely affect the physical and mental well-being of the caregiving spouse. These dependencies may arise from shared environmental conditions, emotional and psychological ties, common economic factors, and lifestyle behaviour. Incorporating such dependencies in future models could enhance the realism and precision of determining net premiums.

Incorporating interdependence between spouses' lifetimes could be effectively achieved using advanced probabilistic techniques, such as copula functions; cf. (Dębicka et al., 2020, 2023; Genest & MacKay, 1986; Nelsen, 2006). These models allow for flexible modelling of dependencies between survival times while preserving the ability to specify marginal distributions independently. This approach is particularly relevant for analysing joint survival among couples and can significantly improve the precision of risk modelling in contracts addressed to households.

A natural extension of the model involves capturing intergenerational dependencies by examining the relation between parental and child lifetimes. This approach would require the construction of a more complex multi-state model, whose probabilistic structure would be modelled by the composition of interdependent Markov chains, referred to as an *Aggregate* or *Composite*

*Markov Model* (see Ahmad et al., 2023). Such a framework would enable the simultaneous modelling of dependent health trajectories of multiple family members, capturing a dynamic representation of changes in health status and mortality risk in the context of their interconnectedness. This method, while complicated and requiring longitudinal data, offers the most accurate representation of risk in family insurance contracts. One alternative approach could be to use for this purpose *Shared Frailty Models*, a specific type of statistical model commonly employed in survival analysis. These models are particularly useful when the data exhibit a group structure, such as individuals within a household. In this context, ‘frailty’ refers to a hidden random variable that indicates an individual’s sensitivity to an event, such as death or disease. Shared frailty models take into account heterogeneity that cannot be observed directly, allowing for the modelling of hidden risk factors, including lifestyle, social status and genetics, which influence sharing mortality but remain unobserved (e.g. Hougaard, 2000; Wienke, 2010).

When examining inter-spouse and intergenerational dependencies, a valuable extension would be to investigate the connection between the future lifespans of household members and the occurrence of a serious long-term illness or disability affecting any one of them.

Considering these interdependencies would enable a more accurate estimate of premiums and insurance reserves, as well as allow for better customisation of the contract to meet the actual needs of the household. Moreover, developing models that account for health and demographic interdependencies among family members could represent a significant advancement in designing new analytical tools for the actuarial evaluation of family contracts. This approach not only fosters a more comprehensive understanding of the risk structure in insurance contracts addressed to couples and families but may also improve the stability and adequacy of pension and disability insurance systems in the face of changing demographics and family life patterns.

### **3.4.3 *Contract Flexibility: Incorporating Newborn Children during the Contract Term***

In the development of actuarial models that consider changes in family structure, further research is needed on dynamic insurance contract designs that accommodate the birth of another child during the term of the policy. One potential solution is to implement a mechanism similar to existing contracts that allow parents to extend coverage by purchasing an additional option. This option would enable them to update the newborn child in the composition of a household while the insurance contract is still active. Such a mechanism would allow for the ‘reactivation’ or adjustment of coverage levels in response to evolving family needs, similar to the *Accelerated Death Benefit* option found in life insurance policies.

Another possibility involves modifying the BHLI contract proposed in Section 2.4 to apply to households with an *open child population* instead of a *closed child population*. From a mathematical modelling perspective, such a solution would require adding additional transitions between the states of the  $\{H(t)\}$  process (i.e.  $(C, B)$  and  $(0, B)$ ) as well as changing the duration of the process in all states.

Implementing these changes would impact the probabilistic structure of the entire model and result in the pricing of premiums; therefore, a significant direction for further research could involve integrating fertility and procreation models with classical actuarial models. For example, family life tables or cohort-based fertility analyses could be incorporated to more accurately capture the dynamics of household life cycle and its impact on risk assessment (cf. Frątczak et al., 2011; Frątczak & Kozłowski, 2005).

While life tables remain a fundamental tool in demographic and actuarial modelling, they may prove insufficient when accounting for the wide range of external factors that influence fertility decisions and the timing of childbirth. In particular, it is important to consider the following categories of determinants: socio-demographic factors (e.g. education, employment), economic factors (e.g. income and housing), cultural and social factors (including pro-family norms, religiosity, pro-family policies), and biological factors (e.g. maternal age and reproductive health).

An interdisciplinary approach that combines tools from actuarial science, demographics, economics, and social sciences is essential for understanding the dynamics of procreation and its impact on insurance risk. Further research in this area may support the development of more flexible insurance products that better reflect the evolving needs of modern families and align more closely with current demographic realities.

#### **3.4.4 Incorporation of Stochastic Interest Rate Models in BHLI Valuation**

The use of stochastic interest rate models in the valuation of life insurance contracts would be a valuable extension to the deterministic framework applied in this monograph. Such models allow for a more nuanced representation of the uncertainty and volatility in financial markets by capturing the random evolution of interest rates over time (Brigo & Mercurio, 2006). This consideration becomes particularly relevant in the context of long-term contracts, for instance BHLI, where even modest fluctuations in interest rates can influence the present value of future benefits and premiums (Bowers et al., 1986).

Various approaches to modelling the interest rate process have been developed, each offering specific advantages depending on the application context (e.g. Brigo & Mercurio, 2006; Cox et al., 1985; Dębicka et al., 2025; Dębicka et al., 2022; Gibson et al., 2010; Mehalla, 2021; Vasicek, 1977). The application of a stochastic interest rate framework enables more robust risk

management, enhanced capital requirement assessment and better consistency with market-based valuation principles, such as those required by Solvency II regulations.

Therefore, incorporating stochastic interest rate dynamics into the valuation of BHLI contracts could enhance model realism, support scenario-based projections and facilitate sensitivity analyses. It would also allow for the actuarial evaluation of contract variants with embedded financial options, including guaranteed annuity options, premium indexation, surrender values, and profit participation.

### **3.4.5     *An Option-Theoretic Perspective on Annually Renewable BHLI***

Another potential direction for future research could be to analyse the possibility of purchasing BHLI as an annually renewable short-term policy or as a supplementary contract functioning as an option to marital life insurance. This short-term structure could serve as a flexible complement to traditional long-term products, potentially addressing specific transitional needs within the household life cycle.

Such an insurance model would be based on a one-year contract, with the option of annual renewal, also automatically. The premium would be recalculated each year based on the ages of the insured spouses, meaning a systematic increase as they grow older. The benefit would be paid only in the event of the death of one of the policyholders during the contract term. The structure of such a contract is similar to third-party liability insurance – it has no surrender value and is designed solely to provide pure risk protection.

The potential advantages of this form of protection primarily include relatively low premiums during the initial period, which is especially important for younger and/or healthier individuals. These conclusions are particularly relevant for households without young children (referred to as *Scenario 0*). For families with children, the situation is more dynamic: on the one hand, the annual probability of parental death increases, but on the other, as children grow older, the need for additional financial resources to support the household after the death of a spouse decreases.

One additional advantage of short-term coverage is its flexibility. This type of insurance allows for annual adjustments to the insured amount and also to the scope of coverage, accommodating the changing needs of a family, such as the addition of newborns. Furthermore, the absence of a long-term commitment can be beneficial, considering the evolving life and financial circumstances of policyholders. These contracts typically feature simplified contracting procedures and require fewer medical examinations.

Future research could focus on developing decision-making models that help to select optimal insurance coverage structures at various stages of the household life cycle, taking into account the variability of protection needs, demographic risk and budgetary constraints.

### 3.4.6 Socioeconomic and Demographic Influences on BHLI Premiums

Section 3.1 presents a comparative analysis of premium rates in selected EU countries, providing a valuable foundation for understanding cross-country differences. Building on these results, a more in-depth examination of premiums under a BHLI contract for specific countries (considering economic factors such as interest rates and socio-demographic characteristics like family structure, the ages of children and parents, and culturally embedded division of household tasks) could lead to better aligning insurance products with the diverse needs of households across different populations.

From an actuarial and mathematical perspective, a precise valuation enables us to assess how sensitive the premium is to changes in various parameters, for example the effect of a lower interest rate on the contract value. This process helps to minimise the risk of underestimating costs and allows for a more precise definition of risk zones for the insurance company. In turn, from the perspective of insurance product design, such analysis enables the construction of diverse BHLI contract variants (e.g. those with partial or full, temporary or lifetime payments) tailored to meet the individual needs of families.

Moreover, these analyses can serve as a valuable foundation for recommendations aimed at developing social and commercial insurance designed specifically for families with children and provide a starting point for further research on risk modelling that takes household structure into account.

To sum up, the BHLI contract presented in this monograph is a proposal designed to protect families from the effects of household disruption during the first three years following the death of a spouse/parent. Although this type of contract is not currently available on the market, it offers potentially significant support during this critical period of the family's adjustment to new living conditions. The directions of future research discussed in this section could serve as the next step for advancements in actuarial science and the design of innovative products in the life insurance market and may also inspire social policies aimed at supporting families facing crisis situations.

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# Conclusions

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Traditional concept of life insurance focuses on replacing lost market income after the death of a wage earner but neglects the economic value of unpaid domestic work. This book introduces Bridging Household Life Insurance as a complementary model that offers temporary support to households facing the loss of such labour by sustaining care, enabling the surviving spouse's participation in the labour market, and preventing the socio-economic disruption of the household. It was shown that unpaid domestic labour has measurable economic value, the protection of which through appropriate insurance mechanisms is theoretically substantiated, practically implementable, and ethically justified.

The main findings of this monograph pertain to four key areas: the socio-economic functioning and security of households, assistance-based support models, actuarial modelling, and practical applications. In particular, this book offers several key insights that enhance and deepen the current understanding of *household socio-economic functioning and its security*.

First, the analysis identifies a significant mismatch between traditional assumptions underpinning social protection systems and the actual structure of contemporary households. Benefits structured around the nuclear family and formal relationships frequently fail to account for a wide range of informal household forms, despite these fulfilling equivalent caregiving and organisational roles. A key contribution of this work is the proposal to adopt a broad, functional definition of the household as a foundation for designing inclusive insurance mechanisms.

Second, the authors highlight the impact of demographic shifts and changes across the household life cycle on the demand for care and domestic labour. Recognising evolving household structures and internal dependencies as risk factors allows for a more precise assessment of the losses arising from the disruption of unpaid labour.

Third, a critical insight is the ongoing underrepresentation of unpaid domestic work, despite its measurability and vital role, within insurance frameworks and official economic statistics. However, this form of labour fulfils the classical criteria for insurable risk, thereby supporting the case for developing targeted protective instruments. This book further introduces an innovative approach

that integrates demographic characteristics such as age, parental status, and number of children into the valuation of domestic labour, providing a realistic and equitable framework for the design of compensation mechanisms.

Fourth, the research draws attention to the persistent and systemic gender inequality in the division of domestic labour. Women not only undertake the majority of unpaid care work but also perform it with greater intensity, juggling multiple responsibilities and engaging in repetitive tasks. These factors result in long-term socio-economic consequences, including reduced labour market participation and lower pension entitlements. This highlights the urgent need for structural reforms aimed at reducing gender-based disparities in unpaid care responsibilities.

A notable finding of this study is the potential for *assistance-driven support models* (already well established within consumer sectors such as automotive and travel) to be effectively applied to the domain of unpaid domestic labour. These models offer practical, service-oriented solutions that correspond to users' preferences for immediacy and efficiency in times of household crisis. Market data, particularly from Poland, confirm the widespread acceptance and high user satisfaction regarding such services, indicating strong societal readiness to adopt comparable forms of support for domestic responsibilities.

The analysis also identifies a broader trend of outsourcing household tasks, driven by the ongoing servicisation of the economy. In many countries, the formalisation and professionalisation of domestic services, partly fuelled by labour migration, have created the infrastructure necessary for implementing assistance-based insurance products. This development fosters innovation within insurance markets as well as reflects deeper cultural and economic shifts in the organisation of domestic life.

The findings also reveal notable limitations of the assistance model, particularly regarding emotional and caregiving tasks such as childcare and eldercare. These responsibilities are often seen as requiring personal trust and strong relationships. This stresses the need for flexible insurance solutions that integrate service provision with the option of cash benefits, thereby enabling beneficiaries to choose the form and provider of support that best suit their needs.

Ultimately, to gain public trust in these new insurance contracts, it is essential to ensure high-quality service delivery and transparency regarding benefit conditions. In the long term, such models may enhance household security and also contribute to broader cultural shifts in how unpaid domestic labour is recognised and organised.

From a methodological point of view, this work delivers a *coherent and flexible actuarial framework*. One of the contributions of this study is constructing a multi-state model specifically designed to capture the complexity of the Bridging Household Life Insurance contract. The model extends traditional life insurance frameworks by incorporating both the future lifetimes of adult household members and the evolving structure of the household, in particular the number and age of dependent children throughout the coverage

period. This life-cycle perspective uniquely suits the model to assess risks associated with unpaid domestic labour.

The use of matrix notation for actuarial values provides a compact and factorised representation of the probabilistic and financial aspects of the model (including interest rates and benefits), and this approach facilitates the valuation of insurance contracts. While the monograph primarily focuses on the pricing of the contracts, the introduced matrices can also be applied in the calculation of gross premiums, reserves, and the overall profitability of the insurance portfolio.

Regarding the *applicational perspective*, an important outcome of this book is the empirical evaluation of the proposed insurance contract for national model households that reflects the variety of family models in different European countries. A representative household type for each country was defined based on publicly available Eurostat data, which captures key life-course transition points (in particular the average age at which individuals leave the parental home and the age at which they typically have children) for both women and men. This analysis highlights the heterogeneity of household structures within the EU and illustrates how this diversity affects the pricing of the BHLI contract. Moreover, it serves as a benchmark for assessing the affordability and adequacy of the proposed insurance contract under typical early-family conditions within the EU.

A comparative analysis of the premium amount for typical (in terms of the number and age of members) European households was also carried out. A mixed-age household was selected for analysis, consisting of a 34-year-old woman and a 36-year-old man. Premiums were calculated for both childless couples and couples with dependent children. In this case, the household was assumed to include children aged four, two, and one. The study showed how differences in future lifetimes of its members directly influence the valuation of household protection. The comparison of premium levels between EU countries provides indirect insight into national mortality and overall population health. These results are particularly relevant in light of recent systemic shocks, such as the COVID-19 pandemic, which have exposed and intensified existing social and health disparities. This is evident in the differences in premium levels calculated for the years 2019, 2021, and 2023.

The results significantly enhance academic discussions on unpaid household work, with implications for social policy, family policy, the legal framework, and jurisprudence. Additionally, the authors introduce a novel approach for modelling the probabilistic structure of insurance contracts.

The insights gained from analysing the value of unpaid work for households reach beyond the domain of insurance.

Firstly, within the context of *social policy* and demographics, recognising the value of domestic work (including childcare, eldercare, and routine household tasks) is essential for developing more equitable social benefit systems. Recognising these activities as socially and economically valuable leads to better-designed pension schemes and long-term care benefits. This is particularly significant in light of population ageing and the growing proportion

of individuals living alone. Furthermore, acknowledging the value of unpaid care enables more accurate predictions of the economic impacts of demographic change and encourages greater acceptance of paid care services. This approach also helps to overcome psychological and cultural barriers, particularly among older adults who may associate external assistance with a loss of personal autonomy.

Secondly, within the domain of family policy, recognising the value of unpaid labour enables more accurate modelling of the costs and outcomes of programmes designed to support families, particularly those aimed at supporting young mothers in re-entering the workforce. This approach allows for a more precise accounting of the actual costs of informal care, which is often provided by family members. As a result, it enhances the effectiveness and relevance of public policy interventions, additionally acknowledging that the economic value of caregiving has ethical and social importance as it draws attention to the significant contributions made by parents and caregivers – often overlooked in traditional financial analyses.

Thirdly, from the *economic and national accounting* perspective, incorporating the valuation of unpaid work into extended measures of gross domestic product (GDP) enables a more realistic assessment of actual household productivity and the socio-economic consequences of structural change. This approach takes into account demographic dynamics and the variety of household structures, thereby contributing to a more nuanced understanding of inequality, particularly with regard to the gendered division of domestic labour. The inclusion of unpaid work emphasises the economic significance of the contributions made by women and migrants to the functioning of the economy, with important implications for both equality and integration policies.

A further implication arises in the fields of law and jurisprudence, particularly concerning the valuation of unpaid work. This valuation can help the courts in making more accurate determinations of compensation and the division of assets, especially in cases involving personal injury, medical negligence, or divorce. Accounting for the value of lost contributions to household functioning enables fairer adjudication by more closely reflecting the actual losses and needs of the parties involved. In this context, the methodology developed in this study may serve as a valuable tool for the judiciary in recognising and compensating domestic labour that has long remained invisible in legal and financial assessments.

From the perspective of *modelling insurance contracts*, this monograph proposes a new approach. It becomes evident that the survival and death probabilities related solely to the future lifetime of the insured adults are inadequate for households with dependent children when describing the transition matrix for the modified multi-state model of Bridging Household Life Insurance, since the probabilistic structure is influenced by two key factors: the future lifetime of the insured adults and the external factor that accounts for the future lifetime of the children and the number of children in the household.

These external elements define the stage of the household development cycle. Consequently, the probabilistic structure of the model exhibits a dual stochastic nature, described by a Markov chain with a random contextual component, which is referred to in this book as the Contextual Markov Chain.

The authors recognise that the research presented in this book is subject to certain *limitations*, which primarily arise from the limited availability of comprehensive data and the varying methodologies employed to define and conceptualise households across various contexts.

A major limitation in international analyses is the pronounced scarcity of *timely and comprehensive statistical data*. Pan-European comparative sources, including Eurostat and OECD statistics, are frequently published several years after the reference period, thereby significantly impeding the analysis of current social and economic trends. For example, time-use surveys are conducted infrequently in many countries where they may occur as rarely as once per decade. Such data collection frequency is insufficient to capture the dynamic changes in daily activity patterns resulting from accelerated automation in household life, the growing importance of care services in the market, and evolving family life models.

An additional challenge arises from *methodological differences* across countries, particularly regarding how *households* are defined and the classification of activities deemed as *unpaid work*. Depending on the definitions used and the scope of observation, data may not be comparable, which complicates the accurate estimation of the value of domestic work on an international scale. These issues are further intensified by the underrepresentation of certain social groups in surveys, such as older adults, migrants and unregistered caregivers, whose contributions to household functioning are often overlooked in official statistics. Thus, the full picture of informal economic activity remains incomplete, impacting not only the quality of academic research but also the effectiveness of social and economic policy design.

Another significant issue is that statistical frameworks often rely on the *model of the traditional nuclear family*, which does not reflect the diverse range of contemporary family and caregiving arrangements. The experiences and needs of individuals in informal partnerships, blended families, and single caregivers are often overlooked in available data sets. Moreover, even when time spent on domestic work is recorded, there is a *lack of in-depth qualitative data* that would enable an understanding of the physical and emotional burden associated with daily caregiving and household management. Simply logging hours does not capture the intensity of the work, the psychological stress involved, or its impact on quality of life.

The issue of *digital inequality* is also of considerable relevance. As social research becomes increasingly conducted online, certain groups (such as older adults, individuals with low incomes, and those living in areas with limited Internet access) are often excluded. Paradoxically, these groups frequently engage in substantial unpaid domestic and caregiving work; however, their

absence from empirical data leads to significant blind spots in both public policy and economic analysis, leaving many aspects of everyday household functioning unrepresented and unaddressed.

One of the modelling assumptions applied in this work was the use of a *closed child population*. Although this approach limits the inclusion of children born after the insurance contract begins, it significantly enhances model transparency and simplifies the actuarial valuation process by eliminating the need to forecast future births, which would introduce considerable uncertainty. Additionally, this method aligns with traditional life insurance modelling practices, which usually assume a closed insured group, and reinforces consistency with established actuarial methodologies and market conventions. Given the innovative nature of the proposed contract, this choice of modelling is both justified and practical, yet the possibility of extending the framework to include an open child population is acknowledged and discussed as a potential direction for future research.

Another key modelling assumption adopted in this work concerns the *independence of future lifetimes among household members*. This simplification facilitates modelling and uses widely available individual mortality data. While this assumption is practical and often sufficiently accurate, especially in annuity contexts, it overlooks known positive correlations, particularly among spouses or across generations. Incorporating dependency structures could improve valuation precision and risk management, especially under regulatory frameworks, for example Solvency II. However, empirical limitations (such as scarce and inconsistent data on joint survival and the lack of standardised family-based life tables) remain a major challenge, while the dynamic nature of household composition also complicates the definition of stable insurable units. Despite these obstacles, modelling intergenerational dependencies remains a promising direction for future research.

Other possible extensions of the research presented in this monograph, along with alternative modelling approaches, are discussed in detail in the last chapter. It highlights the potential advantages and limitations of implementing the proposed insurance contract within the insurance market and related social policy and financial protection areas.

In addition to its conceptual and socio-economic contributions, this book is a valuable resource for academic and professional education. By integrating household-level analysis with advanced actuarial modelling (including multi-state models and the treatment of unpaid labour within satellite accounting and life-cycle frameworks), its content can be selectively applied to insurance mathematics, risk theory, personal finance, and social protection systems courses. This renders the textbook relevant not only to researchers and doctoral students interested in innovative approaches to life insurance but also to trainee actuaries and professionals preparing for certification examinations.

The authors contend that the interdisciplinary nature of this work enhances its relevance across multiple fields and inspires future advancements in household risk protection theory and practice. The monograph broadens the conceptual boundaries of life insurance and encourages reconsidering how social and economic value is recognised within contemporary risk-sharing mechanisms.

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